Complete quantum teleportation by nuclear magnetic resonance

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Quantum mechanics provides spectacular new information processing abilities[1,2]. One of the most unexpected is a procedure called *quantum teleportation*[3] that allows the quantum state of a system to be transported from one location to another, without moving through the intervening space. Partial implementations of teleportation[4,5] over macroscopic distances have been achieved using optical systems,

but omit the final stage of the teleportation procedure. Here we report an experimental implementation of the full quantum teleportation operation over inter-atomic distances using liquid state nuclear magnetic resonance (NMR). The inclusion of the final stage enables for the first time a teleportation implementation which may be used as a *subroutine* in larger quantum computations, or for quantum communication. Our experiment also demonstrates the use of *quantum process tomography*, a procedure to completely characterize the dynamics of a quantum system. Finally, we demonstrate a controlled exploitation of decoherence as a tool to assist in the performance of an experiment.

In classical physics, an object can be teleported, in principle, by performing a measurement to completely characterize the properties of the object. That information can then be sent to another location, and the object reconstructed. Does this provide a complete reconstruction of the original object? No: all physical systems are ultimately quantum mechanical, and quantum mechanics tells us that it is impossible to completely determine the state of an unknown quantum system, making it impossible to use the classical measurement procedure to move a quantum system from one location to another.

Bennett *et al*[3] have suggested a remarkable procedure for teleporting quantum states. Quantum teleportation may be described abstractly in terms of two parties, Alice and Bob. Alice has in her possession an *unknown* state $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ of a single quantum bit (qubit) – a two level quantum system. The goal of teleportation is to transport the state of that qubit to Bob. In addition, Alice and Bob each possess one qubit of a two qubit entangled state,

$$|\Psi\rangle_A \left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right), \tag{1}$$

where subscripts A are used to denote Alice's systems, and subscripts B to denote Bob's system. Here and throughout we omit overall normalization factors from our equations.

This state can be rewritten in the *Bell basis* $((|00\rangle \pm |11\rangle), (|01\rangle \pm |10\rangle))$ for the first two qubits and a conditional unitary transformation of the state $|\Psi\rangle$ for the last one, that is,

$$\begin{aligned}&(|00\rangle + |11\rangle)\,|\Psi\rangle + (|00\rangle - |11\rangle)\,\sigma_z|\Psi\rangle + \\ &(|01\rangle + |10\rangle)\,\sigma_x|\Psi\rangle + (|01\rangle - |10\rangle)\,(-i\sigma_y|\Psi\rangle)\,,\end{aligned} \tag{2}$$

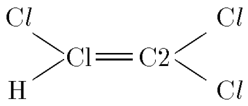
where $\sigma_x, \sigma_y, \sigma_z$ are the Pauli sigma operators [6], in the $|0\rangle, |1\rangle$ basis. A measurement is performed on Alice's qubits in the Bell basis. Conditional on these measurement outcomes it is easy to verify from the previous equation that Bob's respective states are

$$|\Psi\rangle; \quad \sigma_z|\Psi\rangle; \quad \sigma_x|\Psi\rangle; \quad -i\sigma_y|\Psi\rangle. \tag{3}$$

Alice sends the outcome of her measurement to Bob, who can then recover the original state $|\Psi\rangle$ by applying the appropriate unitary transformation I, σ_x, σ_z, or $i\sigma_y$, conditional on Alice's measurement outcome. Notice that the quantum state transmission has not been accomplished faster than light because Bob must wait for Alice's measurement result to arrive before he can recover the quantum state.

Recent demonstrations of quantum teleportation [4,5] omitted the final stage of teleportation, the unitary operators applied by Bob conditional on the result of Alice's measurement. This prevents complete recovery of the original state. Instead, the earlier experiments relied on classical post-processing of the data after completion of the experiment to check that the results were consistent with what one would expect *if the conditional operations had, in fact, been performed.* Our experiment implements the full teleportation operation. The most important implication of the inclusion of this difficult extra stage is that our teleportation procedure can, in principle, be used as a *subroutine* in the performance of other quantum information processing tasks. Teleportation as a subroutine is important in potential applications to quantum computation and communication[7,8], although in the present system, moving the C2 qubit to H may be accomplished more efficiently by techniques other than teleportation.

Our implementation of teleportation is performed using liquid state nuclear magnetic resonance (NMR), applied to an ensemble of molecules of labeled trichloroethylene (TCE). The structure of the TCE molecule may be depicted as



(4)

To perform teleportation we make use of the Hydrogen nucleus (H), and the two Carbon 13 nuclei (C1 and C2), teleporting the state of C2 to H. Figure 1(a) illustrates the teleportation process we used. The circuit has three inputs, which we will refer to as the *data* (C2), *ancilla* (C1), and *target* (H) qubits. The goal of the circuit is to teleport the state of the data qubit so that it ends up on the target qubit. We are therefore only teleporting the qubit a few angstroms, making this a demonstration of the method of teleportation, rather than a practical means for transmitting qubits over long distances.

State preparation is done in our experiment using the gradient-pulse techniques described by Cory *et al*[9], and phase cycling[10,11]. The unitary operations performed during teleportation may be implemented in a straightforward manner in NMR, using non-selective rf pulses tuned to the Larmor frequencies of the nuclear spins, and delays allowing entanglement to form through the interaction of neighboring nuclei[9,12]. Other demonstrations of quantum information processing with three qubits using NMR are described in[14,15,16], and with two qubits in [17,18,19,20].

An innovation in our experiment was the method used to implement the Bell basis measurement. In NMR, the measurement step allows us to measure the expectation values of σ_x and σ_y for each spin, averaged over the ensemble of molecules, rather than performing a projective measurement in some basis. For this reason, we must modify the projective measurement step in the standard description of teleportation, while preserving the remarkable teleportation effect.

We use a procedure inspired by Brassard *et al* [21], who suggest a two-part procedure for performing the Bell basis measurement. Part one of the procedure is to rotate from the Bell basis into the computational basis, $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. We implement this step in NMR by using the natural spin-spin coupling between the Carbon nuclei, and rf pulses. Part two of the procedure is to perform a projective measurement in the computational basis. As Brassard *et al* point out, the effect of this two part procedure is equivalent to performing the Bell basis measurement, and leaving the data and ancilla qubits in one of the four states, $|00\rangle, |01\rangle, |10\rangle, |11\rangle$, corresponding to the different measurement results.

We cannot directly implement the second step in NMR. Instead, we exploit the natural phase decoherence occurring on the Carbon nuclei to achieve the same effect. Recall that phase decoherence completely randomizes the phase information in these nuclei and thus will destroy coherence between the elements of the above basis. Its effect on the state of the Carbon nuclei is to diagonalize the state in the computational basis,

$$\begin{aligned} \rho \quad \longrightarrow \quad & |00\rangle\langle 00|\rho|00\rangle\langle 00| + |01\rangle\langle 01|\rho|01\rangle\langle 01| + |10\rangle\langle 10|\rho|10\rangle\langle 10| \\ & +|11\rangle\langle 11|\rho|11\rangle\langle 11|. \end{aligned} \tag{5}$$

As emphasized by Zurek [22], the decoherence process is indistinguishable from a measurement in the computational basis for the Carbons accomplished by the environment. We do not observe the result of this measurement explicitly, however the state of the nuclei selected by the decoherence process contains the measurement result, and therefore we can do the final transformation conditional on the particular state the environment has selected. As in the scheme of Brassard *et al*, the final state of the Carbon nuclei is one of the four states, $|00\rangle, |01\rangle, |10\rangle, |11\rangle$, corresponding to the four possible results of the measurement.

Our experiment exploits the natural decoherence properties of the TCE molecule. The phase decoherence times (T_2) for C1 and C2 are approximately $0.4s$ and $0.3s$. All other T_2 and T_1 times for all three nuclei are much longer, with a T_2 time for the Hydrogen of approximately $3s$, and relaxation times (T_1) of approximately $20 - 30s$ for the Carbons, and $5s$ for the Hydrogen.

Thus, for delays on the order of $1s$, we can approximate the total evolution by exact phase decoherence on the Carbon nuclei. The total scheme therefore implements a measurement in the Bell basis, with the result of the measurement stored as classical data on the Carbon nuclei following the measurement. We can thus teleport the state from the Carbon to the Hydrogen and verify that the state in the final state decays at the Hydrogen rate and not the Carbon rate.

Re-examining figure 1(a) we see how remarkable teleportation is from this point of view. During the stage labeled "Measure in the Bell basis" in figure 1(a), we allow the C1 and C2 nuclei to decohere and thus be measured by the environment, destroying all phase information on the data and ancilla qubits. Experimentally, a standard NMR technique known as *refocusing* employs rf pulses to ensure that the data qubit effectively does not interact with the target qubit. Classical intuition therefore tells us that the phase information about the input state, $|\Psi\rangle$, has been lost forever. Nevertheless, quantum mechanics predicts that we are still able to recover the complete system after this decoherence step, by quantum teleportation.

We implemented this scheme in TCE using a Bruker DRX-500 NMR spectrometer. Experimentally, we determined the Larmor and coupling frequencies for the Hydrogen, C1 and C2 to be:

$$\omega_H \approx 500.133491\text{MHz}; \qquad \omega_{C1} \approx 125.772580\text{MHz}; \qquad \omega_{C2} \approx \omega_{C1} - 911\text{Hz} \tag{6}$$

$$J_{H\,C1} \approx 201\text{Hz}; \qquad J_{C1\,C2} \approx 103\text{Hz}. \tag{7}$$

The coupling frequencies between H and C2, as well as the Chlorines to H, C1 and C2, are much lower, on the order of ten Hertz for the former, and less than a Hertz for the latter. Experimentally, these couplings are suppressed by multiple refocusings, and will be ignored in the sequel. Note that C1 and C2 have slightly different frequencies, due to the different chemical environments of the two atoms.

We performed two separate sets of experiments. In one set, the full teleportation process was executed, making use of a variety of decoherence delays in place of the measurement. The readout was performed on the Hydrogen nucleus, and a figure of merit – the entanglement fidelity – was calculated for the teleportation process. The entanglement fidelity is a quantity in the range 0 to 1 which measures the combined strength of *all* noise processes occurring during the process [23,24]. In particular, an entanglement fidelity of 1 indicates perfect teleportation, while an entanglement fidelity of 0.25 indicates total randomization of the state. Perfect *classical transmission* corresponds to an entanglement fidelity of 0.5 [23,24], so entanglement fidelities greater than 0.5 indicates that teleportation of some quantum information is taking place.

The second set of experiments was a control set, illustrated in figure 1(b). In those experiments, only the state preparation and initial entanglement of H and C1 were performed, followed by a delay for decoherence on C1 and C2. The readout was performed in this instance on C2, and the entanglement fidelity was calculated for the process.

The results of our experiment are shown in figure 2, where the entanglement fidelity is plotted against the decoherence delay. Errors in our experiment arise from the strong coupling effect, imperfect calibration of rf pulses, and rf field inhomogeneities. The estimated uncertainties in the entanglement fidelities are less than ± 0.05, and are due primarily to rf field inhomogeneity and imperfect calibration of rf pulses.

To determine the entanglement fidelities for the teleportation and control experiments, we performed *quantum process tomography* [25,26], a procedure for obtaining a complete description of the dynamics of a quantum system, as follows: The linearity of quantum mechanics implies that the single qubit input and output for the teleportation process are related by a linear quantum operation [27]. By preparing a complete set of four linearly independent initial states, and measuring the corresponding states output from the experiment, we may completely characterize the quantum process, enabling us to calculate the entanglement fidelity for the process [26].

Three elements ought to be noted in figure 2. First, for small decoherence delays, the entanglement fidelity for the teleportation experiments significantly exceeds the value of 0.5 for perfect classical transmission of data, indicating successful teleportation of quantum information from C2 to H, with reasonable fidelity. Second, the entanglement fidelity decays very quickly for the control experiments as the delay is increased. Theoretically, we expect this to be the case, due to a $T2$ time for C2 of approximately $0.3s$. Third, the decay of the entanglement fidelity for the teleportation experiments occurs much more slowly. Theoretically, we expect this decay to be due mainly to the effect of phase decoherence and relaxation on the *Hydrogen*. Our experimental observations are consistent with this prediction, and provide more support for the claim that quantum data is being teleported in these experiments.

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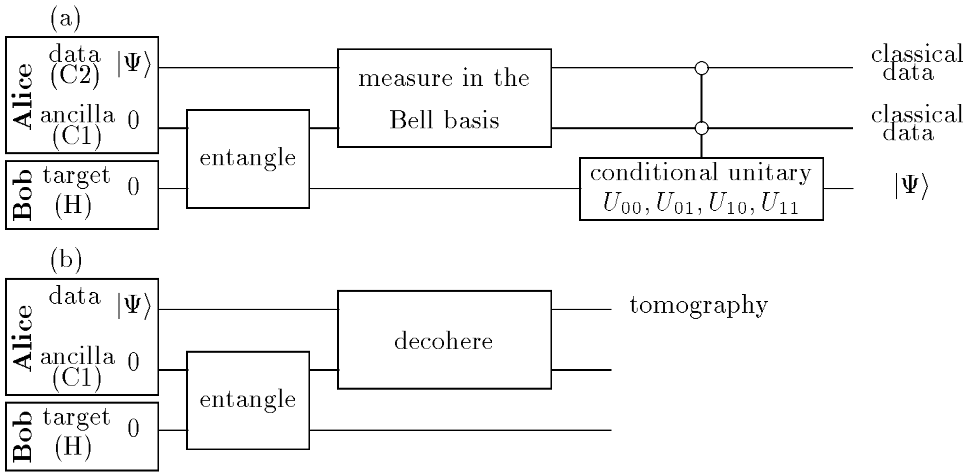
Figure Captions

Figure 1: Schematic circuits for (a) the quantum teleportation experiment, and (b) the control experiment. The teleportation circuit is based upon that suggested by Brassard *et al* [21]. Note that the control circuit simply omits two elements of the teleportation experiment – rotation from the Bell basis into the computational basis, immediately before the decoherence step, and the conditional unitary operation. Commented pulse sequences for our experiment may be obtained on the world wide web [13].

Figure 2: Entanglement fidelity (a measure of how well quantum information is preserved) is plotted as a function of decoherence time. The bottom curve is a control run where the information remains in C2. The curve shows a decay time of approximately 0.5s. The top curve represents the fidelity of the quantum teleportation process. The decay time is approximately 2.6s. The information is preserved for a longer time, corresponding approximately to the combined effects of decoherence and relaxation for the Hydrogen, confirming the prediction of teleportation.



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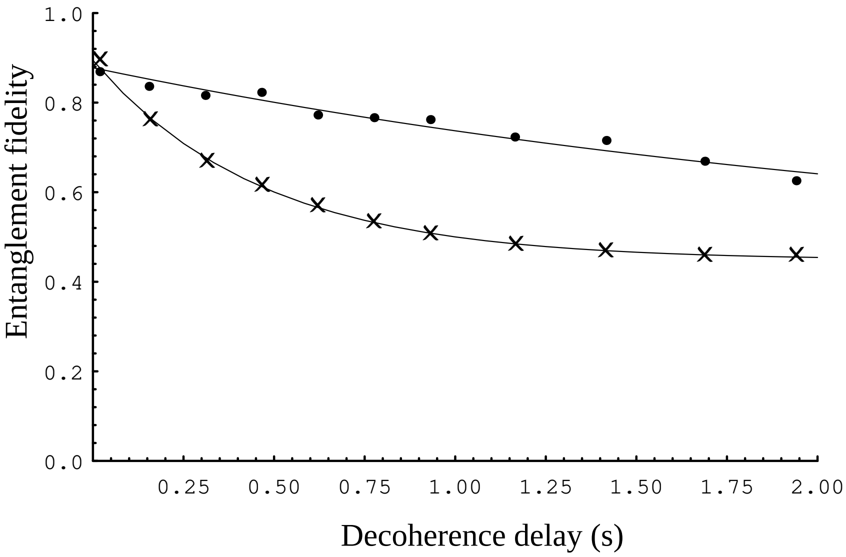

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