Compiler Design

Lecture 19:
Instruction Selection via Tree-pattern matching

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(EaC-11.3)

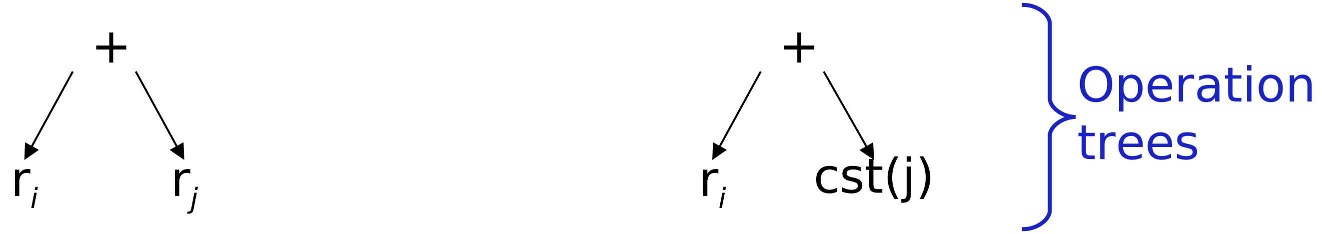
The Concept

Many compilers use tree-structured IRs

- Abstract syntax trees generated in the parser
- Trees or DAGs for expressions

These systems might well use trees to represent target ISA

Consider the add operators

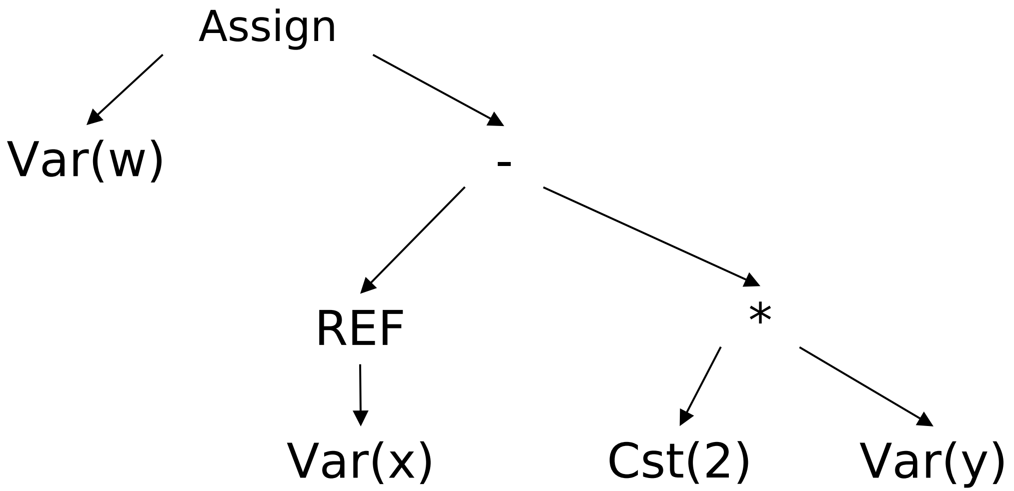


`add` $r_i, r_j \Rightarrow r_k$ `addI` $r_i, j \Rightarrow r_k$

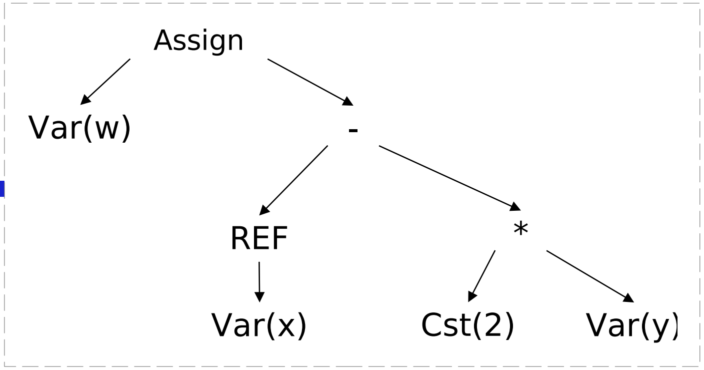
What if we could match these “pattern trees” against IR tree?

The Concept

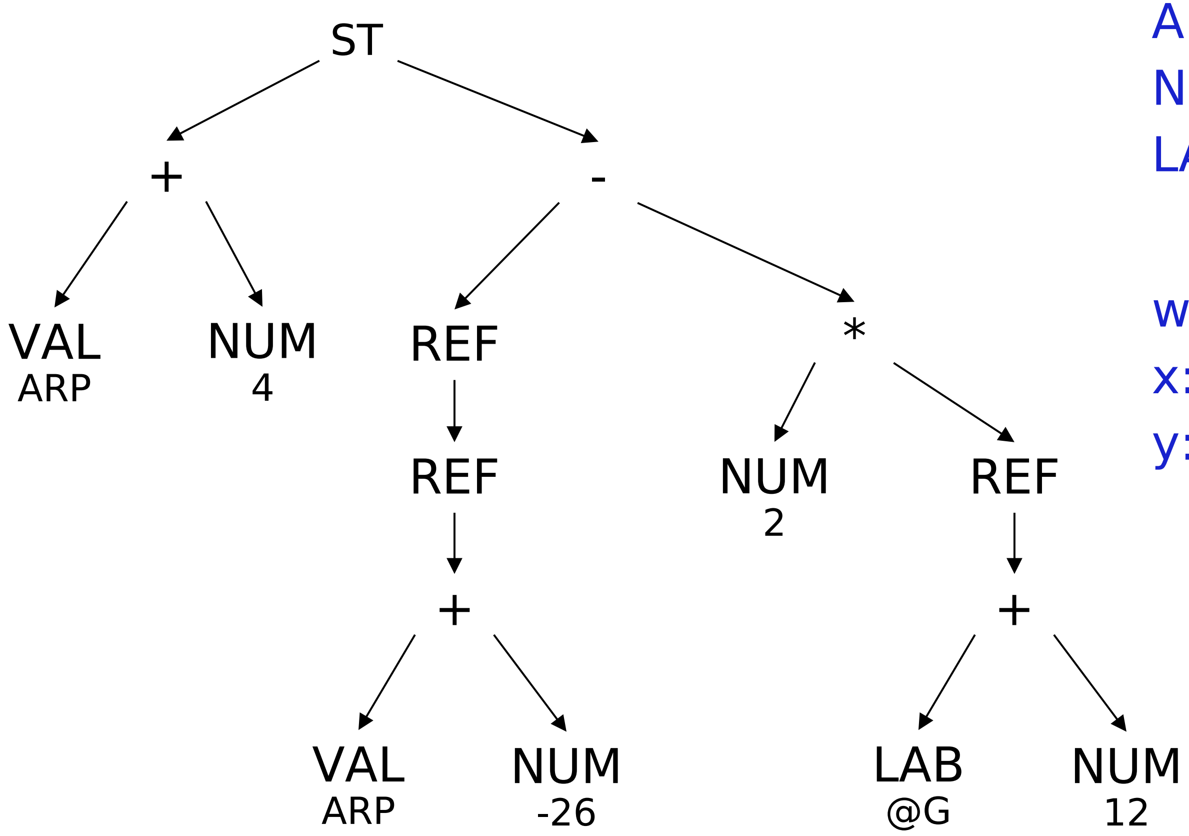
AST for w ← (*x) - 2 * y



The Concept



Low-level AST for w ← (*x) - 2 * y

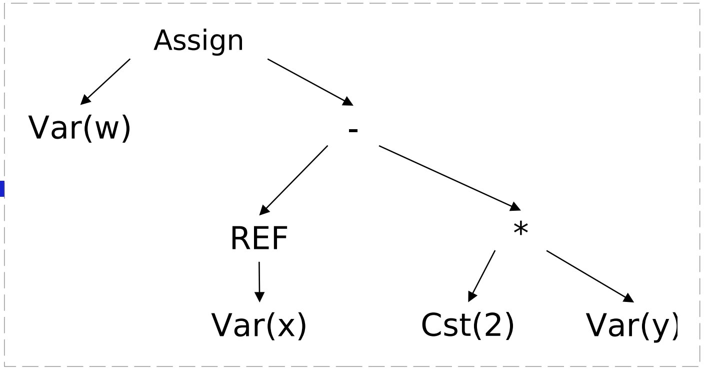


ARP: $fp
NUM: constant
LAB: ASM label

w: at ARP+4
x: at ARP-26
y: at @G+12

ARP = Activation Record Pointer = **frame pointer**

The Concept



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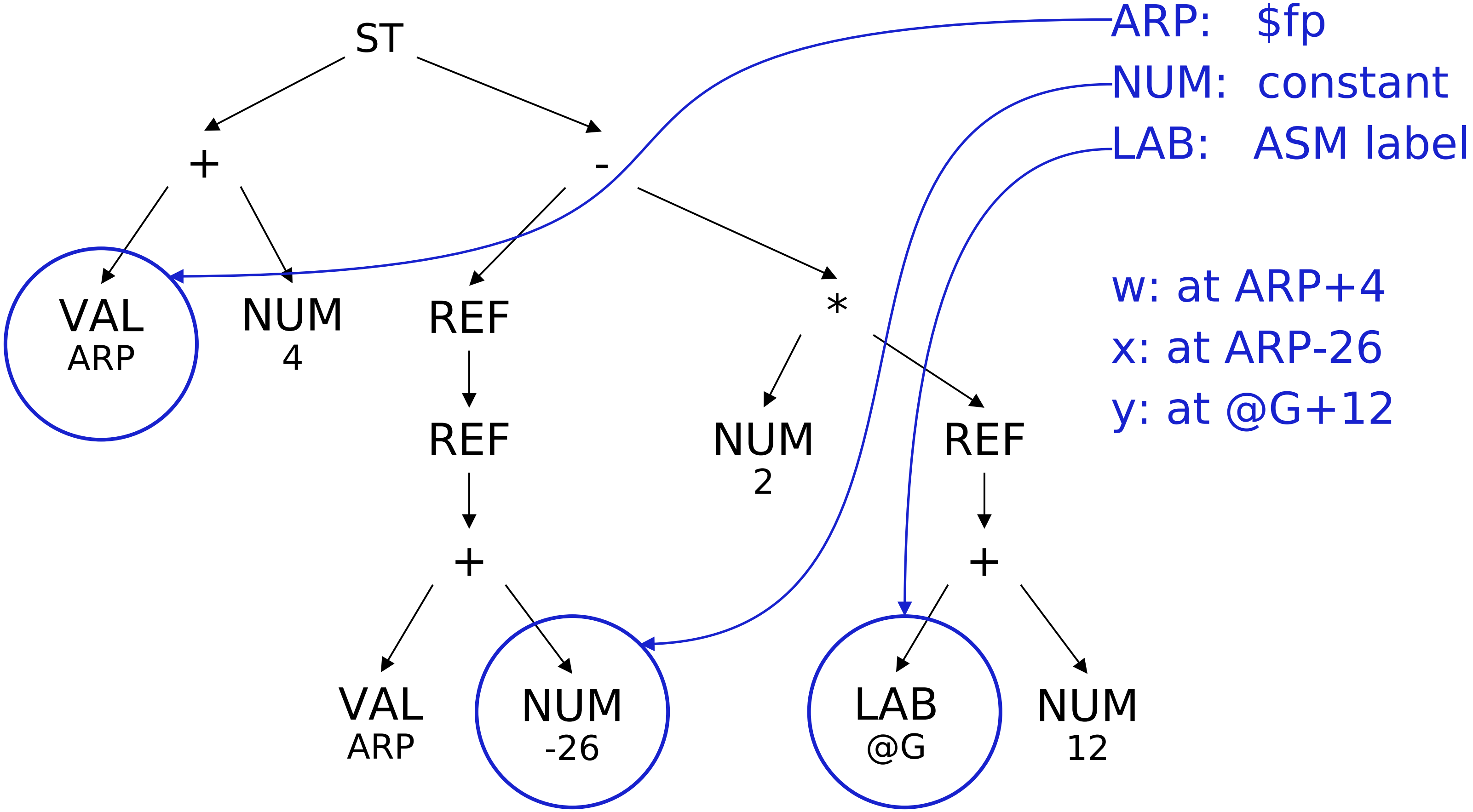


w: at ARP+4
x: at ARP-26
y: at @G+12

ARP = Activation Record Pointer = **frame pointer**

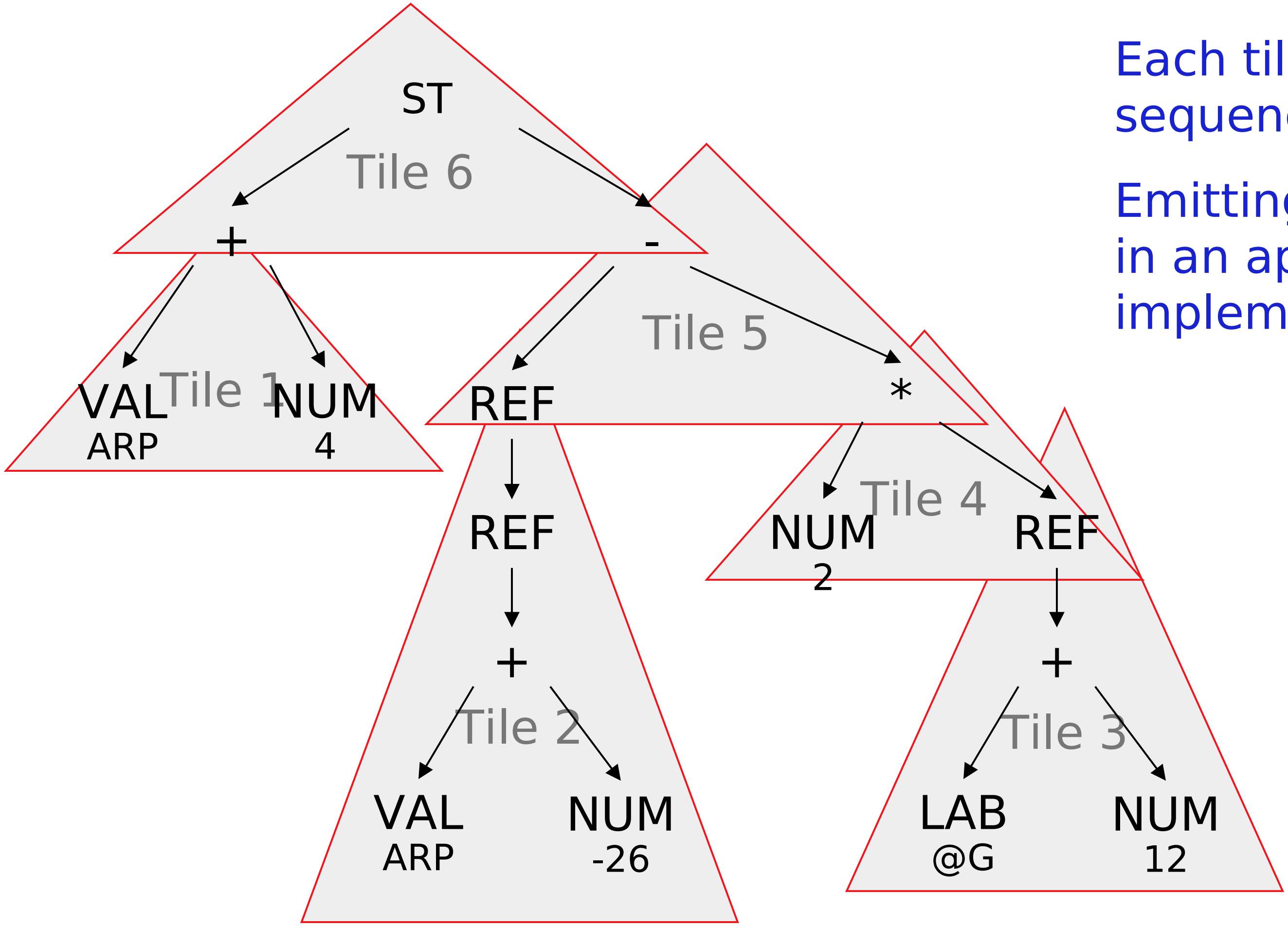
Tree-pattern matching

Goal is to "tile" AST with operation trees

- A tiling is collection of <*ast,op* > pairs
  - *ast* is a node in the low-level AST
  - *op* is an operation tree
  - <*ast*, *op* > means that *op* could implement the subtree at *ast*

- A tiling 'implements" an AST if it covers every node in the AST and the overlap between any two trees is limited to a single node
  - <*ast, op*> ∈ tiling means *AST* is also covered by a leaf in another operation tree in the tiling, unless it is the root
  - Where two operation trees meet, they must be compatible (expect the value in the same location)

Tiling the Tree



Each tile corresponds to a sequence of operations

Emitting those operations in an appropriate order implements the tree.

Generating Code

Given a tiled tree

- Postorder treewalk, with node-dependent order for children
  - → Right child of ← before its left child
  - → Might impose "most demanding first" rule …
- Emit code sequence for tiles, in order
- Tie boundaries together with register names
  - → Tile 6 uses registers produced by tiles 1 & 5
  - → Tile 6 emits "`store` $r_{tile\ 5} \Rightarrow r_{tile\ 1}$"
  - → Can incorporate a "real" register allocator or just use virtual registers

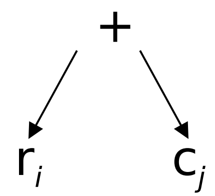
So, What's Hard About This?

Finding the matches to tile the tree

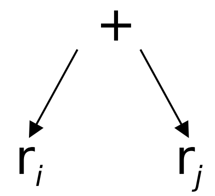
- Compiler writer connects operation trees to AST subtrees
  - → Encode tree syntax, in linear form
  - → Provides a set of rewrite rules
  - → Associated with each is a code template

Notation

To describe these trees, we need a concise notation



$+(r_i,c_j)$



$+(r_i,r_j)$

Linear prefix form

Notation

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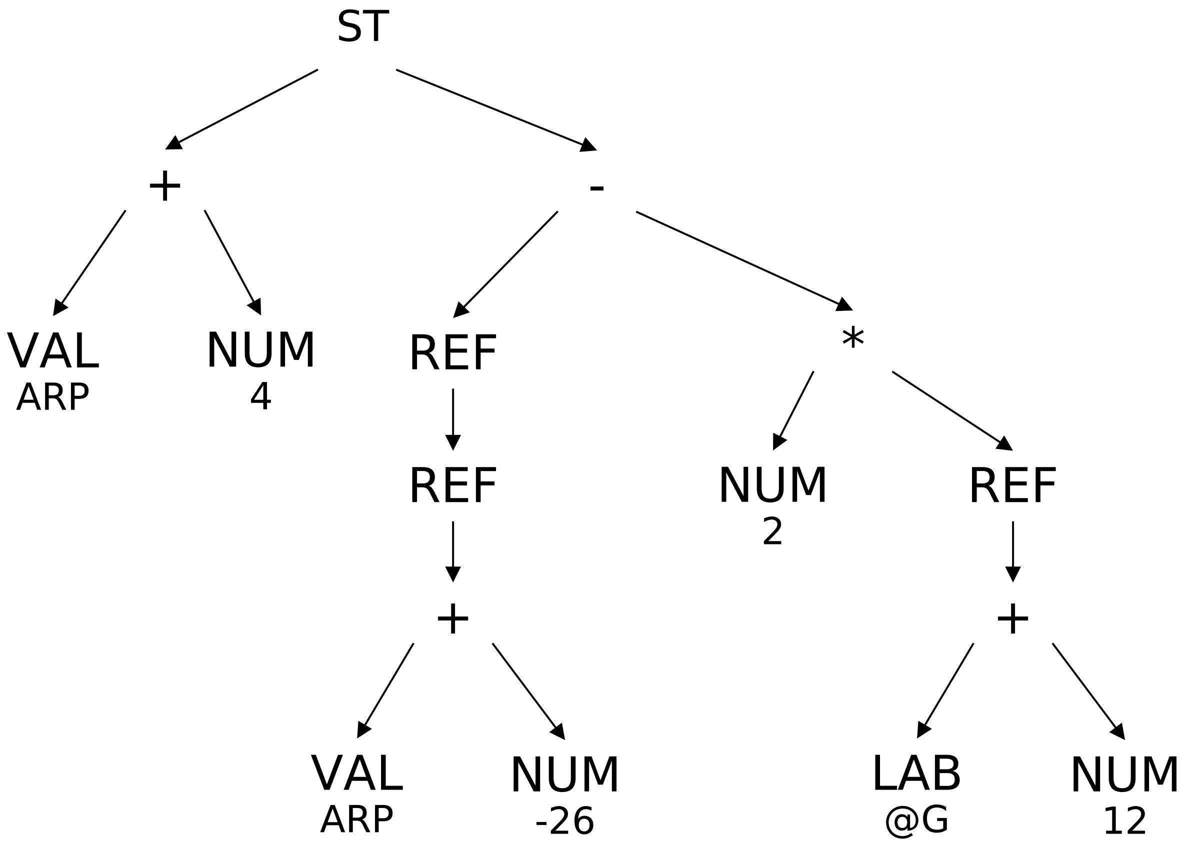

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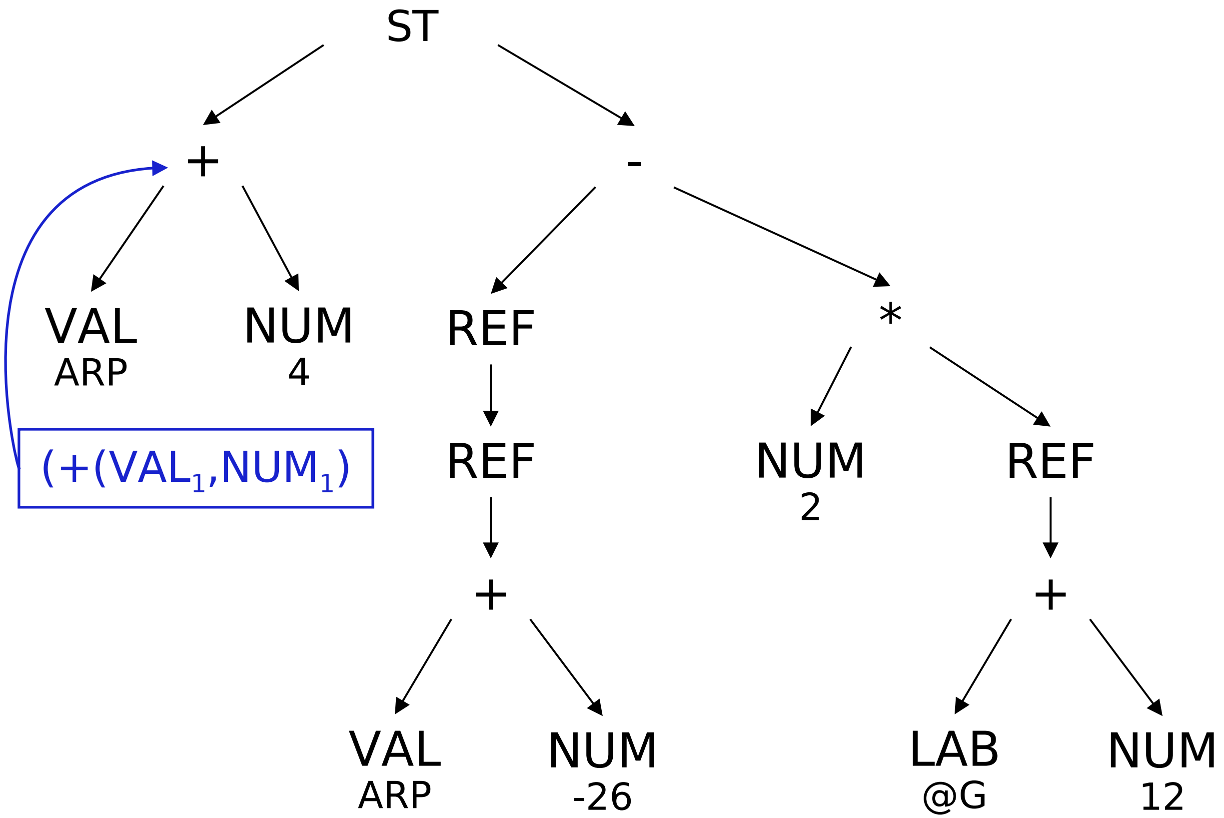

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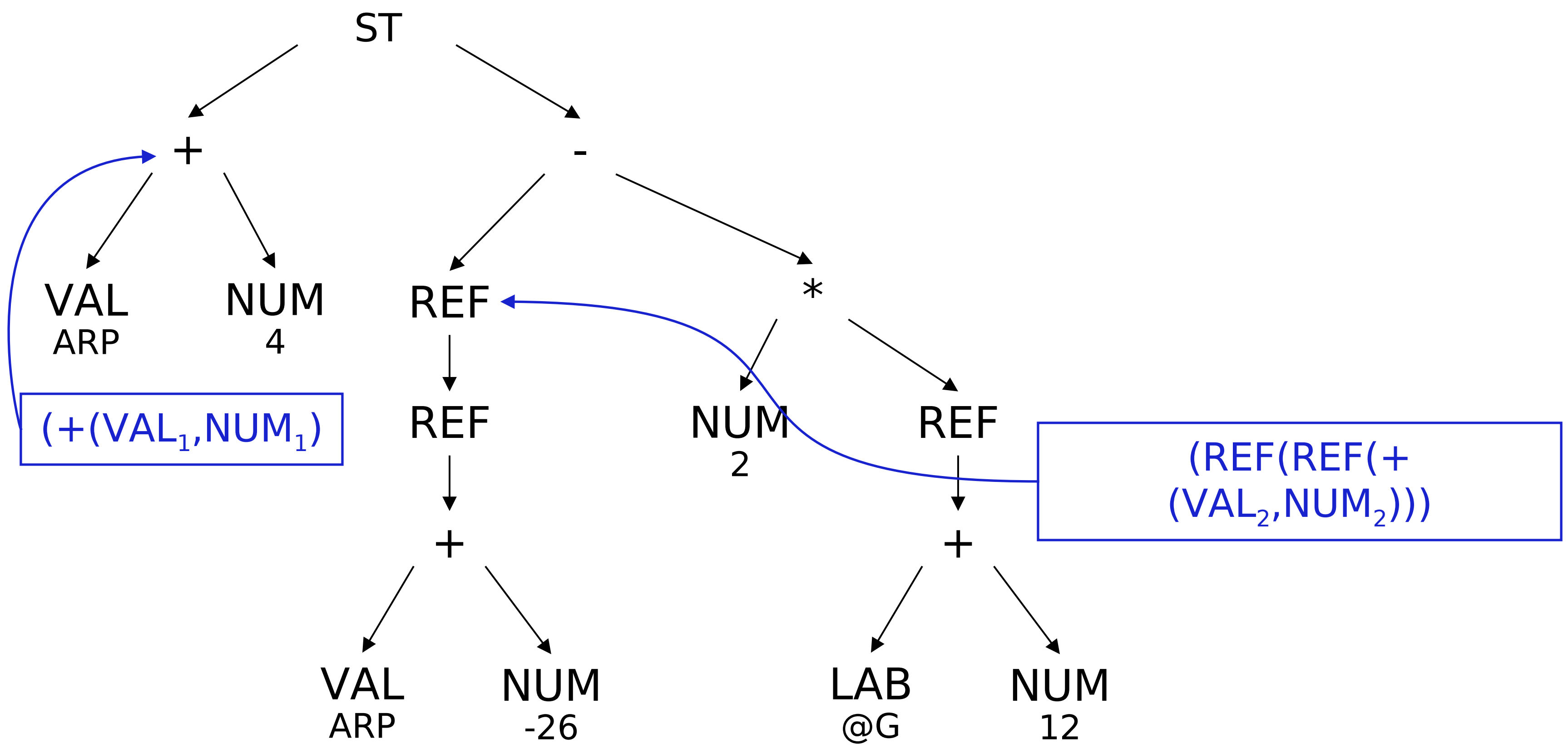

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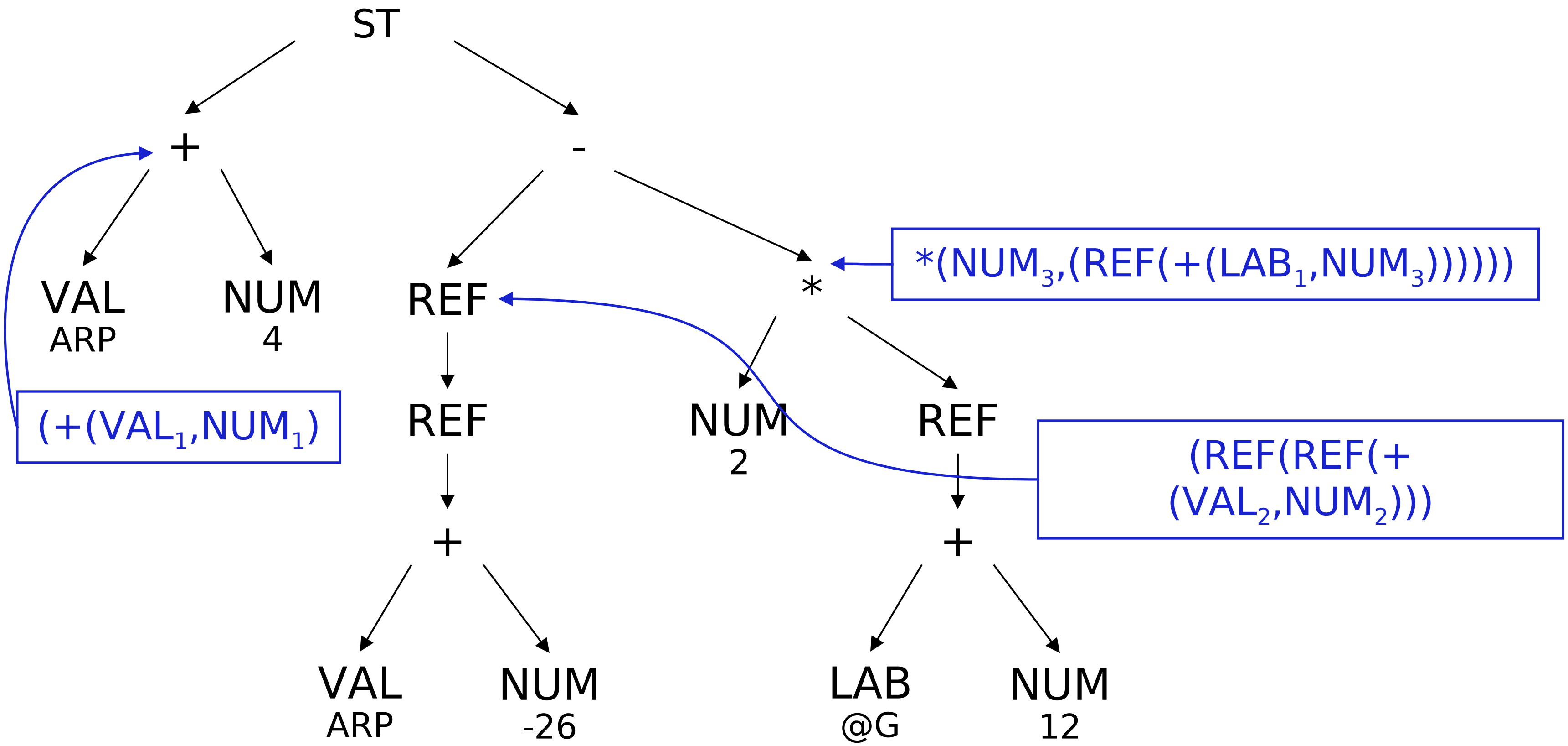

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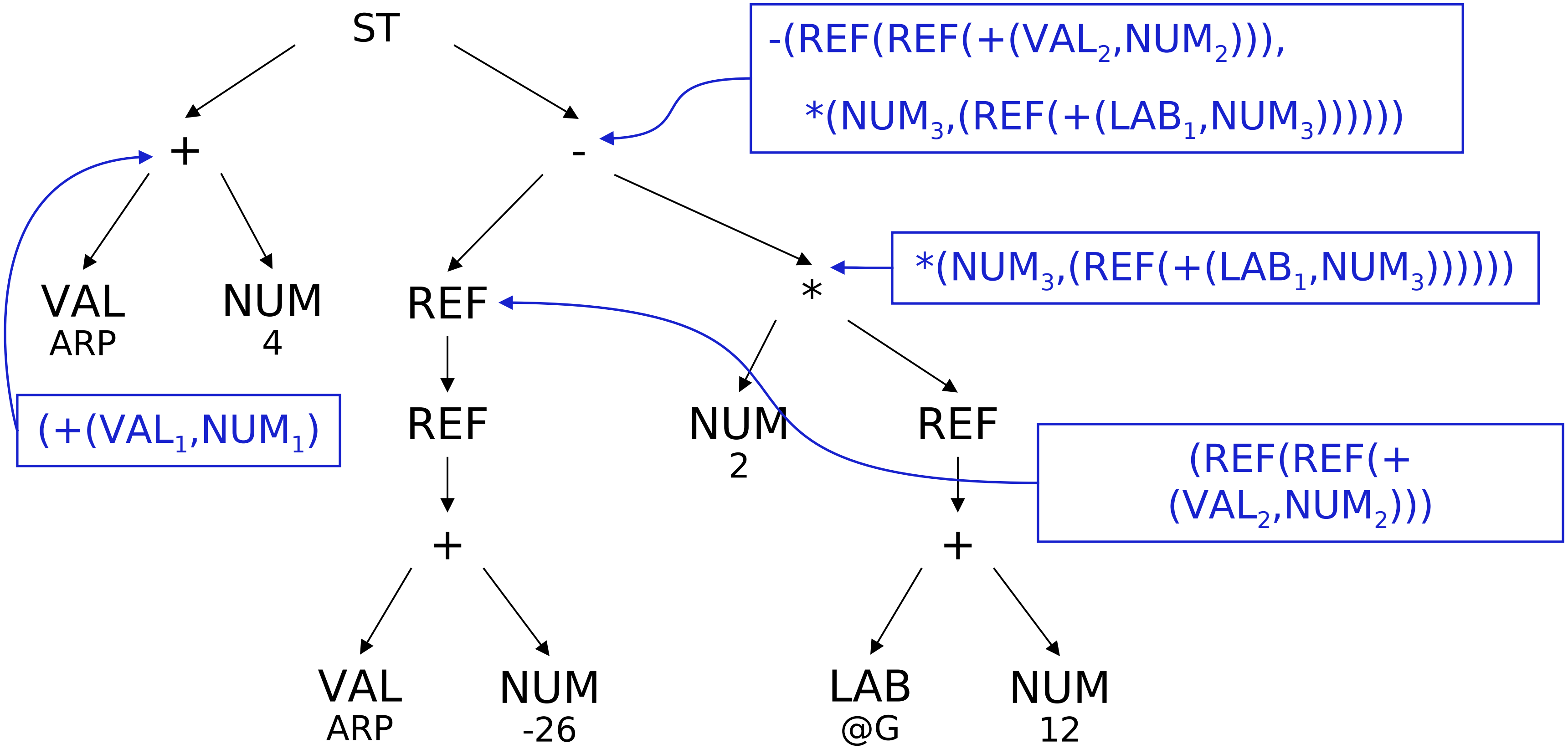

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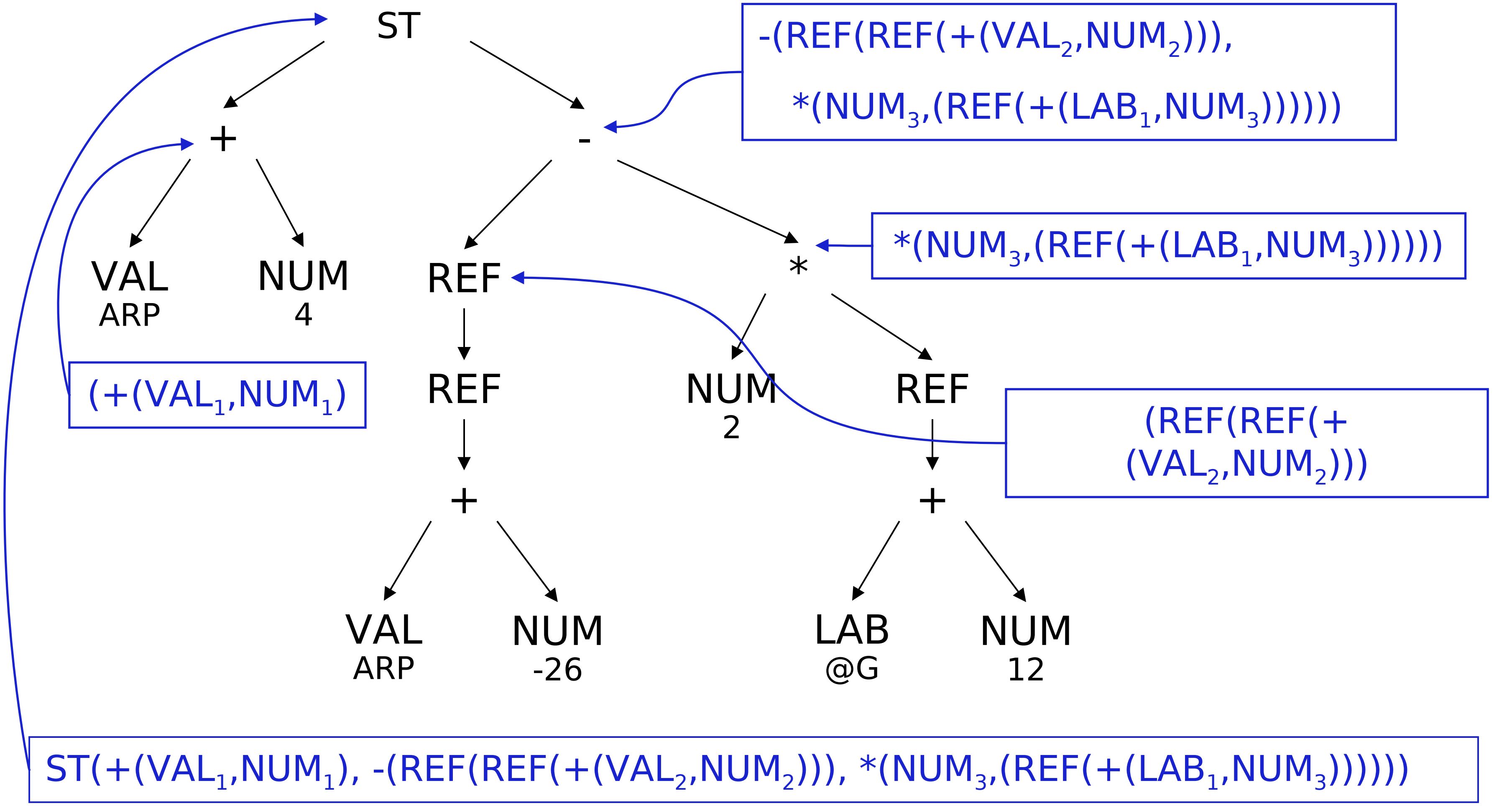


Rewrite rules: LL Integer AST into ILOC

	Rule	Cost	Template
1	Goal → Assign	0	
2	Assign → ST(Reg_1,Reg_2)	1	`store` $r_2 \Rightarrow r_1$
3	Assign → ST(+(Reg_1,Reg_2),Reg_3)	1	`storeAO` $r_3 \Rightarrow r_1, r_2$
4	Assign → ST(+(Reg_1,NUM_2),Reg_3)	1	`storeAI` $r_3 \Rightarrow r_1, n_2$
5	Assign → ST(+(NUM_1,Reg_2),Reg_3)	1	`storeAI` $r_3 \Rightarrow r_2, n_1$
6	Reg → LAB_1	1	`loadI` $l_1 \Rightarrow r_{new}$
7	Reg → VAL_1	0	
8	Reg → NUM_1	1	`loadI` $n_1 \Rightarrow r_{new}$
9	Reg → REF(Reg_1)	1	`load` $r_1 \Rightarrow r_{new}$
10	Reg → REF(+ (Reg_1,Reg_2))	1	`loadAO` $r_1, r_2 \Rightarrow r_{new}$
11	Reg → REF(+ (Reg_1,NUM_2))	1	`loadAI` $r_1, n_2 \Rightarrow r_{new}$
12	Reg → REF(+ (NUM_1,Reg_2))	1	`loadAI` $r_2, n_1 \Rightarrow r_{new}$

Rewrite rules: LL Integer AST into ILOC (*part II*)

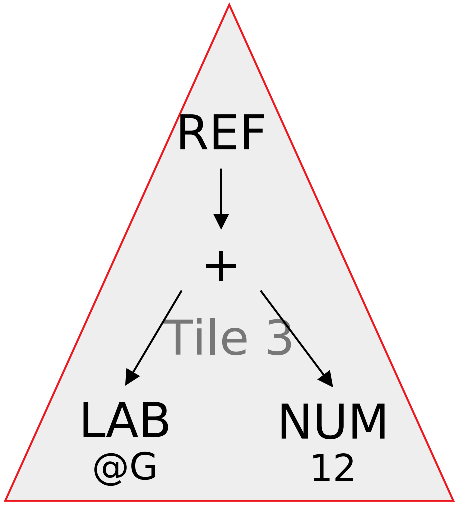
	Rule	Cost	Template
13	Reg → REF(+ (Reg_1,Lab_2))	1	`loadAI` $r_1, l_2 \Rightarrow r_{new}$
14	Reg → REF(+ (Lab_1,Reg_2))	1	`loadAI` $r_2, l_1 \Rightarrow r_{new}$
15	Reg → + (Reg_1,Reg_2)	1	`addI` $r_1, r_2 \Rightarrow r_{new}$
16	Reg → + (Reg_1,NUM_2)	1	`addI` $r_1, n_2 \Rightarrow r_{new}$
17	Reg → + (NUM_1,Reg_2)	1	`addI` $r_2, n_1 \Rightarrow r_{new}$
18	Reg → + (Reg_1,Lab_2)	1	`addI` $r_1, l_2 \Rightarrow r_{new}$
19	Reg → + (Lab_1,Reg_2)	1	`addI` $r_2, l_1 \Rightarrow r_{new}$
20	Reg → - (NUM_1,Reg_2)	1	`rsubI` $r_2, n_1 \Rightarrow r_{new}$
…	…	…	…

A real set of rules would cover more than signed integers …

So, What's Hard About This?

Need an algorithm to AST subtrees with the rules

Consider tile 3 in our example



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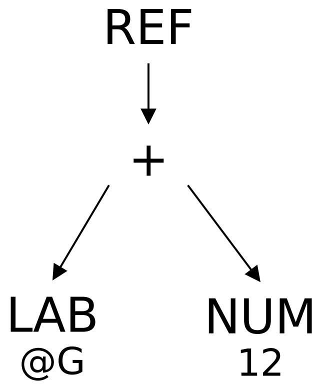
What rules match tile 3?



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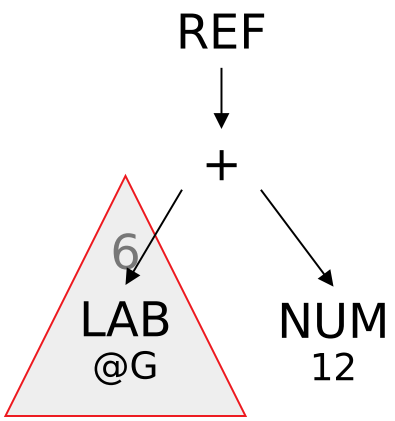

What rules match tile 3?

6: Reg → LAB_1 tiles the lower left node

So, What's Hard About This?

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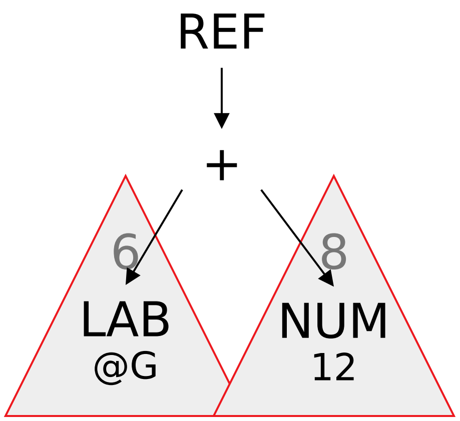

What rules match tile 3?

6: Reg → LAB_1 tiles the lower left node

8: Reg → NUM_1 tiles the bottom right node

So, What's Hard About This?

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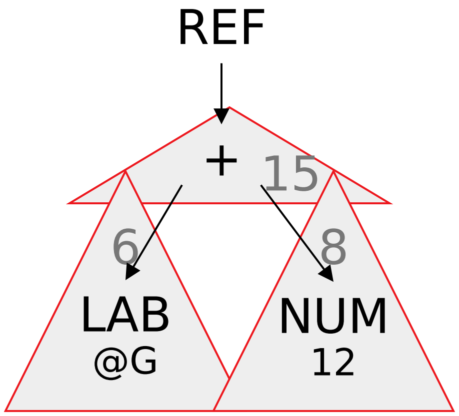
Consider tile 3 in our example



What rules match tile 3?

6: Reg $\rightarrow$ LAB_1 tiles the lower left node

8: Reg $\rightarrow$ NUM_1 tiles the bottom right node

15: Reg $\rightarrow$ + (Reg_1,Reg_2) tiles the + node

So, What's Hard About This?

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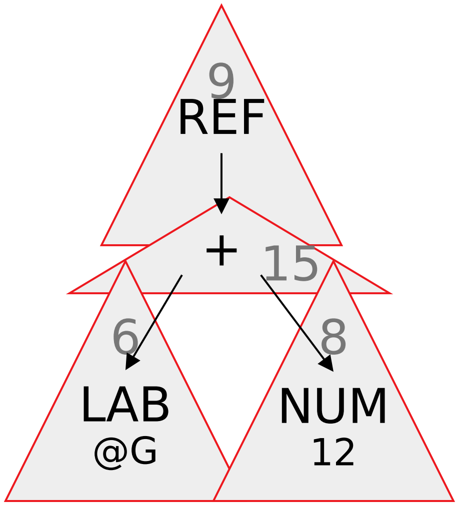

What rules match tile 3?

6: Reg → LAB_1 tiles the lower left node

8: Reg → NUM_1 tiles the bottom right node

15: Reg → + (Reg_1,Reg_2) tiles the + node

9: Reg → REF(Reg_1) tiles the REF

So, What's Hard About This?

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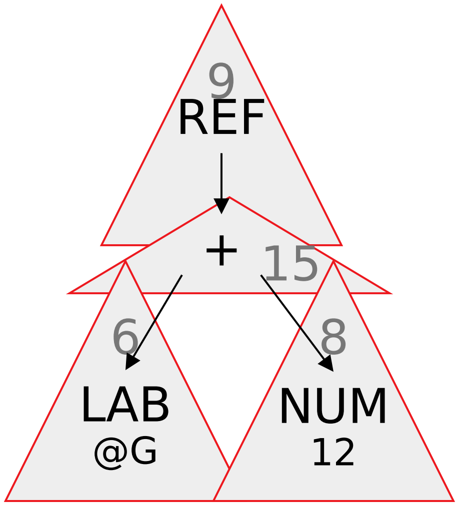

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9: Reg $\rightarrow$ REF(Reg_1) tiles the REF

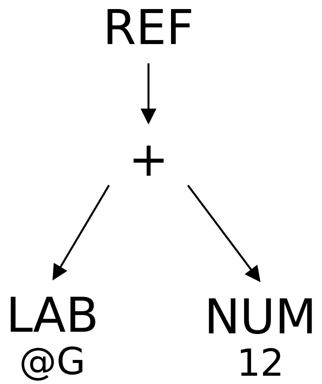
We denote this match as <6,8,15,9>

Of course, it implies <8,6,15,9>

Both have a cost of 4

Finding matches

Many Sequences Match Our Subtree



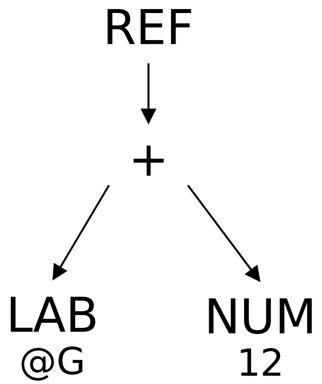
Cost	Sequences			
2	6,11	8,14		
3	6,8,10	8,6,10	6,16,9	8,19,9
4	6,8,15,9	8,6,15,9		

In general, we want the low cost sequence

- Each unit of cost is an operation (1 cycle)
- We should favour short sequences

Finding matches

Low Cost Matches



Sequences with Cost of 2	
6: Reg → LAB_1 11: Reg → REF(+(Reg_1,NUM_2))	`loadI @G ⇒ r`$_i$ `loadAI r`$_i$`,12 ⇒ r`$_j$
8: Reg → NUM_1 14: Reg → REF(+(LAB_1,Reg_2))	`loadI 12 ⇒ r`$_i$ `loadAI r`$_i$`,@G ⇒ r`$_j$

These two are equivalent in cost

6,11 might be better, because @G may be longer than the immediate field

Tiling the Tree

Still need an algorithm

- Assume each rule implements one operator
- Assume operator takes 0, 1, or 2 operands

Now, …

Tiling the Tree

Tile(n)
  Label(n) ← Ø
  if n has two children then
    Tile (left child of n)
    Tile (right child of n)
    for each rule r that implements n
      if (left(r) ∈ Label(left(n)) and
        (right(r) ∈ Label(right(n))
        then Label(n) ← Label(n) ∪ { r }

Match binary nodes against binary rules

  else if n has one child
    Tile(child of n)
    for each rule r that implements n
      if (left(r) ∈ Label(child(n))
        then Label(n) ← Label(n) ∪ { r }

Match unary nodes against unary rules

  else / n is a leaf */*
    Label(n) ← {all rules that implement n }

Handle leaves with lookup in rule table

Notes:

- left and right refer to the children of the AST node or left/right-hand sides of a rule
- implements: e.g. rule 9 implements REF

Tiling the Tree

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  else  /* n is a leaf */
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```

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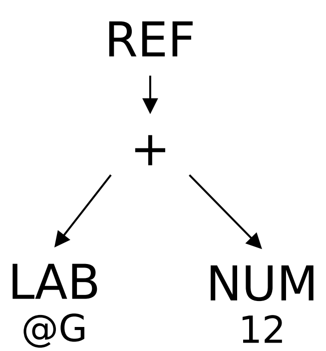
19	Reg → + (Lab$_1$,Reg$_2$)	1	addI r$_2$,l$_1$ ⇒ r$_{new}$
20	Reg → - (NUM$_1$,Reg$_2$)	1	rsubI r$_2$,n$_1$ ⇒ r$_{new}$



Label(Ref) =

Label(+) =

Label(Lab) =

Label(Num) =

Tiling the Tree

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```

This algorithm

- Finds all matches in rule set
- Labels node n with that set
- Can keep lowest cost match at each point for each type of nodes → Dynamic programming
- Spends its time in the two matching loops

The Big Picture

- Tree patterns represent AST and ASM
- Can use matching algorithms to find low-cost tiling of AST
- Can turn a tiling into code using templates for matched rules
- Techniques (& tools) exist to do this efficiently

Hand-coded matcher like *Tile*	Avoids large sparse table Lots of work
Encode matching as an automaton	O(1) cost per node Tools like BURS (bottom-up rewriting system), BURG
Use parsing techniques	Uses known technology Very ambiguous grammars
Linearize tree into string and use string searching algorithm (Aho-Corasick)	Finds all matches

Next Lecture

- Object Oriented Programming Support