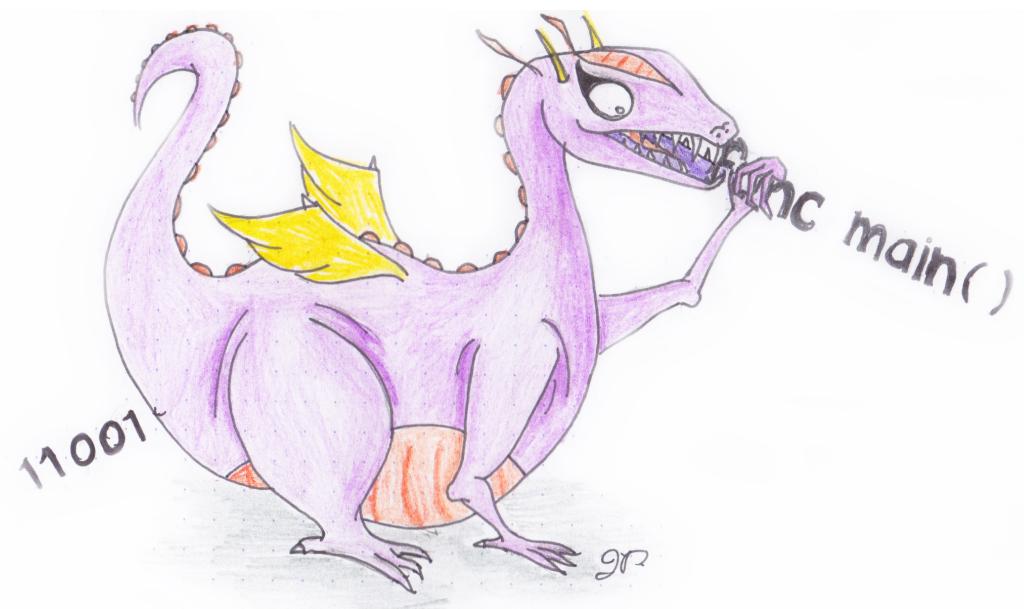
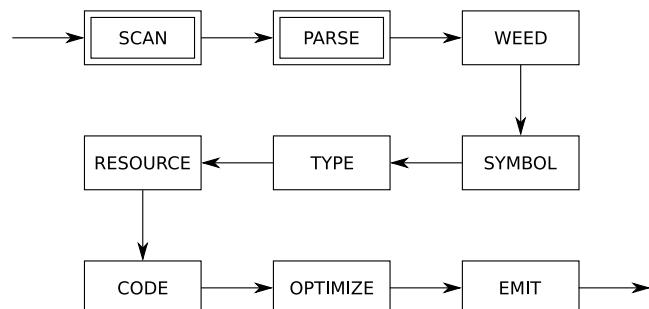


# Parsing

COMP 520: Compiler Design (4 credits)

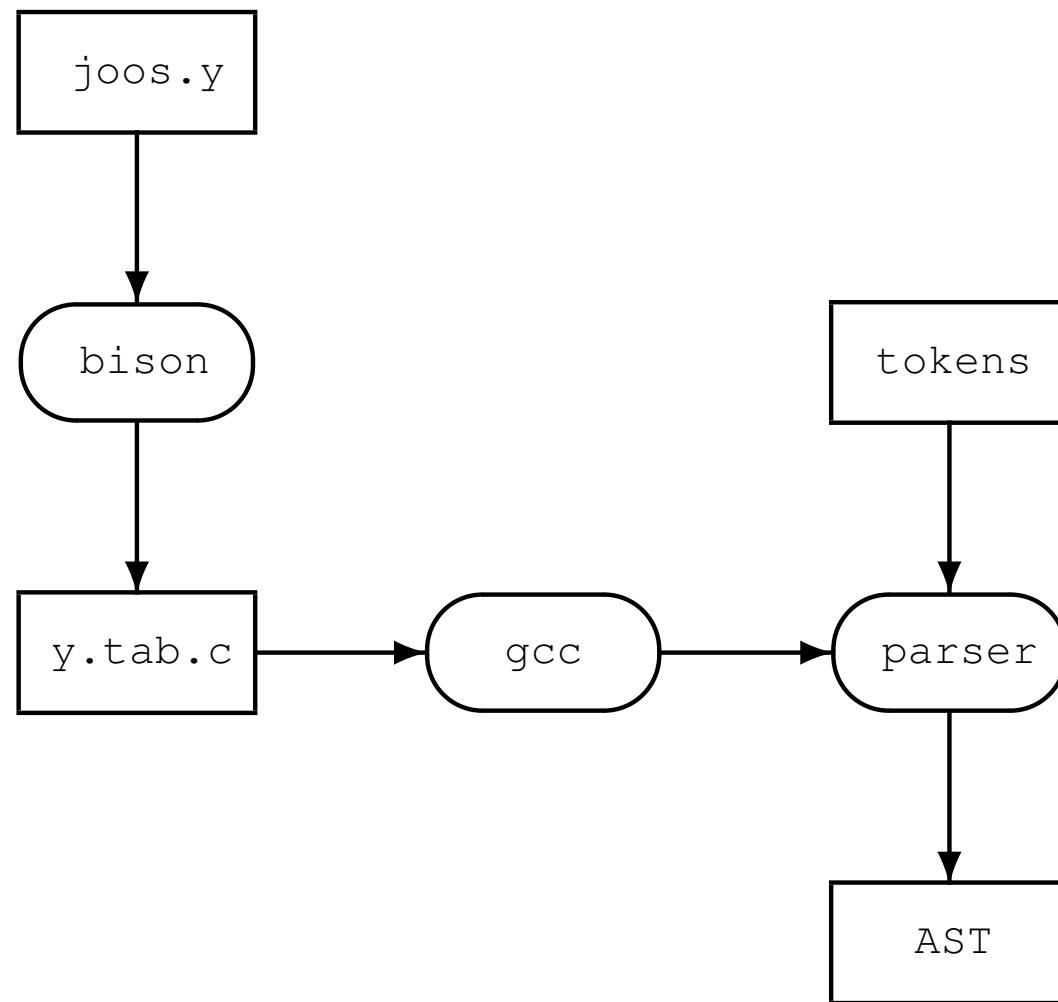
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**A *parser* transforms a string of tokens into a parse tree, according to some grammar:**

- it corresponds to a *deterministic push-down automaton*;
- plus some glue code to make it work;
- can be generated by `bison` (or `yacc`), CUP, ANTLR, SableCC, Beaver, JavaCC, ...



A **context-free grammar** is a 4-tuple  $(V, \Sigma, R, S)$ , where we have:

- $V$ , a set of *variables* (or *non-terminals*)
- $\Sigma$ , a set of *terminals* such that  $V \cap \Sigma = \emptyset$
- $R$ , a set of *rules*, where the LHS is a variable in  $V$  and the RHS is a string of variables in  $V$  and terminals in  $\Sigma$
- $S \in V$ , the start variable

CFGs are stronger than regular expressions, and able to express recursively-defined constructs.

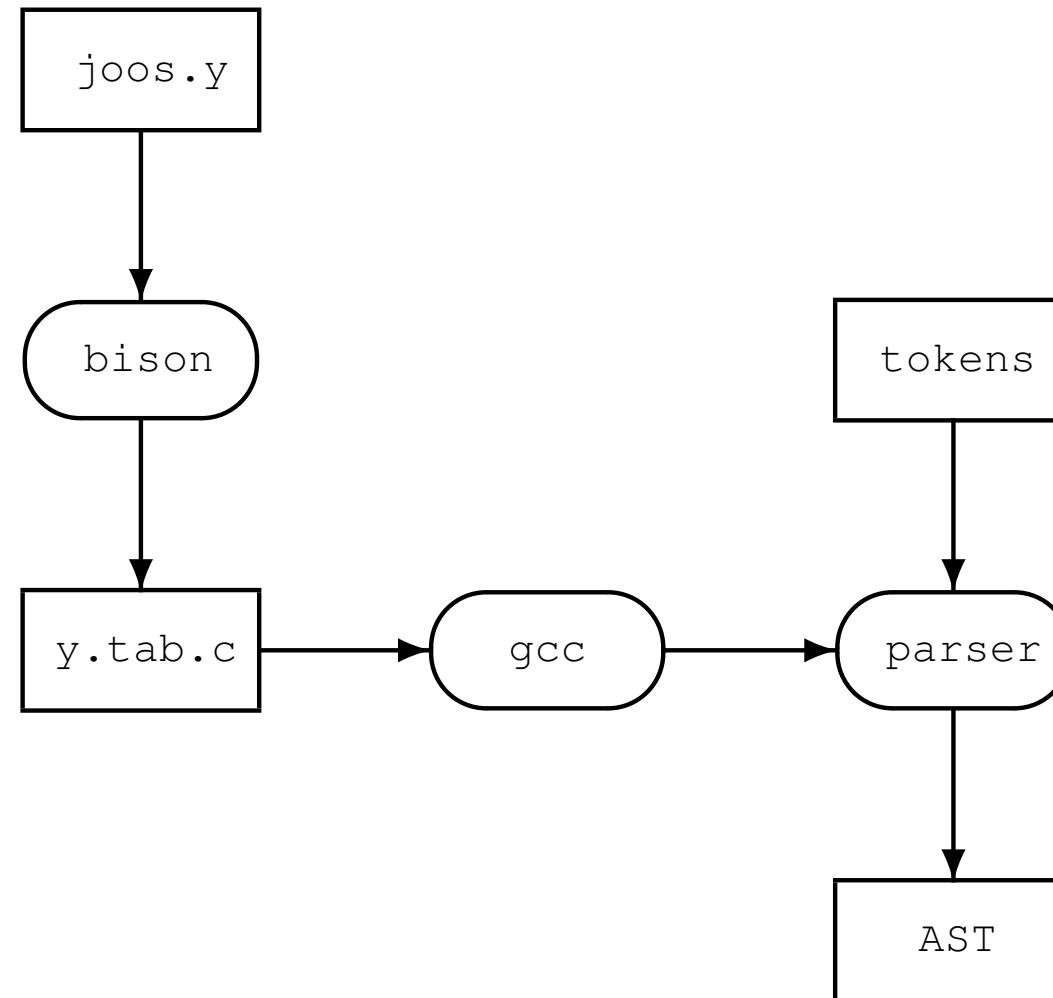
Example: we cannot write a regular expression for any number of matched parentheses:

$(()), ((())), (((()))), \dots$

Using a CFG:

$$E \rightarrow (E) \mid \epsilon$$

**Automatic parser generators use CFGs as input and generate parsers using the machinery of a deterministic pushdown automaton.**



By limiting the kind of CFG allowed, we get efficient parsers.

Simple CFG example:

$$A \rightarrow a B$$

$$A \rightarrow \epsilon$$

$$B \rightarrow b B$$

$$B \rightarrow c$$

Alternatively:

$$A \rightarrow a B \mid \epsilon$$

$$B \rightarrow b B \mid c$$

In both cases we specify  $S = A$ . Can you write this grammar as a regular expression?

We can perform a *rightmost derivation* by repeatedly replacing variables with their RHS until only terminals remain:

A

a B

a b B

a b b B

a b b c

**Different grammar formalisms.** First, consider BNF (Backus-Naur Form):

```
stmt ::= stmt_expr ";" |  
       while_stmt |  
       block |  
       if_stmt  
  
while_stmt ::= WHILE "(" expr ")" stmt  
  
block ::= "{" stmt_list "}"  
  
if_stmt ::= IF "(" expr ")" stmt |  
          IF "(" expr ")" stmt ELSE stmt
```

We have four options for stmt\_list:

1.  $\text{stmt\_list} ::= \text{stmt\_list } \text{stmt} \mid \epsilon$  (0 or more, left-recursive)
2.  $\text{stmt\_list} ::= \text{stmt } \text{stmt\_list} \mid \epsilon$  (0 or more, right-recursive)
3.  $\text{stmt\_list} ::= \text{stmt\_list } \text{stmt} \mid \text{stmt}$  (1 or more, left-recursive)
4.  $\text{stmt\_list} ::= \text{stmt } \text{stmt\_list} \mid \text{stmt}$  (1 or more, right-recursive)

## Second, consider EBNF (Extended BNF):

BNF	derivations		EBNF
$A \rightarrow A a \mid b$ (left-recursive)	b	<u><math>A</math></u> a <u><math>A</math></u> a a b a a	$A \rightarrow b \{ a \}$
$A \rightarrow a A \mid b$ (right-recursive)	b	a <u><math>A</math></u> a a <u><math>A</math></u> a a b	$A \rightarrow \{ a \} b$

where '{' and '}' are like Kleene \*'s in regular expressions.

**Now, how to specify stmt\_list:**

Using EBNF repetition, our four choices for stmt\_list

1.  $\text{stmt\_list} ::= \text{stmt\_list } \text{stmt} \mid \epsilon \quad (\text{0 or more, left-recursive})$
2.  $\text{stmt\_list} ::= \text{stmt } \text{stmt\_list} \mid \epsilon \quad (\text{0 or more, right-recursive})$
3.  $\text{stmt\_list} ::= \text{stmt\_list } \text{stmt} \mid \text{stmt} \quad (\text{1 or more, left-recursive})$
4.  $\text{stmt\_list} ::= \text{stmt } \text{stmt\_list} \mid \text{stmt} \quad (\text{1 or more, right-recursive})$

become:

1.  $\text{stmt\_list} ::= \{ \text{stmt} \}$
2.  $\text{stmt\_list} ::= \{ \text{stmt} \}$
3.  $\text{stmt\_list} ::= \{ \text{stmt} \} \text{stmt}$
4.  $\text{stmt\_list} ::= \text{stmt} \{ \text{stmt} \}$

**EBNF also has an *optional*-construct.** For example:

```
stmt_list ::= stmt stmt_list | stmt
```

could be written as:

```
stmt_list ::= stmt [ stmt_list ]
```

And similarly:

```
if_stmt ::= IF "(" expr ")" stmt |
           IF "(" expr ")" stmt ELSE stmt
```

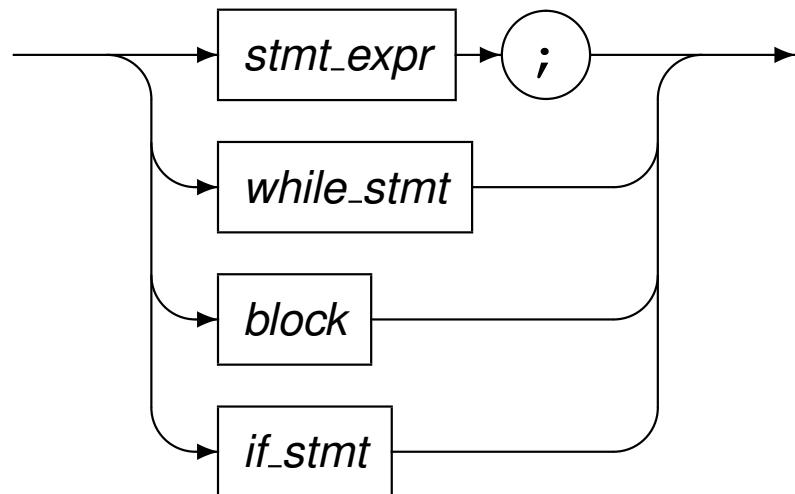
could be written as:

```
if_stmt :=
           IF "(" expr ")" stmt [ ELSE stmt ]
```

where '[' and ']' are like '?' in regular expressions.

Third, consider “railroad” syntax diagrams: (thanks rail.sty!)

*stmt*



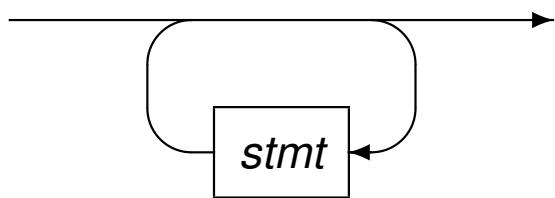
*while\_stmt*



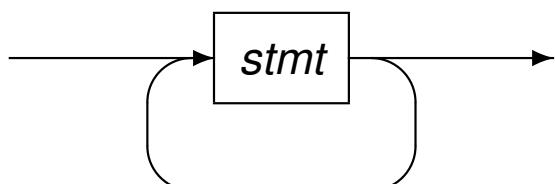
*block*

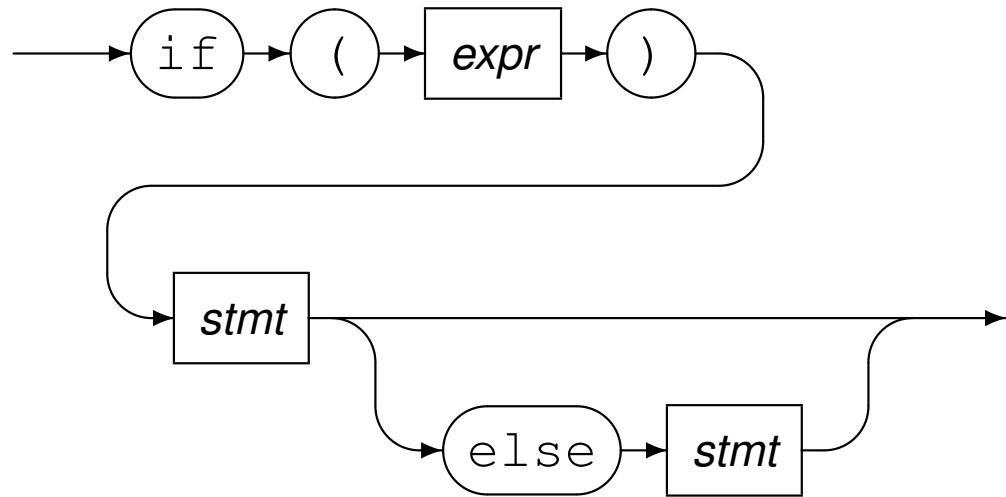


*stmt\_list* (0 or more)



*stmt\_list* (1 or more)



*if\_stmt*

$$\begin{array}{lll}
 S \rightarrow S ; S & E \rightarrow \text{id} & L \rightarrow E \\
 S \rightarrow \text{id} := E & E \rightarrow \text{num} & L \rightarrow L , E \\
 S \rightarrow \text{print} ( L ) & E \rightarrow E + E \\
 & E \rightarrow ( S , E )
 \end{array}$$

a := 7;  
 b := c + (d := 5 + 6, d)

S (rightmost derivation)

$$\begin{array}{l}
 S; \underline{S} \\
 S; \text{id} := \underline{E} \\
 S; \text{id} := E + \underline{E} \\
 S; \text{id} := E + (S, \underline{E}) \\
 S; \text{id} := E + (\underline{S}, \text{id}) \\
 S; \text{id} := E + (\text{id} := \underline{E}, \text{id})
 \end{array}$$

$$\begin{array}{l}
 S; \text{id} := E + (\text{id} := E + \underline{E}, \text{id}) \\
 S; \text{id} := E + (\text{id} := \underline{E} + \text{num}, \text{id}) \\
 S; \text{id} := \underline{E} + (\text{id} := \text{num} + \text{num}, \text{id}) \\
 \underline{S}; \text{id} := \text{id} + (\text{id} := \text{num} + \text{num}, \text{id}) \\
 \text{id} := \underline{E}; \text{id} := \text{id} + (\text{id} := \text{num} + \text{num}, \text{id}) \\
 \text{id} := \text{num}; \text{id} := \text{id} + (\text{id} := \text{num} + \text{num}, \text{id})
 \end{array}$$

$$S \rightarrow S ; S$$

$$E \rightarrow \text{id}$$

$$S \rightarrow \text{id} := E$$

$$E \rightarrow \text{num}$$

$$S \rightarrow \text{print} ( L )$$

$$E \rightarrow E + E$$

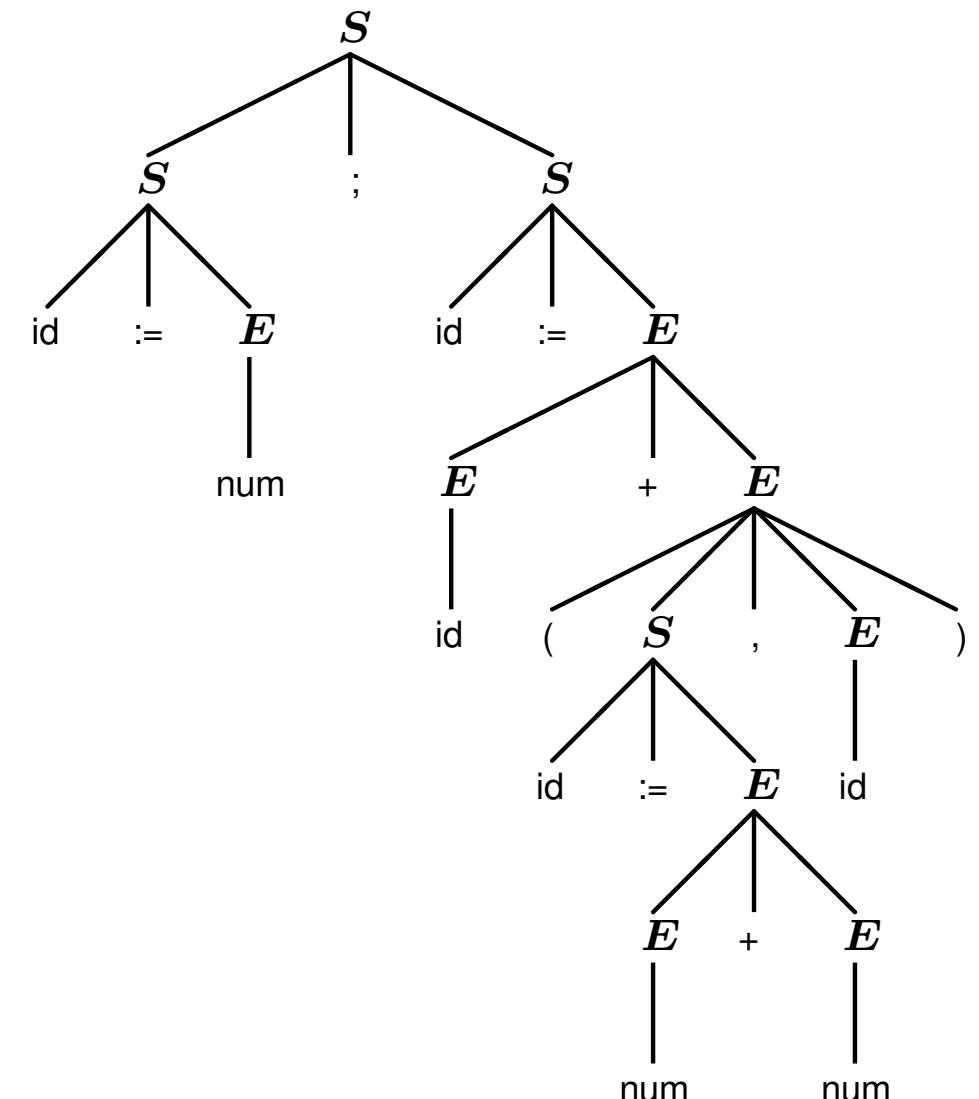
$$E \rightarrow ( S , E )$$

$$L \rightarrow E$$

$$L \rightarrow L , E$$

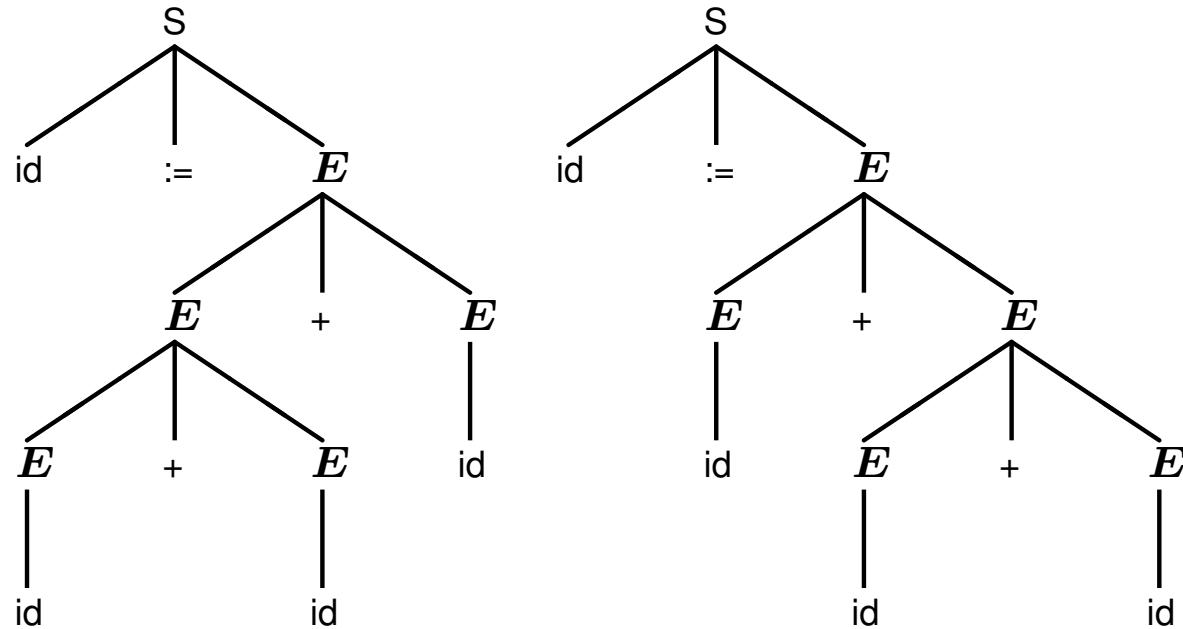
a := 7;

b := c + (d := 5 + 6, d)



A grammar is *ambiguous* if a sentence has different parse trees:

$\text{id} := \text{id} + \text{id} + \text{id}$



The above is harmless, but consider:

$\text{id} := \text{id} - \text{id} - \text{id}$

$\text{id} := \text{id} + \text{id} * \text{id}$

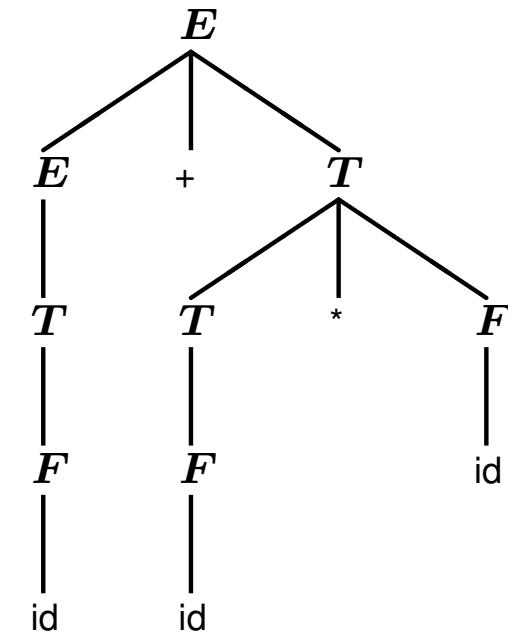
Clearly, we need to consider associativity and precedence when designing grammars.

An ambiguous grammar:

$$\begin{array}{lll}
 E \rightarrow \text{id} & E \rightarrow E / E & E \rightarrow ( E ) \\
 E \rightarrow \text{num} & E \rightarrow E + E \\
 E \rightarrow E * E & E \rightarrow E - E
 \end{array}$$

may be rewritten to become unambiguous:

$$\begin{array}{lll}
 E \rightarrow E + T & T \rightarrow T * F & F \rightarrow \text{id} \\
 E \rightarrow E - T & T \rightarrow T / F & F \rightarrow \text{num} \\
 E \rightarrow T & T \rightarrow F & F \rightarrow ( E )
 \end{array}$$



There are fundamentally two kinds of parser:

1) Top-down, *predictive* or *recursive descent* parsers. Used in all languages designed by Wirth, e.g. Pascal, Modula, and Oberon.

One can (easily) write a predictive parser by hand, or generate one from an LL( $k$ ) grammar:

- Left-to-right parse;
- Leftmost-derivation; and
- $k$  symbol lookahead.

Algorithm: look at beginning of input (up to  $k$  characters) and unambiguously expand leftmost non-terminal.

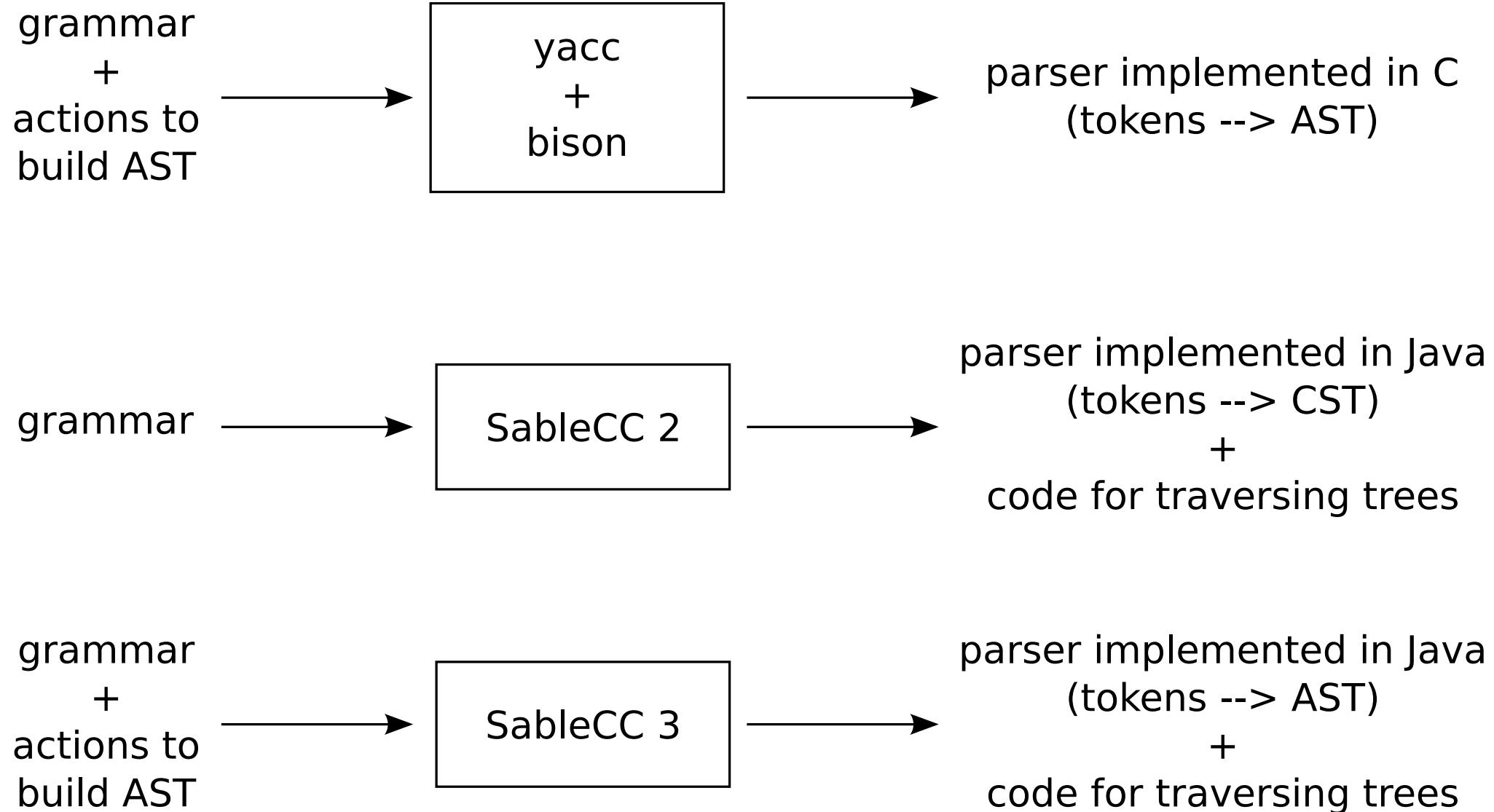
## 2) Bottom-up parsers.

Algorithm: look for a sequence matching RHS and reduce to LHS. Postpone any decision until entire RHS is seen, plus  $k$  tokens lookahead.

Can write a bottom-up parser by hand (tricky), or generate one from an LR( $k$ ) grammar (easy):

- Left-to-right parse;
- Rightmost-derivation; and
- $k$  symbol lookahead.

## LALR Parser Tools



## The ***shift-reduce*** bottom-up parsing technique.

1) Extend the grammar with an end-of-file \$, introduce fresh start symbol  $S'$ :

$$S' \rightarrow S\$$$

$$S \rightarrow S ; S \quad E \rightarrow \text{id} \quad L \rightarrow E$$

$$S \rightarrow \text{id} := E \quad E \rightarrow \text{num} \quad L \rightarrow L , E$$

$$S \rightarrow \text{print} ( L ) \quad E \rightarrow E + E$$

$$E \rightarrow ( S , E )$$

2) Choose between the following actions:

- shift:

move first input token to top of stack

- reduce:

replace  $\alpha$  on top of stack by  $X$

for some rule  $X \rightarrow \alpha$

- accept:

when  $S'$  is on the stack

	$a := 7 ;$	$b := c + (d := 5 + 6, d) \$$	shift
id	$:= 7 ;$	$b := c + (d := 5 + 6, d) \$$	shift
id :=	$7 ;$	$b := c + (d := 5 + 6, d) \$$	shift
id := num	$;$	$b := c + (d := 5 + 6, d) \$$	$E \rightarrow \text{num}$
id := $E$	$;$	$b := c + (d := 5 + 6, d) \$$	$S \rightarrow \text{id} := E$
$S$	$;$	$b := c + (d := 5 + 6, d) \$$	shift
$S ;$		$b := c + (d := 5 + 6, d) \$$	shift
$S ; id$		$:= c + (d := 5 + 6, d) \$$	shift
$S ; id :=$		$c + (d := 5 + 6, d) \$$	shift
$S ; id := id$		$+ (d := 5 + 6, d) \$$	$E \rightarrow \text{id}$
$S ; id := E$		$+ (d := 5 + 6, d) \$$	shift
$S ; id := E +$		$(d := 5 + 6, d) \$$	shift
$S ; id := E + ($		$d := 5 + 6, d) \$$	shift
$S ; id := E + ( id$		$:= 5 + 6, d) \$$	shift
$S ; id := E + ( id :=$		$5 + 6, d) \$$	shift
$S ; id := E + ( id := \text{num}$		$+ 6, d) \$$	$E \rightarrow \text{num}$
$S ; id := E + ( id := E$		$+ 6, d) \$$	shift
$S ; id := E + ( id := E +$		$6, d) \$$	shift
$S ; id := E + ( id := E + \text{num}$		$, d) \$$	$E \rightarrow \text{num}$
$S ; id := E + ( id := E + E$		$, d) \$$	$E \rightarrow E + E$

$S; id := E + ( id := E + E$	,	d) \$	$E \rightarrow E + E$
$S; id := E + ( id := E$	,	d) \$	$S \rightarrow id := E$
$S; id := E + ( S$	,	d) \$	shift
$S; id := E + ( S,$		d) \$	shift
$S; id := E + ( S, id$	)	\$	$E \rightarrow id$
$S; id := E + ( S, E$	)	\$	shift
$S; id := E + ( S, E )$		\$	$E \rightarrow (S; E)$
$S; id := E + E$		\$	$E \rightarrow E + E$
$S; id := E$		\$	$S \rightarrow id := E$
$S; S$		\$	$S \rightarrow S; S$
$S$		\$	shift
$S \$$		\$	$S' \rightarrow S \$$
$S'$			accept

$$\begin{array}{ll}
 _0 S' \rightarrow S\$ & _5 E \rightarrow \text{num} \\
 _1 S \rightarrow S ; S & _6 E \rightarrow E + E \\
 _2 S \rightarrow \text{id} := E & _7 E \rightarrow ( S , E ) \\
 _3 S \rightarrow \text{print} ( L ) & _8 L \rightarrow E \\
 _4 E \rightarrow \text{id} & _9 L \rightarrow L , E
 \end{array}$$

Use a DFA to choose the action; the stack only contains DFA states now.

Start with the initial state ( $s_1$ ) on the stack.

Lookup (stack top, next input symbol):

- $\text{shift}(n)$ : skip next input symbol and push state  $n$
- $\text{reduce}(k)$ : rule  $k$  is  $X \rightarrow \alpha$ ; pop  $|\alpha|$  times; lookup (stack top,  $X$ ) in table
- $\text{goto}(n)$ : push state  $n$
- $\text{accept}$ : report success
- $\text{error}$ : report failure

DFA state	terminals						non-terminals					
	id	num	print	;	,	+	$\coloneqq$	( )	\$	<b>S</b>	<b>E</b>	<b>L</b>
1	s4	s7								g2		
2			s3		a							
3	s4	s7								g5		
4				s6								
5			r1	r1		r1						
6	s20	s10			s8					g11		
7					s9							
8	s4	s7					g12					
9								g15	g14			
10			r5	r5	r5	r5						

DFA state	terminals							non-terminals					
	id	num	print	;	,	+	$\coloneqq$	( )	\$	<b>S</b>	<b>E</b>	<b>L</b>	
11								r2	r2	s16			r2
12								s3	s18				
13								r3	r3				r3
14								s19		s13			
15								r8		r8			
16	s20	s10								s8			g17
17						r6	r6	s16			r6	r6	
18	s20	s10								s8			g21
19	s20	s10								s8			g23
20					r4	r4	r4			r4	r4		
21										s22			
22					r7	r7	r7			r7	r7		
23								r9	s16		r9		

Error transitions omitted.

$s_1$	a := 7\$
shift(4)	
$s_1 s_4$	:= 7\$
shift(6)	
$s_1 s_4 s_6$	7\$
shift(10)	
$s_1 s_4 s_6 s_{10}$	\$
reduce(5): $E \rightarrow \text{num}$	
$s_1 s_4 s_6 / \$10$	\$
lookup( $s_6, E$ ) = goto(11)	
$s_1 s_4 s_6 s_{11}$	\$
reduce(2): $S \rightarrow \text{id} := E$	
$s_1 / \$4 / \$6 / \$11$	\$
lookup( $s_1, S$ ) = goto(2)	
$s_1 s_2$	\$
accept	

bison (yacc) is a parser generator:

- it inputs a grammar;
- it computes an LALR(1) parser table;
- it reports conflicts;
- it resolves conflicts using defaults (!); and
- it creates a C program.

Nobody writes (simple) parsers by hand anymore.

The grammar:

$$\begin{array}{lll}
 1 \ E \rightarrow \text{id} & 4 \ E \rightarrow E / E & 7 \ E \rightarrow ( E ) \\
 2 \ E \rightarrow \text{num} & 5 \ E \rightarrow E + E \\
 3 \ E \rightarrow E * E & 6 \ E \rightarrow E - E
 \end{array}$$

is expressed in bison as:

```

%{
/* C declarations */

%}

/* Bison declarations; tokens come from lexer (scanner) */
%token tIDENTIFIER tINTCONST

%start exp

/* Grammar rules after the first %% */

%%

exp : tIDENTIFIER
    | tINTCONST
    | exp '*' exp
    | exp '/' exp
    | exp '+' exp
    | exp '-' exp
    | '(' exp ')'

;
%% /* User C code after the second %% */

```

The grammar is ambiguous:

```
$ bison --verbose exp.y # --verbose produces exp.output  
exp.y contains 16 shift/reduce conflicts.
```

```
$ cat exp.output  
State 11 contains 4 shift/reduce conflicts.  
State 12 contains 4 shift/reduce conflicts.  
State 13 contains 4 shift/reduce conflicts.  
State 14 contains 4 shift/reduce conflicts.
```

[ . . . ]

## With more details about each state

state 11

exp	->	exp . '*' exp	(rule 3)
exp	->	exp '*' exp .	(rule 3) <-- problem is here
exp	->	exp . '/' exp	(rule 4)
exp	->	exp . '+' exp	(rule 5)
exp	->	exp . '-' exp	(rule 6)

' *'	shift, and go to state 6
' /'	shift, and go to state 7
' +'	shift, and go to state 8
' -'	shift, and go to state 9

' *'	[reduce using rule 3 (exp)]
' /'	[reduce using rule 3 (exp)]
' +'	[reduce using rule 3 (exp)]
' -'	[reduce using rule 3 (exp)]
\$default	reduce using rule 3 (exp)

**Rewrite the grammar to force reductions:**

$$E \rightarrow E + T \quad T \rightarrow T * F \quad F \rightarrow \text{id}$$

$$E \rightarrow E - T \quad T \rightarrow T / F \quad F \rightarrow \text{num}$$

$$E \rightarrow T \quad T \rightarrow F \quad F \rightarrow ( E )$$

```
%token tIDENTIFIER tINTCONST
```

```
%start exp
```

```
%%
```

```
exp : exp '+' term
```

```
    | exp '-' term
```

```
    | term
```

```
;
```

```
term : term '*' factor
```

```
    | term '/' factor
```

```
    | factor
```

```
;
```

```
factor : tIDENTIFIER
```

```
    | tINTCONST
```

```
    | '(' exp ')' '
```

```
;
```

```
%%
```

**Or use precedence directives:**

```
%token tIDENTIFIER tINTCONST  
%start exp  
%left '+' '-' /* left-associative, lower precedence */  
%left '*' '/' /* left-associative, higher precedence */  
  
%%  
exp : tIDENTIFIER  
    | tINTCONST  
    | exp '*' exp  
    | exp '/' exp  
    | exp '+' exp  
    | exp '-' exp  
    | '(' exp ')' ;  
%%
```

**Which resolve shift/reduce conflicts:**

Conflict in state 11 between rule 5 and token '+'  
resolved as reduce. <-- Reduce exp + exp . +

Conflict in state 11 between rule 5 and token '-'  
resolved as reduce. <-- Reduce exp + exp . -

Conflict in state 11 between rule 5 and token '\*'  
resolved as shift. <-- Shift exp + exp . \*

Conflict in state 11 between rule 5 and token '/'  
resolved as shift. <-- Shift exp + exp . /

Note that this is not the same state 11 as before.

The precedence directives are:

- `%left` (*left-associative*)
- `%right` (*right-associative*)
- `%nonassoc` (*non-associative*)

When constructing a parse table, an action is chosen based on the precedence of the last symbol on the right-hand side of the rule.

Precedences are ordered from lowest to highest on a linewise basis.

If precedences are equal, then:

- `%left` favors reducing
- `%right` favors shifting
- `%nonassoc` yields an error

This usually ends up working.

```
state 0
    tIDENTIFIER shift, and go to state 1
    tINTCONST    shift, and go to state 2
    '('         shift, and go to state 3
    exp         go to state 4

state 1
    exp -> tIDENTIFIER .   (rule 1)
    $default    reduce using rule 1 (exp)

state 2
    exp -> tINTCONST .   (rule 2)
    $default    reduce using rule 2 (exp)

...
state 14
    exp -> exp . '*' exp   (rule 3)
    exp -> exp . '/' exp   (rule 4)
    exp -> exp '/' exp .  (rule 4)
    exp -> exp . '+' exp   (rule 5)
    exp -> exp . '-' exp   (rule 6)
    $default    reduce using rule 4 (exp)

state 15
    $          go to state 16

state 16
    $default    accept
```

```
$ cat exp.y
```

```
%{  
#include <stdio.h> /* for printf */  
extern char *yytext; /* string from scanner */  
void yyerror() { printf ("syntax error before %s\n", yytext); }  
%}  
%union {  
    int intconst;  
    char *stringconst;  
}  
%token <intconst> tINTCONST  
%token <stringconst> tIDENTIFIER  
%start exp  
%left '+' '-'  
%left '*' '/'  
%%  
exp : tIDENTIFIER { printf ("load %s\n", $1); }  
    | tINTCONST   { printf ("push %i\n", $1); }  
    | exp '*' exp { printf ("mult\n"); }  
    | exp '/' exp { printf ("div\n"); }  
    | exp '+' exp { printf ("plus\n"); }  
    | exp '-' exp { printf ("minus\n"); }  
    | '(' exp ')' { }  
;  
%%
```

```
$ cat exp.l
%{
#include "y.tab.h" /* for exp.y types */
#include <string.h> /* for strlen */
#include <stdlib.h> /* for malloc and atoi */
%}
%%
[ \t\n]+ /* ignore */;
"*"      return '*';
"/"      return '/';
"+"      return '+';
"-"      return '-';
"("      return '(';
")"      return ')';
0|([1-9][0-9]*) {
    yyval.intconst = atoi (yytext);
    return tINTCONST;
}
[a-zA-Z_][a-zA-Z0-9_]* {
    yyval.stringconst =
        (char *) malloc (strlen (yytext) + 1);
    sprintf (yyval.stringconst, "%s", yytext);
    return tIDENTIFIER;
}
.      /* ignore */
%%
```

```
$ cat main.c
void yyparse();
int main (void)
{
    yyparse ();
}
```

Using flex/bison to create a parser is simple:

```
$ flex exp.l
$ bison --yacc --defines exp.y # note compatibility options
$ gcc lex.yy.c y.tab.c y.tab.h main.c -o exp -lfl
```

When input  $a * (b - 17) + 5 / c$ :

```
$ echo "a * (b - 17) + 5 / c" | ./exp
```

our exp parser outputs the correct order of operations:

```
load a  
load b  
push 17  
minus  
mult  
push 5  
load c  
div  
plus
```

You should confirm this for yourself!

If the input contains syntax errors, then the bison-generated parser calls `yyerror` and stops.

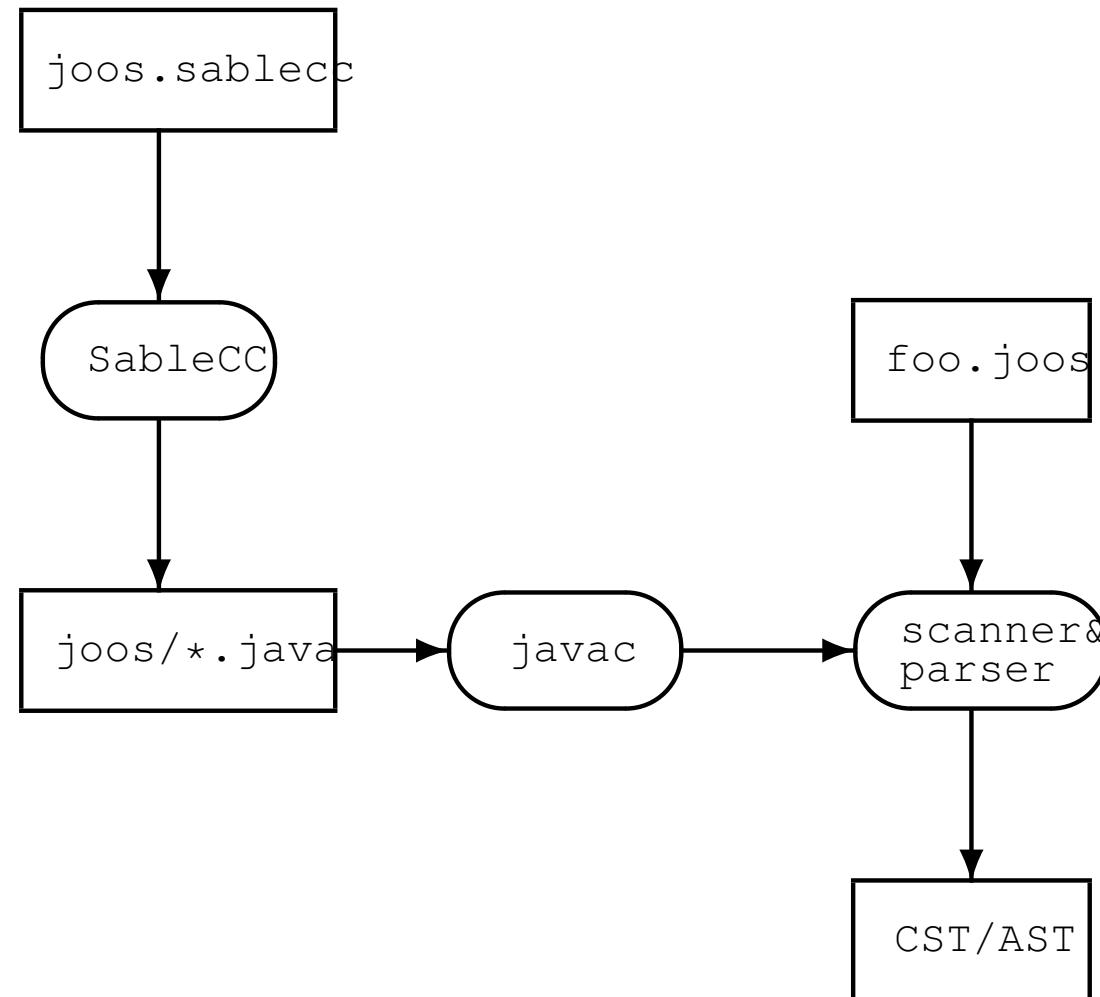
We may ask it to recover from the error:

```
exp : tIDENTIFIER { printf ("load %s\n", $1); }
...
| '(' exp ')'
| error { yyerror(); }
;
```

and on input `a@(b-17) ++ 5/c` get the output:

```
load a
syntax error before (
syntax error before (
syntax error before (
syntax error before b
push 17
minus
syntax error before )
syntax error before )
syntax error before +
plus
push 5
load c
div
plus
```

SableCC (by Etienne Gagnon, McGill alumnus) is a *compiler compiler*: it takes a grammatical description of the source language as input, and generates a lexer (scanner) and parser for it.



## The SableCC 2 grammar for our Tiny language:

```
Package tiny;  
Helpers  
    tab    = 9;  
    cr     = 13;  
    lf     = 10;  
    digit = ['0'...'9'];  
    lowercase = ['a'...'z'];  
    uppercase = ['A'...'Z'];  
    letter = lowercase | uppercase;  
    idletter = letter | '_';  
    idchar  = letter | '_' | digit;  
  
Tokens  
    eol    = cr | lf | cr lf;  
    blank = ' ' | tab;  
    star   = '*';  
    slash  = '/';  
    plus   = '+';  
    minus  = '-';  
    l_par  = '(';  
    r_par  = ')';  
    number = '0' | [digit-'0'] digit*;  
    id     = idletter idchar*;  
  
Ignored Tokens  
    blank, eol;
```

## Productions

```
exp =
  {plus}    exp plus factor |
  {minus}   exp minus factor |
  {factor}  factor;
```

```
factor =
  {mult}    factor star term |
  {divd}    factor slash term |
  {term}   term;
```

```
term =
  {paren}   l_par exp r_par |
  {id}      id |
  {number}  number;
```

Version 2 produces parse trees, a.k.a. concrete syntax trees (CSTs).

## The SableCC 3 grammar for our Tiny language:

Productions

```
cst_exp {-> exp} =
  {cst_plus}    cst_exp plus factor
                {-> New exp.plus(cst_exp.exp, factor.exp) } |
  {cst_minus}   cst_exp minus factor
                {-> New exp.minus(cst_exp.exp, factor.exp) } |
  {factor}       factor {-> factor.exp};
```

```
factor {-> exp} =
  {cst_mult}    factor star term
                {-> New exp.mult(factor.exp, term.exp) } |
  {cst_divd}    factor slash term
                {-> New exp.divd(factor.exp, term.exp) } |
  {term}         term {-> term.exp};
```

```
term {-> exp} =
  {paren}        l_par cst_exp r_par {-> cst_exp.exp} |
  {cst_id}       id {-> New exp.id(id) } |
  {cst_number}   number {-> New exp.number(number) };
```

## Abstract Syntax Tree

```
exp =  
  {plus}      [l]:exp [r]:exp |  
  {minus}     [l]:exp [r]:exp |  
  {mult}      [l]:exp [r]:exp |  
  {divd}      [l]:exp [r]:exp |  
  {id}         id |  
  {number}    number;
```

Version 3 generates abstract syntax trees (ASTs).