Generalized multiperiod MIP model for production scheduling and processing facilities selection and location

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Abstract – *Generally, production planning is concerned with the best use of fixed resources to determine a production schedule for one or more production units supplying one or more markets. Production scheduling determines when each production unit will be used and its production level. However, the production scheduling problems are intimately associated with the location of existing or new production units and processing facilities. Normally, mine location allows only a limited choice; processing plant types and locations are considerably more flexible. This paper presents a generalized multiperiod MIP model as an aid in determining the mine production schedule and the processing plant types and location to satisfy multiple markets. A brief description of MIP and how it has been applied in the mineral and other industries is discussed. A complete description of the model with an application to a coal preparation plant system is presented.*

Introduction

In the mineral industry, the objective of a mining venture is to extract, process, and market a mineral commodity in such a manner that the profit of the venture is maximized. In a mining venture, a company normally has control over mine production scheduling, selection and location of processing facilities (mineral beneficiation plants, smelting plants, refineries, or blending facilities), processed ore blending, and the market selection. The flexibility of the above factors provides many alternatives to satisfy the needs of each market with regard to quantity and quality of ore. This paper presents a mathematical model for determining the alternative that yields the maximum profit.

Production planning is generally concerned with the best use of fixed resources, more specifically, with determining production and inventory levels to meet fluctuating demand requirements. A mine can be any source of ore, such as an entire mine, a production unit within a mine, or an independent supplier. The objective of production scheduling is to determine the times when each mine will be used and to allocate a required level of production. Production planning and scheduling problems are intimately associated with the location of new and existing mines and processing facilities. Although the location of the mines is usually fixed, there can be flexibility in locating new processing facilities, in selecting the processing plant, and in blending the ore before and after processing.

The production scheduling, facility location, processing plant selection, and blending problems have traditionally been resolved through experience, intuition, and judgment. However, it is too difficult and time consuming to evaluate every available alternative. Hence, the first alternative that satisfies the required goals is usually chosen. This alternative may or may not be close to optimal. A better solution technique is required to solve these problems. Due to the nature of the problem, this can best be developed through mathematical modeling.

In the past, linear programming has been used extensively in mining to optimize production scheduling and blending problems (Manula, 1966; Kim, 1967; Gezik, 1967; Ramani, 1970; Johnson, 1969; and Janssen, 1969). However, linear programming is not suited to solving facility location and processing plant assignment problems because these are yes/no decisions. The yes/no decisions must be represented by integer variables, in particular, by zero-one variables. Linear programming allows all solution variables to assume any nonnegative value, and as such it cannot force variables to be integer valued. As a result, linear mixed-integer programming (MIP) must be used to solve the facility location and processing plant assignment along with the production scheduling, blending, and transportation problems.

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In this paper, a multiperiod MIP model is presented that optimizes (1) production schedules for multiple mines, (2) locations of new processing facilities, (3) type of processing facility for each processing site, (4) processed ore blending schedule for each processing facility, and (5) market selection. The model optimizes these by maximizing the total profit before taxes while considering all revenue from delivered ore and the costs associated with production, transportation, blending, processing, and waste disposal.

The model is designed specifically for those involved in production scheduling or purchasing decisions. This includes executives and planners responsible for selecting and locating processing facilities and for scheduling the production and blending of ore.

Review of linear mixed-integer programming models

A linear mixed-integer programming (MIP) model is a subset of the general class called mathematical programming models. In mathematical programming, the variables are constrained to be continuous or integer valued. The relationships between the variables can be linear or nonlinear. The best solution to the problem is obtained by maximizing or minimizing a function of the variables. A linear MIP model is, therefore, a mathematical representation of a system where all constraints are linear and the variables can be continuous or integer valued. The general mathematical structure of an MIP model with M constraints and N variables is:

$$\text{Optimize:} \quad Z = \Sigma C_j X_j$$

Subject to:

$$\sum_j A_{ij} X_j \left(\begin{smallmatrix} \geq \\ = \\ \leq \end{smallmatrix} \right) B_i, \qquad i = 1,2,\ldots M$$

$$X_j \geq 0, \qquad j = 1,2,\ldots N$$

$$X_j = 0 \text{ or } 1, \text{ for some } j\text{'s}$$

where C_j, B_i, and A_{ij} are known coefficients, and X_j are unknown variables.

Several solution methods have been proposed for MIP problems. Two methods are the cutting plane approach and the branch and bound approach. The branch and bound method is more efficient for large problems and is consequently the most common method used in MIP computer solution codes.

MIP applications

Mixed-integer programming has been applied to many types of problems including plant location, scheduling, transportation, knapsack, traveling salesman, assignment, and others. MIP applications in industries other than mining are numerous.

Balinski (1964) was among the first to formulate the discrete plant location problem as a mixed-integer model. Bishwal, Sahu, and Chowdhury (1971) applied MIP to determine the optimal location of a chemical plant in India. Dee, Norbert, and Liebman (1972) used MIP to solve the problem of locating such public facilities as hospitals, police stations, and fire companies. Bednar and Strohmeir (1979) used MIP to determine the location of warehouses and assign consumer zones to the warehouses based on transportation costs. Noonan and Giglio (1977) used an MIP model to determine the optimal electric power generating plant mix and size. Austin and Hogan (1976) used MIP to determine a minimum acceptable quantity for the suppliers of aviation fuel. Devine (1973) minimized the cost allocation of oil deposits to single and dual completion oil wells.

MIP applications in mining are not extensive. Lambert and Mutmansky (1973) used pure integer programming to model the truck and shovel assignment problem in an open-pit mine. Daud and Pariseau (1975) coupled integer programming with simulation to solve the same truck and shovel assignment problem. Two papers discussed the application of MIP to locating coal blending facilities that receive coal from several sources (Ravindran, Bailey, and Hanline, 1976; Ravindran and Hanline, 1980). Finally, various mathematical model formulations were developed for optimizing coal purchasing by Kinston, Mancini, and Wright (1981). They applied MIP to handle two situations: modeling the yes/no decision for installing a scrubber, and modeling discrete quantities of coal shipments such as unit train loads.

Model formulation

General description

A diagram of the modeled flow of ore from the mines to the processing plants to the markets is shown in Fig. 1. This figure shows that (1) the ore produced by any mine can be shipped to any processing facility, (2) any processing facility can be located at any site, and (3) the processed ore can be shipped to any market.

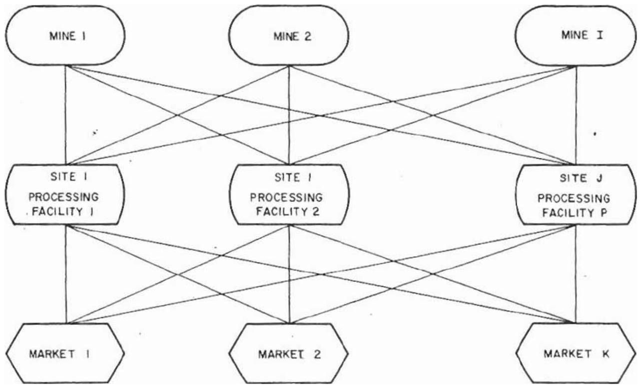

Fig. 1 — Modeled flow of ore from the mines to the processing sites to the markets

Figure 2 shows the modeled flow of ore through the processing facility. For a given processing facility, the ore from all mines is split into various processing streams by size separation, and processed by size and density separation. The processed ore is then blended with other processing streams and shipped to the market. The waste material is shipped to a disposal area.

The goal of the model is to determine for each time period: the amount of ore produced by each mine and shipped to each processing facility; the amount of processed ore produced by each processing stream in each processing facility and shipped to each market; the processed ore blending schedule; the types of processing facilities to be used and their locations; and the selection of markets. This is carried out so that the profit is maximized.

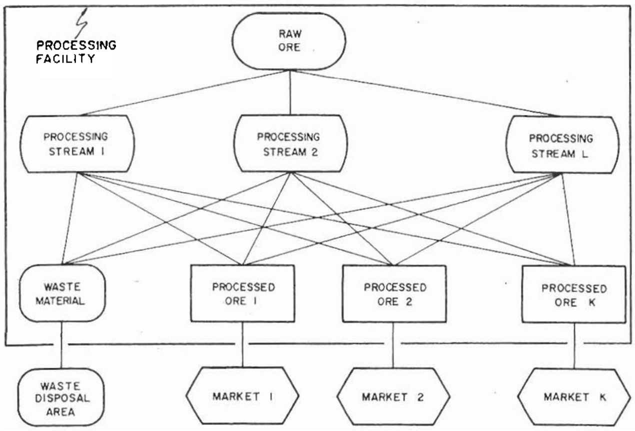

Fig. 2 — Modeled flow of ore through a processing facility

To represent reality accurately, the model considers the following:

- Fixed and variable production costs for each mine;
- Production capacity for each mine;
- Minimum acceptable production levels for each mine;
- Ore quality for each mine;
- Processing characteristics for each mine;
- Transportation cost from each mine to each processing facility;
- Ore blending;
- Fixed and variable processing costs for each processing facility;
- Performance for each processing facility;
- Capacity for each processing facility;
- Waste disposal cost at each processing facility site;
- Capital cost of the processing facility;
- Quantity of ore demanded at each market;
- Upper and lower limits for each attribute at each market;
- Upper limit for the number of mines, processing facility sites, processing facilities, and markets; and
- The fact that certain mines, processing facilities and sites, and markets must be used.

The model is general enough to enable the optimization of a wide variety of mining problems involving one or more of the following: production scheduling, processing facilities selection and location, processed ore blending, and selection of markets. This model can be applied to a single- or multiple-period problem consisting of several mines supplying ore to several markets with intermediate blending or processing.

Mathematical model

Notation: The notations used throughout this paper for subscripts, variables, and constants are shown in Tables 1, 2, and 3, respectively.

Table 1 — Subscript Definition

Subscript	Definition
i	mine
j	processing facility site
p	processing facility type
ℓ	processing stream
k	market
t	time period
q	attribute

Table 2 — Model Variables

Variable	Definition
$F_{ijp\ell kt}$	quantity of ore shipped from mine i to processing facility p, processed in stream ℓ, and sent to market k in time period t
X_{it}	= 1, if mine i produces in time period t = 0, otherwise
W_j	= 1, if a processing facility is located at site j = 0, otherwise
Y_{jp}	= 1, if processing facility p is located at site j = 0, otherwise
Z_{kt}	= 1, if market k is satisfied in time period t = 0, otherwise

Table 3 — Model Constants

Constant	Definition
$DISFAC_t$	the discount factor for time t accounting for both inflation and rate of return
REV_{kt}	selling price of the ore delivered to market k in time period t
C_{it}	cost of production at mine i in time period t
$SC1_{ijt}$	shipping cost from mine i to site j in time period t
$SC2_{jkt}$	shipping cost from site j to market k in time period t
FCM_{it}	fixed cost for mine i in time period t
FCS_{jt}	fixed cost for site j in time period t
$FCPF_{pt}$	fixed cost for processing facility p in time period t
$FCMK_{kt}$	fixed cost for market k in time period t
$PPOC_{p\ell t}$	operating cost for processing stream ℓ of processing facility p in time period t
WDC_{jt}	waste disposal cost for site j in time period t
$FRAC_{ip\ell t}$	fraction of ore from mine i that is processed by facility p that reports to stream ℓ in time period t
$R_{ip\ell t}$	recovery factor of ore from mine i that is processed by processing stream ℓ in processing facility p in time period t
$QUAL_{ip\ell tq}$	quality for attribute q of ore from mine i in time period t after it is processed in stream ℓ of processing facility p
$QUAN_{kt}$	quantity of processed ore demanded by market k in time period t
LBA_{ktq}	lower quality bound for attribute q at market k in time period t
UBA_{ktq}	upper quality bound for attribute q at market k in time period t
LBF_{it}	lower production bound (minimum acceptable production) for mine i in time period t
UBF_{it}	upper production bound (capacity) for mine i in time period t
$CAP_{p\ell}$	upper processing bound (capacity) for processing stream ℓ of facility p
NM_t	maximum number of mines desired in the system in time period t
NS	maximum number of processing sites desired in the system
NPF_j	maximum number of processing facilities to be located at site j
$MAXK_{kt}$	maximum number of markets to be satisfied in time period t

Objectives: The objective is to maximize the total discounted net profit before taxes of the system.

$$\text{Maximize:} \quad Z = \sum_t Z_t \, DISFAC_t$$

where,

$$Z_t = \sum_k QUAN_{kt} \, REV_{kt} \, Z_{kt}$$ revenue for delivered ore

$$- \sum_{i}\sum_{j}\sum_{p}\sum_{\ell}\sum_{k} F_{ijp\ell kt} \, C_{it}$$ production cost per ton

$$- \sum_i FCM_{it} \, X_{it}$$ fixed cost for each production unit

$$- \sum_j FCS_{jt} \, W_j$$ fixed cost for each processing site

$$- \sum_{i}\sum_{j}\sum_{p}\sum_{\ell}\sum_{k} F_{ijp\ell kt} \, SC1_{ijt}$$ shipping cost from mines to processing sites

$$- \sum_{i}\sum_{j}\sum_{p}\sum_{\ell}\sum_{k} F_{ijp\ell kt} \, SC2_{jkt}$$ shipping cost from processing sites to markets

$$R_{ip\ell t}$$

$$- \sum_{i}\sum_{j}\sum_{p}\sum_{\ell}\sum_{k} F_{ijp\ell kt} \, PPOC_{p\ell t}$$ processing plant operating cost

$$- \sum_{j}\sum_{p} FCPF_{pt} \, Y_{jp}$$ fixed cost for each processing facility

$$- \sum_{i}\sum_{j}\sum_{p}\sum_{\ell}\sum_{k} F_{ijp\ell kt} \, (1 - R_{ip\ell t}) \, WDC_{jt}$$ waste disposal cost

$$- \sum_{k} FCMK_{kt} \, Z_{kt}$$ fixed cost associated with markets

Constraints: The following constraints mathematically represent the modeled relationships of the system:

1. If a mine is chosen, its production must be less than its capacity. The production is zero if X_{it} is zero.

$$\sum_{j}\sum_{p}\sum_{\ell}\sum_{k} F_{ijp\ell kt} - UBF_{it} \, X_{it} \leq 0 \text{ for all i and t}$$

2. If the mine is chosen, its production must be greater than the minimum acceptable production level (MAP). The MAP level is zero if X_{it} is zero. This constraint is optional.

$$\sum_{j}\sum_{p}\sum_{\ell}\sum_{k} F_{ijp\ell kt} - LBF_{it} \, X_{it} \geq 0 \text{ for all i and t}$$

3. If a market is chosen, satisfy the quantity demanded. The market demand is zero if Z_{kt} is zero.

$$\sum_{i}\sum_{j}\sum_{p}\sum_{\ell} F_{ijp\ell kt} \, R_{ip\ell t} - QUAN_{kt} \, Z_{kt} = 0$$

for all k and t

4. Maintain material balance through the processing facility. This constraint is only necessary for those processing facilities with more than one processing stream.

$$\sum_{\ell}\sum_{k} F_{ijp\ell kt} \, FRAC_{ip\ell t} - \sum_{k} F_{ijp\ell kt} = 0$$

for all i, j, p, $(\ell - 1)$, and t

5. If a market is chosen, the quality of each attribute must be greater than a specified minimum. This constraint is only necessary if a minimum quality level is required.

$$\sum_{i}\sum_{j}\sum_{p}\sum_{\ell} QUAL_{ip\ell tq} \, F_{ijp\ell kt} \, R_{ip\ell t}$$

$$- LBA_{ktq} \; QUAN_{kt} \; Z_{kt} \geq 0 \text{ for all q, k, and t}$$

6. If a market is chosen, the quality of each attribute must be less than a specified maximum. This constraint is only necessary if maximum quality level is required.

$$\sum_{i}\sum_{j}\sum_{p}\sum_{\ell} QUAL_{ip\ell tq} \, F_{ijp\ell kt} \, R_{ip\ell t}$$

$$- UBA_{ktq} \; QUAN_{kt} \; Z_{kt} \leq 0 \text{ for all q, k, and t}$$

7. Limit the amount of ore processed in each processing stream for each processing facility to less than its capacity. The capacity of all streams in a facility is zero, if Y_{jp} is zero.

$$\sum_{i}\sum_{k} F_{ijp\ell kt} - CAP_{p\ell} \, Y_{jp} \leq 0$$

for all j, p, ℓ, and t

8. If a site is chosen, limit the number of processing facilities located at that site to NPF_j.

$$NPF_j \; W_j - \sum_{p} Y_{jp} \geq 0 \text{ for all j}$$

9. Limit the number of mines in the system. This constraint is optional.

$$\sum_{i} X_{it} \leq NM_t \text{ for all t}$$

10. Limit the number of processing sites in the system. This constraint is optional.

$$\sum_{j} W_j \leq NS$$

11. Limit the number of markets in the system. This constraint is optional.

$$\sum_{k} Z_{kt} \leq MAXK_t \text{ for all t}$$

12. Force mines, processing facilities, or markets into solution if required. These constraints are optional.

$X_{it} = 1$, if required in the solution

$Y_{jp} = 1$, if required in the solution

$Z_{kt} = 1$, if required in the solution

Model capabilities

The above model formulation includes many features that have been used in various models cited in the literature. In addition, it has several new features that have not previously been implemented. Because of these features, the model is termed "general". The general formulation allows this model to be applied to a wide variety of problems associated with scheduling production, locating facilities, and selecting the type of facilities. A partial list of questions that can be answered under the profit maximizing criteria includes:

- When market demand is greater than production capacity, which markets should be satisfied?
- When production capacity is greater than market demand, which mines should be operated?
- When the mined product has to be processed, where should the processing plant be built? What should be the type and size of the facility? What is the optimal combination of processing facility locations and types?

The features that have been used in past models are: multiple mines, multiple blending sites, multiple markets, multiple time periods, maximum production levels for each mine, minimum acceptable production levels for each mine, and the blending of ore. The classic model considers several mines with limited capacity supplying several markets by blending the different ores. The consideration of several blending sites and time periods has rarely been applied to the mineral industry. Also, the consideration of minimum acceptable production levels, which allows the model to eliminate a mine if a minimum production level cannot be attained, has only been used in nonmining applications.

Several new features have been incorporated in this model. All are associated with the processing of ore. As a result of these, the model has the ability to select both the types and locations of processing facilities, and to determine the optimal processed ore blending schedule. This is achieved by including the processing facility performance characteristics and the processing characteristics of the raw ore. Furthermore, the maximum capacity of each processing stream in each processing facility is considered.

Model solution

Since even a small problem is time consuming to solve manually, computers must be used. The two most common commercial MIP codes are IBM's MPSX MIP and CDC's APEX IV. IBM's program was chosen as the computer code for problem solution.

Matrix generator

As the problem size increases, the input data requirements become demanding. For example, the number of input values required by IBM's MPSX MIP program to solve a problem with five mines, three sites, three processing facilities, two processing streams, three markets, one time period, and two quality attributes easily exceeds 16,000 values, of which, many have to be calculated. However, a little more than 300 values describe the problem completely and these can be used to calculate the remaining 16,000 values.

A computer program designed to convert the raw data into a complete data set for the MIP program is called a matrix generator. The raw data is in a form familiar to the planning engineers. Specifically, this means maximum and minimum production levels for each mine, capacity of the individual processing streams, production cost for each mine, transportation cost for each shipping route, processing facility performance characteristics, quality of ore from each mine, etc. A matrix generator is essential for solving large MIP problems.

Report writer

The output on IBM's MPSX MIP is not in a format readily understood by management. Hence a computer program, called a report writer, is used to extract the important information from the reams of output. This program generates reports that summarize the important solution information. With the combination of the matrix generator and the report writer, the evaluation of large complex problems involving production scheduling, processing facility location and selection, or processed ore blending possible becomes much faster, simpler, and less prone to errors.

Model demonstration

The following example of a centralized coal preparation plant system is presented to demonstrate the capabilities of this model. The problem consists of two coal mines, two possible centralized preparation plant sites, a choice of a preparation plant with two processing streams and/or a blending facility, two possible markets with a maximum sulfur limit, and one time period. Figure 3 illustrates the problem.

Summaries of the input data for the mines, processing sites, processing facilities and streams, and the markets are shown in Tables 4, 5, 6, and 7, respectively. In Table 6, the coal preparation plant represents a typical heavy media plant with two processing streams. As seen from Stream 1, the fraction of coal processed is 0.6 for Mine 1 and 0.5 for Mine 2. The fractional recovery from raw coal is 0.9 for Mine 1 and 0.8 for Mine 2. And the sulfur content of the processed coal is 1.2% for Mine 1 and 0.9% for Mine 2. Since the blending plant has only one stream and the preparation plant has two, a dummy stream must be added to the blending plant. This dummy stream has a capacity of zero to force it out of the solution.

The "picture" of the matrix produced by MPSX is shown in Fig. 4. There are 32 continuous variables and 10 integer variables. The variable names, which correspond to those given in the model formulation, are printed vertically at the top of the figure. For instance, the variable, F11111, represents the amount of raw coal produced at Mine 1, shipped to Site 1, processed in Processing Facility 1 and Stream 1, and shipped to Market 1. The integer variables, X, W, Y, and Z, are zero-one variables for the mines, sites, processing facilities, and markets, respectively. The row names are in the left column and the first number in the row name corresponds to the constraint type and the second number is sequential numbering system within each constraint type. The second column represents the type of constraint with the letter N, L, G, or E to designate a nonconstrained, less than or equal, greater than or equal, or equality constraint, respectively.

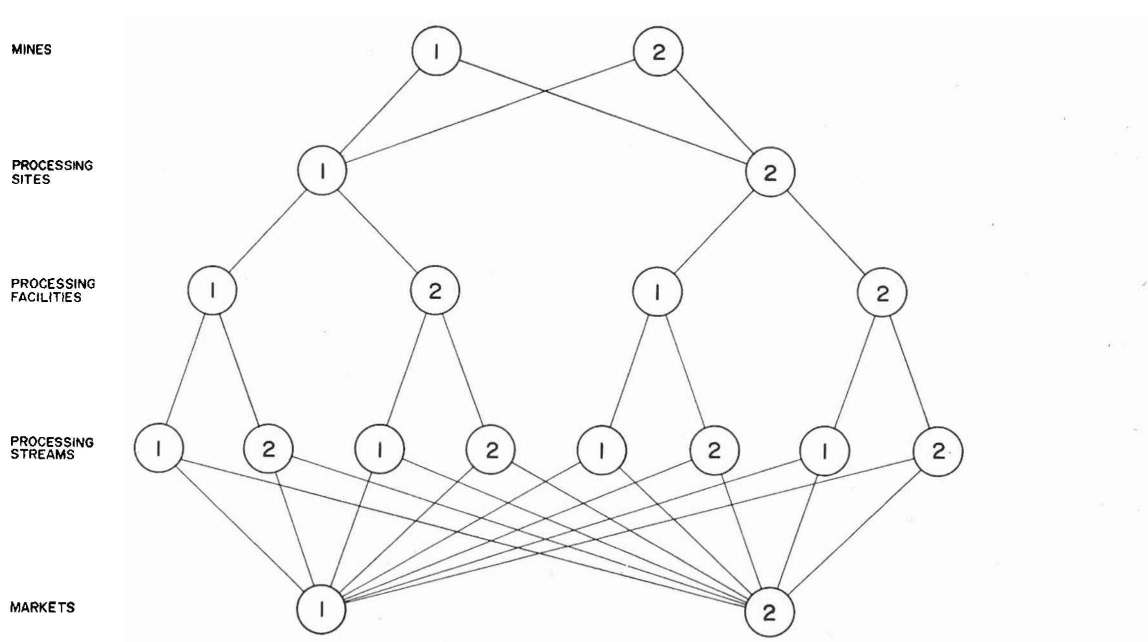

Fig. 3 — Diagram of the example problem's layout and data

Table 4 — Mine Data

	Variable Name	Mine 1	Mine 2
Production Cost ($/t)	C_{it}	$23.00	$26.00
Maximum Production (t)	UBF_{it}	1,000,000	1,000,000
Minimum Production (t)	LBF_{it}	600,000	500,000
Transportation Cost ($/t)	$SC1_{ijt}$		
to Site 1		$ 1.00	$ 3.00
to Site 2		$ 3.00	$ 1.50
Fixed Cost ($)	FCM_{it}	$ 0.00	$ 0.00

Table 5 — Site Data

	Variable Name	Site 1	Site 2
Fixed Cost ($)	FCS_{jt}	$200,000	$200,000
Transportation Cost ($/t)	$SC2_{jkt}$		
to Market 1		$ 1.50	$ 2.00
to Market 2		$ 3.00	$ 1.00
Waste Disposal Cost ($/t)	WDC_{it}	$ 0.90	$ 1.10
Maximum Number of Processing Facilities	NPF_j	2	2

Table 6 — Processing Facility Data

	Variable Name	Facility 1 Preparation Plant		Facility 2 Blending Facility
Annual Capital Cost ($)	$FCPF_{pt}$	$700,000		$100,000
		Stream 1	Stream 2	Stream 1
Processing Cost ($/t)	$PPOC_{p\ell t}$	$ 2.00	$ 2.00	$ 0.25
Processing Capacity (t)	$CAP_{p\ell}$	900,000	700,000	2,000,000
Percent Recovery (%)	$R_{ip\ell}$	(90,80)*	(80,70)	(100,100)
Percent Ore Reporting (%)	$FRAC_{ip\ell t}$	(60,50)	(40,50)	(100,100)
Sulfur Content in Processed Ore (%)	$QUAL_{ip\ell tq}$	(1.2,0.9)	(0.8,0.6)	(1.6,1.3)

The values inside the parentheses correspond to the ore from Mine 1 and Mine 2, respectively.

Table 7 — Market Data

	Variable Name	Market 1	Market 2
Tonnage Demand (t)	$QUAN_{kt}$	600,000	700,000
Maximum Sulfur (%)	UBA_{ktq}	1.0	1.2
Selling Price ($/t)	REV_{kt}	$40.00	$35.00

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LOWER BOUND
UPPER BOUND                                                                         1 1 1 1 1 1 1 1 1 1
                                                                               I                     I
               F F F F F F F F F F F F F F F F F F F F F F F F F F F F F F F F N                     N
               1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 T                     T V
               1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 O                     E A
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               1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 G X X W W 1 1 2 2 Z Z D U
               1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 ' 1 2 1 2 1 2 1 2 1 2 ' E
OBJ         N -B-B-B-B-B-B    -B-B-B-B-B-B    -B-B-B-B-B-B    -B-B-B-B-B-B          -F-F-F-E-F-E G G
ROW1.1      L  1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1                                  -F
ROW1.2      L                                  1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1    -F
ROW2.1      G  1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1                                  -F
ROW2.2      G                                  1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1    -F
ROW3.1      E  T   T   1       T   T   1       T   T   1       T   T   1                          -F
ROW3.2      E    T   T   1       T   T   1       T   T   1       T   T   1                          -F
ROW4.1      E -T-T T T
ROW4.2      E              1 1
ROW4.3      E                -T-T T T
ROW4.4      E                              1 1
ROW4.5      E                                -T-T T T
ROW4.6      E                                            1 1
ROW4.7      E                                                -T-T T T
ROW4.8      E                                                            1 1
ROW6.1      L  A   T   A       A   T   A       T   T   A       T   T   A                          -F
ROW6.2      L    A   T   A       A   T   A       T   T   A       T   T   A                          -F
ROW7.1      L  1 1                             1 1                                      -F
ROW7.2      L      1 1                             1 1                                  -F
ROW7.3      L          1 1                             1 1                                -G
ROW7.4      L              1 1                             1 1
ROW7.5      L                  1 1                             1 1                          -F
ROW7.6      L                      1 1                             1 1                      -F
ROW7.7      L                          1 1                             1 1                    -G
ROW7.8      L                              1 1                             1 1
ROW8.1      G                                                                         A   -1-1
ROW8.2      G                                                                           A     -1-1
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Initial tableau.

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                        SUMMARY OF MATRIX
SYMBOL                       RANGE                              COUNT (INCL.RHS)
  Z                 LESS THAN               .000001
  Y          .000001  THRU                  .000009
  X          .000010                        .000099
  W          .000100                        .000999
  V          .001000                        .009999
  U          .010000                        .099999
  T          .100000                        .999999                  44
  1         1.000000                       1.000000                 116
  A         1.000001                      10.000000                  14
  B        10.000001                     100.000000                  24
  C       100.000001                   1,000.000000
  D     1,000.000001                  10,000.000000
  E    10,000.000001                 100,000.000000                   2
  F   100,000.000001               1,000,000.000000                  16
  G            GREATER THAN        1,000,000.000000                   4
MINIMUM = .400000E+00    MAXIMUM = .245000E+08
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Legend

Fig. 4 — Example problem formulation in matrix form

To solve this problem, IBM's MPSX MIP program was run on an IBM 370/3081 computer system. The optimal solution is to use coal from both Mine 1 and Mine 2, to locate a preparation plant at Site 1 and a blending facility at Site 2, and to supply both Market 1 and Market 2. The maximum feasible total profit of the system is $5.7 million on $48.5 million in sales (Table 10). The raw coal production schedule is as follows:

	Total Production (t)	Ship to Prep Plant at Site 1 (t)	Ship to Blending Facility at Site 2 (t)
Mine 1	961,921	961,921	0
Mine 2	500,000	109,009	390,991

The cleaned coal blending schedule is summarized in Table 8. For example, this table shows that of the 962 kt (106,000 st) of raw coal from Mine 1 and shipped to the preparation plant, 577 kt (636,000 st) were processed in Stream 1. This produces 519 kt (572,000 st) of clean coal, of which, 308 kt (340,000 st) and 211 kt (233,000 st) were shipped to Market 1 and Market 2, respectively.

The summary of the solution results that were constrained are shown in Table 9. Those factors at their limit levels will cause the solution and the objective function value to change, if the limiting values are changed.

A financial analysis of the solution is shown in Table 10. This table reveals that the production costs are the major portion of the total costs (82%), while the transportation (10%), processing (5%), and the other costs (3%) complete the cost breakdown.

In addition, the model can be used to evaluate the effect that any changes have on the solution. Some of the items that can be evaluated are changes in:

- production capacity or minimum acceptable quantity;
- processing facility capacities;
- market requirements; and
- any costs including production, transportation, processing, waste disposal, or the fixed costs associated with the mines, sites, processing facilities, or markets.

This ability to quickly evaluate these changes allows management to ask more "What if ..." questions than ever before. The answers to these questions can greatly improve management's understanding of the degree of risk involved and can improve their negotiating position in a competitive situation.

Table 9 — Solution Results and the Corresponding Upper and Lower Limit Values

	Solution Value	Lower Limit	Upper Limit
Production (t)			
Mine 1	961,921	600,000	1,000,000
Mine 2	500,000*	500,000	1,000,000
Processing Capacities (t)			
Preparation Plant			
Stream 1	631,657	None	900,000
Stream 2	439,272	None	700,000
Blending Facility	390,991	None	2,000,000
Delivered Coal (t)			
Market 1			
Tonnage	600,000*	600,000	600,000
Sulfur Quality	1.00*	None	1.00
Market 2			
Tonnage	700,000*	700,000	700,000
Sulfur Quality	1.20*	None	1.20

* *Value at limit level*

Table 10 — Revenue and Cost Breakdown

Item	Amount
Revenue	$48,500,000
Variable Costs:	
Production	35,124,183
Raw Coal Transportation	1,875,434
Processing	2,239,608
Clean Coal Transportation	2,218,018
Waste Disposal	161,921
Fixed Costs:	
Mines	0
Sites	400,000
Processing Facilities	800,000
Markets	0
Total Cost	$42,819,165
Total Profit	$ 5,680,835

Remarks

This paper has presented a generalized multiperiod MIP model for production scheduling, selecting and locating new processing facilities, processed ore blending scheduling, and the selection of markets. An MIP model has two requirements on the problem formulation — all relationships are linear and all coefficients are known with certainty. Therefore, the following assumptions have been made in model formulation:

- The raw ore from each mine is processed separately. This model considers the blending of the different ores after they have been processed. A non-

Table 8 — Production, Blending, and Processing Schedule for the Optimal Solution

	Total Raw Coal Tonnage (t)	Total Clean Coal Tonnage (t)	Clean Coal Market 1 Tonnage (t)	Market 1 Sulfur Percent (%)	Market 2 Tonnage (t)	Market 2 Sulfur Percent (%)
Processing Plant						
Mine 1 Coal	961,921					
to Stream 1	577,153	519,438	308,176	1.20	211,262	1.20
to Stream 2	384,768	307,814	210,067	0.80	97,747	0.80
Mine 2 Coal	109,009					
to Stream 1	54,504	43,604	43,604	0.90	0	0
to Stream 2	54,504	38,153	38,153	0.60	0	0
Mine 1 and Mine 2	1,070,930	909,009	600,000	1.00	309,009	1.07
Blending Facility						
Mine 1 Coal	0	0	0	0	0	0
Mine 2 Coal	390,991	390,991	0	0	390,991	1.30
Total	1,461,921	1,300,000	600,000	1.00	700,000	1.20

linear model is required to consider both raw and processed ore blending in the same model. However, if the quality of ore is not considered, a linear MIP model could handle both raw and processed ore blending.

- The transportation costs are known with certainty before optimization. Therefore, economies of scale are not directly considered in the optimization. However, the model can be rerun with new transportation costs to reflect the current optimal tonnages until the new optimal tonnage closely corresponds to the new transportation costs that were input into the model. This procedure allows the model to indirectly consider nonlinear economies of scale.

- The processing facility performance estimates based on a steady-state operation.

This model has the potential for solving problems common to most mineral producers and buyers. The general formulation allows it to model many different situations ranging from very simple to very complex problems. In short, this model allows management to evaluate several operating schemes for extracting, processing, and marketing a mineral commodity in its search for the best scheme. ■

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Appendix A

The method of calculating the estimates for the performance of a processing facility given the performance of each processing device within the facility and characteristics of the ore is described here. It is assumed that all material flow relationships through the processing facility are linear.

To calculate the processing facility performance, the ore is not treated as a single product but rather as the sum of several products, each with its own size and specific gravity group. With this formulation, the ore analysis may have any number of groups. The more groups the ore is broken into, the more accurate the estimate of the performance of a processing facility.

The characteristics of the ore can be described by two variables:

f_{abit} — the fraction by weight of ore from mine i in time period t that is contained in size group a and specific gravity group b. The sum of f_{abit} for all size and specific gravity groups must be equal to one.

$$\sum_{a}\sum_{b} f_{abit} = 1 \text{ for all i and t}$$

q_{abitq} — the quality of attribute q for ore from mine i in time period t in size group a and specific gravity group b.

The performance of the processing equipment can be described by two variables:

$t_{abp\ell}$ — the fraction of ore in size group a, specific gravity group b, processing stream ℓ in processing facility p that reports to waste.

$\phi_{ap\ell}$ — the fraction of ore in size group a that reports to processing stream ℓ in processing facility p. The sum of $\phi_{ap\ell}$ for all processing streams must equal one.

$$\sum_{\ell} \phi_{ap\ell} = 1 \text{ for all a and p}$$

The fraction of ore from mine i that reports to stream ℓ in processing facility p in time period t is the sum for each size and specific gravity group of the product of the fraction of ore that reports to stream ℓ and the fraction of ore within each size and specific gravity group:

$$FRAC_{ip\ell t} = \sum_{a}\sum_{b} f_{abit}\, \phi_{ap\ell} \qquad (1)$$

The recovery fraction of ore from mine i that reports to stream ℓ in processing facility p in time period t is the sum over all size and specific gravity groups of the product of the fraction of ore that reports to stream ℓ, the fraction of ore within the group, and the fraction of ore that reports to the clean ore. This can be written as:

$$R_{ip\ell t} = \sum_{a}\sum_{b} f_{abit}\, \phi_{ap\ell}\, (1 - t_{abp\ell}) \qquad (2)$$

The attribute level in each group can be multiplied before the summation to yield the processed ore quality. This ore quality for attribute q of the ore from mine i, processed in stream ℓ of processing facility p in time period t can be written:

$$QUAL_{ip\ell tq} = \sum_{a}\sum_{b} f_{abit}\, \phi_{ap\ell}\, (1 - t_{abp\ell})\, q_{abitq} \qquad (3)$$

These three equations describe the relationships between the raw and processed ore quantity and quality. For a more complete description of the processing facility performance, refer to Barbaro, Ramani, and Luckie (1982).