

CS 252B Assignment 4

Due at start of class on Tuesday March 22nd

1. *Working with adjacency matrices.* A node s of a directed graph $G = (V, E)$ is called a *sink* if for every node $v \in V$, $v \neq s$, the edge (v, s) exists, whereas the edge (s, v) does not exist. Write an algorithm that can detect the presence of a sink in G in a (worst-case) time $\mathcal{O}(n)$, where $n = |V|$. Your algorithm should accept the graph represented by its adjacency matrix. Notice that a running time of $\mathcal{O}(n)$ is remarkable, given that the instance takes a space $\Omega(n^2)$ merely to write down.
2. *MSTs and edges of negative weight.* The problem of finding a subset T of the edges of a connected graph G such that all the nodes remain connected when only the edges in T are used, and the sum of the weights of the edges in T is as small as possible, still makes sense even if G may have edges with negative weights. However, the solution may no longer be a tree. Adapt either Kruskal's algorithm or Prim's algorithm to work on a graph that may include edges of negative lengths.
3. *Bottle-neck spanning trees.* Exercise 23-3, part *a* only, on page 577 of CLRS.
4. *Slot-size bound for chaining.* Exercise 11-2, parts *a, b, c* and *e*, on page 250 of CLRS. Part *d* has been modified and is now a bonus question (question 7). If you can't solve one part of the question, you can still assume that what you had to show is true, and use that to solve the other parts. In particular, you need the result from question 7 to solve part *e*.
5. *The cost of sending the Huffman tree along with the encoding of a text.* Exercise 16.3-5, on page 392 of CLRS.
6. *Proving your instructor wrong.* I said in class that the greedy algorithm we saw for making change would always find the least possible amount of coins needed to pay any given amount, as long as we had a coinage in which every coin is worth at least twice as much as the coin with the next lower value. Find a coinage that has this "twice-as-much" property, and that includes a 1-unit coin, but such that the greedy algorithm sometimes fails to find an optimal solution. (Hint : A small modification to the Canadian coinage will do the trick...)
7. Bonus Question! (This is a slightly modified version of Exercise 11-2, part *d* of CLRS. You should use the definitions and results from parts *a, b* and *c*.) We want to find a "cutoff" value k_0 such that $P_k = \Pr\{M = k\}$ is small for

any $k > k_0$. First show that for n large enough, there exists a constant $c > 1$ such that $Q_{k_0} < 1/n^3$, where $k_0 = c \log n / \log \log n$. Next, show that Q_k is a decreasing function of k . Conclude that $P_k < 1/n^2$ for any $k > k_0$, where again $k_0 = c \log n / \log \log n$.