Compiler Design

Lecture 10: Semantic Analysis: part II Types

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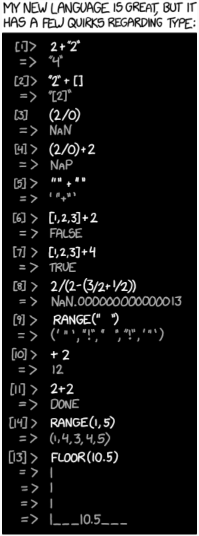
Specification

What are types used for?

Checking that identifiers are declared and used correctly is not the only thing that needs to be verified in the compiler.

In most programming languages, expressions have a type.

Types are here to ensure that expressions are compatible with one another to guarantee some level of correctness.



source: https://xkcd.com/1537/ (CC BY-NC 2.5)

Examples: typing rules of our Mini-C language

- The operands of `+` must be integers
- The operands of `==` must be compatible (`int` with `int`, `char` with `char`)
- The number of arguments passed to a function must be equal to the number of parameters
- ...

Type Systems

Type properties

Typing properties

Strong/weak typing

A language is said to be strongly typed if the violation of a typing rule results in an error.

A language is said to be weakly typed or not typed in other cases — in particular if the program behaviour becomes unspecified after an incorrect typing.

Strong/weak typing is about how strictly types are distinguished (e.g. implicit conversion).

Static/dynamic typing

A language is said to be statically typed if there exists a type system that can detect incorrect programs before execution.

A language is said to be dynamically types in other cases.

Static/dynamic typing is about when type information is available

⚠ A strongly typed language does not imply static typing. ⚠

Language examples

	strong	weak
static	Java	C/C++
dynamic	Python	JavaScript

Java (static/strong)

```java
class A {}
class B {}
B b = new B();
A a = (A) b;
// compile-time error
```

Python (dynamic/strong)

```python
1+'a'
# run-time error
```

C (static/weak)

```c
int * p1;
char ** p2;
p1 = (int*) p2;
// no error
```

JavaScript (dynamic/weak)

```javascript
3 + '6'; // '36'
3 * '6'; // 18

num = 11;
num.toUpperCase();
// run-time error
```

Weak dynamic typing: the worst of the worst!

JavaScript

```
num = 11;
num.toUpperCase();
// run-time error
```

```
3 + '6'; // '36'
3 * '6'; // 18
// no error
```

source: http://gunshowcomic.com/648

Goal

We want to give an exact specification of the language.

- We will formally define this, using a mathematical notation.
- Programs who pass the type checking phase are well-typed since they corresponds to programs for which is it possible to give a type to each expression.

This mathematical description will fully specify the typing rules of our language.

Inference Rules

Suppose that we have a small language expressing constants (integer literal), the `+` binary operation and the type **int**.

Example: language for arithmetic expressions

Constants	i	=	*a number (integer literal)*
Expressions	e	=	i
		|	e_1 + e_2
Types	T	=	**int**

Type judgement

We want to define a type *judgement* (*a.k.a.* statement):

$$\vdash e : \tau$$

In english: I can "conclude" that expression e has type τ.

Inference Rules

Inference Rules

An expression e is of type T iff:

- it's an expression of the form i and T = **int** or
- it's an expression of the form $e_1 + e_2$, where e_1 and e_2 are two expressions of type **int** and T = **int**

To represent such a definition, it is convenient to use inference rules which in this context is called a typing rule:

Typing rules

$$\textsc{IntLit}\ \frac{}{\vdash i : \mathbf{int}} \qquad\qquad \textsc{BinOp}\ \frac{\vdash e_1 : \mathbf{int} \qquad \vdash e_2 : \mathbf{int}}{\vdash e_1 + e_2 : \mathbf{int}}$$

Typing rules

$$\textsc{IntLit}\ \frac{}{\vdash i : \mathbf{int}} \qquad\qquad \textsc{BinOp}\ \frac{\vdash e_1 : \mathbf{int} \qquad \vdash e_2 : \mathbf{int}}{\vdash e_1 + e_2 : \mathbf{int}}$$

An inference rule is composed of:

- a horizontal line
- a name on the left or right of the line
- a list of premisses placed above the line
- a conclusion placed below the line

An inference rule where the list of premisses is empty is called an axiom.

An inference rule can be read bottom up:

Example

$$\textsc{BinOp}\ \frac{\vdash e_1 : \textbf{int} \qquad \vdash e_2 : \textbf{int}}{\vdash e_1 + e_2 : \textbf{int}}$$

“To show that an expression of the form $e_1 + e_2$ has type **int**, we need to show that $e1$ and $e2$ have the type **int**”.

- To show that the conclusion of a rule holds, it is enough to prove that the premisses are correct
- This process stops when we encounter an axiom.

Using the inference rule representation, it possible to see whether an expression is well-typed.

Example: (1+2)+3

$$
\text{BinOp}\ \dfrac{\text{BinOp}\ \dfrac{\text{IntLit}\ \dfrac{}{\vdash 1 : \mathsf{int}} \qquad \text{IntLit}\ \dfrac{}{\vdash 2 : \mathsf{int}}}{\vdash 1 + 2 : \mathsf{int}} \qquad \text{IntLit}\ \dfrac{}{\vdash 3 : \mathsf{int}}}{\vdash (1 + 2) + 3 : \mathsf{int}}
$$

Such a tree is called a derivation tree.

Conclusion

An expression e has type T iff there exist a derivation tree whose conclusion is $\vdash e : T$.

Inference Rules

Environments

Identifiers

Let's add identifiers to our language.

Example: language for arithmetic expressions

Identifiers	x	=	*a name (string literal)*
Constants	i	=	*a number (integer literal)*
Expressions	e	=	i
		|	e_1 + e_2
		|	x
Types	T	=	**int**

To determine if an expression such as `x+1` is well-typed, we need to have information about the type of `x`.

We add an environment Γ to our typing rules which associates a type for each identifier.

Our type judgement are now written as: $\Gamma \vdash e : \tau$.

In english: given Γ, I can conclude e has type τ.

Environment

A typing environment Γ is list of pairs of an identifier x and a type T.

It can be implemented in two ways in the compiler:

- As a symbol table;
- Or directly encoded in the AST nodes:
  e.g. VarExpr node has a reference to the declaration (filled in during *Name Analysis*)

We can add an inference rule to decide when an expression containing an identifier is well-typed:

$$\textsc{Ident}\ \frac{x : T \in \Gamma}{\Gamma \vdash x : T}$$

Example: $x + 1$

In the environment $\Gamma = \{x : \mathbf{int}\}$, it is possible to type check $x + 1$

$$\textsc{BinOp}\ \frac{\textsc{Ident}\ \dfrac{x : \mathbf{int} \in \Gamma}{\Gamma \vdash x : \mathbf{int}} \qquad \textsc{IntLit}\ \dfrac{}{\Gamma \vdash 1 : \mathbf{int}}}{\Gamma \vdash x + 1 : \mathbf{int}}$$

Inference Rules

Function Call

Function call

We need to add a notation to talk about the type of the functions.

Example: language for arithmetic expressions

Identifiers	x	=	*a name (string literal)*
Constants	i	=	*a number (integer literal)*
Expressions	e	=	i
		|	e_1 + e_2
		|	x
Types	T,U	=	**int**
		|	$(U_1, \ldots, U_n) \rightarrow T$

where $(U_1, \ldots, U_n) \rightarrow T$ represents a function type.

Function call inference rule

$$\textsc{FunCall}(f)\ \frac{\Gamma \vdash f : (U_1, \ldots, U_n) \to T \qquad \Gamma \vdash x_1 : U_1 \qquad \ldots \qquad \Gamma \vdash x_n : U_n}{\Gamma \vdash f(x_1, \ldots, x_n) : T}$$

In plain English:

- each argument x_i must be of type U_i
- the function f is defined in the environment Γ as a function taking parameters of types $U_1, \ldots, U_n$ and a return type T.

Example: int foo(int, int)

$$\textsc{FunCall}(\text{foo})\ \frac{\Gamma \vdash foo : (\text{int}, \text{int}) \to \text{int} \qquad \Gamma \vdash x_1 : \text{int} \qquad \Gamma \vdash x_2 : \text{int}}{\Gamma \vdash foo(x_1, x_2) : \text{int}}$$

Implementation with Pattern-Matching

TypeChecker

```
class TypeChecker {

  Type visit (ASTnode node) {
    return switch(node) {
      ...
    }
  }

}
```

The visit method returns the type inferred for the AST node (if any).

$$\textsc{BinOp}(+)\ \frac{\vdash e_1 : \mathsf{int} \qquad \vdash e_2 : \mathsf{int}}{\vdash e_1 + e_2 : \mathsf{int}}$$

TypeChecker : binary operation

```
...
case BinOp bo → {
  Type lhsT = bo.lhs.visit();
  Type rhsT = bo.rhs.visit();
  if (bo.op == ADD) {
    if (lhsT == Type.INT && rhsT == Type.INT) {
      bo.type = Type.INT; // set the type
      yield Type.INT;     // returns it
    } else
      error();
      yield Type.INVALID;
  }
}
...
```

TypeChecker: variables

```
case VarDecl vd → {
  if (vd.type == VOID)
    error();
  yield Type.NONE;
}

case Var v → {
  v.type = v.vd.type;
  yield v.vd.type;
}
```

Not just analysis!

The type checker does more than analysing the AST: it also remembers the result of the analysis directly in the AST node.

$$\textsc{FunCall}(f)\ \frac{\Gamma \vdash f : (U_1, \ldots, U_n) \to T \qquad \Gamma \vdash x_1 : U_1 \qquad \ldots \qquad \Gamma \vdash x_n : U_n}{\Gamma \vdash f(x_1, \ldots, x_n) : T}$$

Exercise: write the case for function call

```
case FunCall fc → {



}
```

Conclusion

- Typing rules can be formally defined using inference rules.
- We saw how to implement them with a pattern-matching

Next lecture:

- An introduction to MIPS Assembly