Two Carrier Phase Based Approaches for Autonomous Fault Detection and Exclusion

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BIOGRAPHIES

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ABSTRACT

This paper presents two carrier phase based approaches for receiver autonomous fault detection and exclusion integrity monitoring (FDE). Some models based on carrier phase measurements are established. Any typical snapshot FDE algorithm based on a linear model can be applied directly to the models.

The first approach uses only measurements of consecutive epochs. For a single difference measurement model, if 4 epochs are used, at least 6 satellites are required to detect a fault and 7 are required to identify the faulty satellite.

The second approach combines the measurements of the current epoch and estimates of integer ambiguities and error covariances obtained from the positioning algorithm of Van Graas and Lee. This approach is more suitable for computing the horizontal protection levels. If single difference measurements are used, at least 5 satellites are required for fault detection and 6 are required for identification.

1 INTRODUCTION

Receiver autonomous fault detection and exclusion integrity monitoring (FDE) is crucial for some GPS applications. FDE consists of two distinct parts: satellite fault detection/exclusion and bad geometry detection/exclusion, see Kelly [4]. The term "exclusion" includes "identification" or "isolation" (see [5]). We will use "identification" in the context of this paper, even though we use the more general term "exclusion" in the title. Most existing fault detection and identification methods are for code based linear models and are roughly equivalent. In essence, for fault detection and identification, they just remove an equation from the model and check the reduction of the residual, see for example Kelly [4].

It is important to distinguish between models and methods in this area. For any model there may be several methods we can apply in order to carry out a given integrity test. Two subclasses of methods will be of particular interest:– *equation removal methods*, and *satellite removal methods*. The former just removes an equation from the model for fault detection and identification, while the latter removes all equations corresponding to a given satellite. The usual code based model for integrity tests is quite simple. It is just a set of linearized measurement equations at one time step.

Since there is exactly one equation per satellite in the observation model, removing an equation is equivalent to removing a satellite. Therefore the two subclasses of methods are identical. But we will see later that for some linear models the two subclasses are distinct.

In some GPS applications, such as aircraft precision approach and landing, high precision is required. This can be accomplished by using carrier phase based differential GPS. But precision is not the only crucial factor for an application like aircraft landing. Safety imposes strict requirements for the integrity of positioning solutions. Since carrier phase data have higher resolution than code phase data, carrier phase can provide better integrity monitoring, see for example Pervan et al [6].

The goal of this paper is to present two carrier phase based approaches for FDE. Specifically, we will give two carrier phase based linear models. Then it will be shown how some typical code based FDE methods can be applied to the linear models.

In Sections 6–7 we will give a first approach to obtaining models for testing based solely on consecutive epochs. It ignores the usual position estimates and their error covariance matrices, apart from using such position estimates to compute matrix elements. In theory this approach is satisfactory for fault detection and identification, especially when there are slowly increasing (ramp) faults, but although it can be used to compute the horizontal protection level (HPL), it probably is not useful for this.

A second approach is given in Section 8. It is potentially more reliable, and is also suitable for the calculation of HPL. This improvement is obtained by combining the estimates and error covariances of [8] with the single difference equations.

Notation used. We work with reals only, and use i, j, k, l, m, n to denote indices and dimensions (superscript i will refer to the i-th satellite, subscript k to the k-th time step), while lower case Greek letters will denote scalars. Other lower case Roman letters will denote vectors, while upper case Roman will denote matrices. We will use $(a)_i$ to denote the i-th element of a given vector a. Superscript T will denote transpose, and $A^\dagger$ is the Moore-Penrose generalized inverse of A. The unit matrix will be denoted by I and its i-th column by e_i, while $e \equiv (1, 1, \ldots, 1)^T$ (we use $\equiv$ to mean 'is defined to be'). I_n will denote the $n \times n$ unit matrix. Throughout we use $\|x\| = \|x\|_2 \equiv \sqrt{x^T x}$ for vectors. We will use $\mathcal{E}\{\cdot\}$ to denote the expected value, and $\mathrm{cov}\{\cdot\}$ to denote the covariance, that is $\mathrm{cov}\{x\} = \mathcal{E}\{(x - \mathcal{E}\{x\})(x - \mathcal{E}\{x\})^T\}$. $u \sim N(\bar{u}, U)$ will mean u is a a normally distributed random vector with mean $\bar{u}$ and covariance U.

2 CARRIER PHASE POSITION ESTIMATION

We first introduce the single, double and triple differenced carrier phase measurement equations, then we give the van Graas and Lee position algorithms. Our two carrier phase based approaches for FDE will be based on the material presented here.

2.1 Single, double, and triple differences

The first differences are between (values at) the stationary receiver and the roving receiver, creating single differences $(\cdot)^{SD}$. Let λ denote the carrier wavelength. At a given time step k, let there be m_s single differences η_k^i of integrated Doppler measurements (in wavelengths, one value for each of m_s satellites) and corresponding integer ambiguities α^i, with $y_k^{SD} \equiv (\eta_k^1, \ldots, \eta_k^{m_s})^T$ and $a^{SD} \equiv (\alpha^1, \ldots, \alpha^{m_s})^T$. At time step k the single difference equations for the baseline vector x_k from the stationary receiver to the roving receiver are

$$\begin{bmatrix} \eta_k^1 \\ \cdot \\ \eta_k^{m_s} \end{bmatrix} = \lambda^{-1} \begin{bmatrix} (e_k^1)^T \\ \cdot \\ (e_k^{m_s})^T \end{bmatrix} x_k + \begin{bmatrix} \alpha^1 \\ \cdot \\ \alpha^{m_s} \end{bmatrix} + \begin{bmatrix} 1 \\ \cdot \\ 1 \end{bmatrix} \Delta\beta_k + \begin{bmatrix} \nu_k^1 \\ \cdot \\ \nu_k^{m_s} \end{bmatrix},$$

where e_k^i is the unit vector from mid-baseline to the i-th satellite and $\Delta\beta_k$ is the difference between the two receivers' clock biases in units of carrier wavelengths. In the usual model, the ν_k^i are assumed to be unbiased independently distributed random noises with the same normal distribution, so at time step k we can write the above equations as

$$\begin{aligned} y_k^{SD} &= \lambda^{-1} E_k^{SD} x_k + a^{SD} + e\Delta\beta_k + v_k^{SD}, \quad (1) \\ v_k^{SD} &\sim N(0, \sigma^2 I_{m_s}), \end{aligned}$$

where the integer ambiguities are fixed, but the clock biases can vary with time. The almost identical equation appears in van Graas and Lee [8, (1)], see also Hofmann-Wellenhof et al [3, p.189, (8.34)].

The second differences are between (values at) different satellites, creating double differences $(\cdot)^{DD}$. With this double differencing, one satellite is chosen to be special. In theory it can be any satellite, but to simplify the presentation we will assume it is the first. Define the $(m_s - 1) \times m_s$ matrix J and the $(m_s - 1)$-vector y_k^{DD} of double differences via

$$J \equiv \begin{bmatrix} 1 & -1 & & \\ \cdot & & \cdot & \\ 1 & & & -1 \end{bmatrix}, \quad y_k^{DD} \equiv J y_k^{SD} = \begin{bmatrix} \eta_k^1 - \eta_k^2 \\ \cdot \\ \eta_k^1 - \eta_k^{m_s} \end{bmatrix}.$$

Then defining

$$E_k^{DD} \equiv JE_k^{SD}, \quad a^{DD} \equiv Ja^{SD}, \quad v_k^{DD} \equiv Jv_k^{SD},$$

we have (by multiplying (1) by J and noticing that $Je = 0$) the double difference equation at time step k, where for later use we will assume that E_k^{DD} has full column rank:

$$\begin{aligned} y_k^{DD} &= \lambda^{-1} E_k^{DD} x_k + a^{DD} + v_k^{DD}, \qquad (2) \\ v_k^{DD} &\sim N(0, \sigma^2 JJ^T). \end{aligned}$$

For later use, we define $G_k \equiv [\lambda^{-1} E_k^{DD}, I]$, then (2) can be written

$$y_k^{DD} = G_k \begin{bmatrix} x_k \\ a^{DD} \end{bmatrix} + v_k^{DD}. \qquad (3)$$

The main purpose of double differencing is to eliminate the unknown $\Delta\beta_k$ in (1), which means that the receiver's clock biases will not affect the position estimates.

The third differences are between two time steps (creating triple differences). Based on double difference equations (2), the vector of triple differences of measurements is defined by

$$t_k \equiv y_k^{DD} - y_{k-1}^{DD} = \lambda^{-1}[E_k^{DD} x_k - E_{k-1}^{DD} x_{k-1}] + v_k^{DD} - v_{k-1}^{DD} \qquad (4)$$

which eliminates the unknown integer ambiguities from the expression. Therefore

$$E_k^{DD} x_k = E_{k-1}^{DD} x_{k-1} + \lambda t_k + \lambda(v_k^{DD} - v_{k-1}^{DD}). \qquad (5)$$

Remark 2.1 *Assuming the model (1) is correct, this is all rigorous so far. However this is a nonlinear model, since $E_k^{DD} \equiv E^{DD}(x_k)$ is a function of x_k. Here we will assume the estimate $\hat{x}_k^-$ (below) of x_k is accurate enough so the approximation $E^{DD}(\hat{x}_k^-)$ for E_k^{DD} causes no meaningful inaccuracy. We have to take such approximations since we will not know the true x_k. Thus we know $\mathcal{E}\{E_k^{DD}\hat{x}_k^-\} = E_k^{DD}\mathcal{E}\{\hat{x}_k^-\}$ is true, since $E_k^{DD} \equiv E^{DD}(x_k)$ is fixed even though we will not know it accurately, but if $\hat{x}_k^-$ is the least squares solution of $E_k^{DD}\hat{x}_k^- \approx c_k$, for some given c_k, we will assume $(E_k^{DD})^T(E_k^{DD}\hat{x}_k^- - c_k) = 0$ for the* correct *E_k^{DD}, even though to compute our $\hat{x}_k^-$ we must use the approximation $E^{DD}(\hat{x}_k^-)$ for E_k^{DD}, and so can only have exactly (after iterating to convergence) $(E^{DD}(\hat{x}_k^-))^T(E^{DD}(\hat{x}_k^-)\hat{x}_k^- - c_k) = 0$. We will usually skim over such approximations without further comment.*

Suppose we have an estimate $\hat{x}_{k-1}$ such that $\tilde{x}_{k-1} \equiv \hat{x}_{k-1} - x_{k-1}$ satisfies $\mathcal{E}\{\tilde{x}_{k-1}\} = 0$. We can solve the nonlinear least squares problem (NLLSP), see (5),

$$E_k^{DD}\hat{x}_k^- \approx E_{k-1}^{DD}\hat{x}_{k-1} + \lambda t_k$$

for $\hat{x}_k^-$, our preliminary estimate for x_k, giving with $(E_k^{DD})^\dagger \equiv [(E_k^{DD})^T(E_k^{DD})]^{-1}(E_k^{DD})^T$,

$$\hat{x}_k^- = (E_k^{DD})^\dagger(E_{k-1}^{DD}\hat{x}_{k-1} + \lambda t_k). \qquad (6)$$

Replacing t_k in the above equation by the right hand side of (4), we have

$$\hat{x}_k^- = x_k + (E_k^{DD})^\dagger E_{k-1}^{DD}\tilde{x}_{k-1} + \lambda(E_k^{DD})^\dagger(v_k^{DD} - v_{k-1}^{DD}).$$

Thus with $\tilde{x}_k^- \equiv \hat{x}_k^- - x_k$,

$$\mathcal{E}\{\tilde{x}_k^-\} = 0, \qquad (7)$$

i.e., $\hat{x}_k^-$ is an unbiased estimate of x_k.

The above completes the background theory we give to motivate the van Graas and Lee positioning algorithm presented in [8].

2.2 The van Graas and Lee positioning algorithm

The positioning algorithm presented in [8] can be implemented as follows.

- Assume we have unbiased estimates $\hat{x}_{k-1}$ of x_{k-1}, and $\hat{a}_{k-1}^{DD}$ of a^{DD}, obtained at time $k-1$, with error covariance matrix

$$\breve{H}_{k-1} \equiv \mathrm{cov}\{\begin{bmatrix} \hat{x}_{k-1} \\ \hat{a}_{k-1}^{DD} \end{bmatrix}\} = \begin{bmatrix} H_{k-1}^{(11)} & (H_{k-1}^{(21)})^T \\ H_{k-1}^{(21)} & H_{k-1}^{(22)} \end{bmatrix}. \qquad (8)$$

- Compute the nonlinear least squares solution, see (6),

$$\hat{x}_k^- = (E_k^{DD})^\dagger(E_{k-1}^{DD}\hat{x}_{k-1} + \lambda t_k). \qquad (9)$$

  This can be implemented as follows:

  - Compute $\hat{x}_k^* = \hat{x}_{k-1} + (E_{k-1}^{DD})^\dagger \lambda t_k$.
  - Compute $E^{DD}(\hat{x}_k^*)$.
  - Compute $\hat{x}_k^- = (E_k^{DD}(\hat{x}_k^*))^\dagger(E_{k-1}^{DD}\hat{x}_{k-1} + \lambda t_k)$.

- Construct G_k in (3) using $E_k^{DD} = E^{DD}(\hat{x}_k^-)$ and compute the gain matrix

$$K_k \equiv \breve{H}_{k-1} G_k^T (G_k \breve{H}_{k-1} G_k^T + H^{mn})^{-1}, \qquad (10)$$

  where the measurement noise covariance matrix $H^{mn} \equiv \mathrm{cov}\{v_k^{DD}\} = \sigma^2 JJ^T$, see (2). (in [8], H^{mn} is chosen to be a diagonal matrx and σ is taken to be $0.001m$.)

- Compute the innovation:

$$r_k = y_k^{DD} - G_k \begin{bmatrix} \hat{x}_k^- \\ \hat{a}_{k-1}^{DD} \end{bmatrix}. \qquad (11)$$

- Update the state vector:

$$\begin{bmatrix} \hat{x}_k \\ \hat{a}_k^{DD} \end{bmatrix} = \begin{bmatrix} \hat{x}_k^- \\ \hat{a}_{k-1}^{DD} \end{bmatrix} + K_k r_k. \qquad (12)$$

- Update the error covariance matrix:

$$\breve{H}_k = (I - K_k G_k)\breve{H}_{k-1} + H^{pn}, \qquad (13)$$

where H^{pn} is the process noise covariance matrix. [8, p.609] chooses H^{pn} to be a diagonal matrix, and uses 10^{-4} to 10^{-3} m^2 for position coordinate variances and 10^{-10} m^2 for integer ambiguity variances.

Let $\tilde{x}_k \equiv \hat{x}_k - x_k$ and $\tilde{x}_k^- \equiv \hat{x}_k^- - x_k$ as before, and $\tilde{a}_{k-1}^{DD} \equiv \hat{a}_{k-1}^{DD} - a^{DD}$. Assume $\hat{x}_{k-1}$ and $\hat{a}_{k-1}^{DD}$ are unbiased estimates of x_{k-1} and a^{DD}, respectively. We now show $\hat{x}_k$ and $\hat{a}_k^{DD}$ are unbiased estimates of x_k and a^{DD}, respectively. From (11) and the measurement equation (3), we have

$$\begin{aligned} r_k &= y_k^{DD} - \lambda^{-1}E_k^{DD}\hat{x}_k^- - \hat{a}_{k-1}^{DD} \\ &= \lambda^{-1}E_k^{DD}x_k + a^{DD} + v_k^{DD} - \lambda^{-1}E_k^{DD}\hat{x}_k^- - \hat{a}_{k-1}^{DD} \\ &= -\lambda^{-1}E_k^{DD}(\hat{x}_k^- - x_k) - (\hat{a}_{k-1}^{DD} - a^{DD}) + v_k^{DD} \\ &= -\lambda^{-1}E_k^{DD}\tilde{x}_k^- - \tilde{a}_{k-1}^{DD} + v_k^{DD}. \qquad (14) \end{aligned}$$

Since $\mathcal{E}\{\tilde{x}_k^-\} = 0$ from (7), and $\mathcal{E}\{\tilde{a}_{k-1}^{DD}\} = \mathcal{E}\{v_k^{DD}\} = 0$, we have

$$\mathcal{E}\{r_k\} = 0. \qquad (15)$$

It follows from (12) that

$$\mathcal{E}\{\tilde{x}_k\} = 0, \qquad \mathcal{E}\{\tilde{a}_k^{DD}\} = 0. \qquad (16)$$

Thus with $\tilde{x}_k \equiv \hat{x}_k - x_k$ and $\tilde{a}_k^{DD} = \hat{a}_k^{DD} - a^{DD}$

$$\begin{bmatrix} \tilde{x}_k \\ \tilde{a}_k^{DD} \end{bmatrix} \sim N\left(0, \breve{H}_k \equiv \begin{bmatrix} H_k^{(11)} & (H_k^{(21)})^T \\ H_k^{(21)} & H_k^{(22)} \end{bmatrix}\right), \qquad (17)$$

where $\breve{H}_k$ is the version of (8) to be used for the next step.

3 SOME CODE BASED FDE METHODS

In this section we give a brief presentation of typical existing fault detection and identification methods for a usual code based linear model. The ideas of these methods can be applied to the two carrier phase based linear models we will establish in Sections 6-8.

Suppose we have a linear model:

$$y = Gz + v + b, \qquad v \sim N(0, \sigma^2 I), \qquad (18)$$

where G is $m \times n$ of rank n with $m = m_s$ here , the i-th element of y is obtained from measurements involving satellite-i only, and $b = e_i\beta$ for some $\beta \neq 0$ if there is a fault in satellite-i, $1 \leq i \leq m_s$, otherwise $b = 0$.

Let the QR factorization of G be $G = Q_1 R$ where $Q \equiv [Q_1, Q_2]$ is an orthogonal matrix and R is upper triangular, the parity vector is:

$$\begin{aligned} p \equiv Q_2^T y = Q_2^T(v + b), \qquad (19) \\ \mathcal{E}\{p\} = Q_2^T b, \quad \text{cov}\{p\} = \sigma^2 I_{m-n}. \end{aligned}$$

Note that parity vectors are not unique, but their 2-norms are, and that is what concerns us. The least squares (LS) estimate $\hat{z} = G^{\dagger}z = R^{-1}Q_1^T y$ (it is also the Best, or Minimum Variance, Linear Unbiased Estimate when $b = 0$), with residual r:

$$\begin{aligned} r \equiv y - G\hat{z} = Q_2Q_2^T y = Q_2 p = Q_2Q_2^T(v + b), \qquad (20) \\ \mathcal{E}\{\hat{r}\} = Q_2Q_2^T b, \quad \text{cov}\{\hat{r}\} = \sigma^2 Q_2Q_2^T. \end{aligned}$$

The typical fault detection and identification algorithms based on the model (18) are the standard parity space algorithm, the Parkinson single deletion algorithm, the Brenner parity space algorithm, and the maximum residual method. These four algorithms are roughly equivalent, see for example [4]. Essentially the statistics used by these algorithms are as follows:

- **Fault detection.**

$$\delta^2 = \|r\|_2^2 = \|p\|_2^2,$$

or

$$\delta_{ii}^2 = \|p\|_2^2 - \|p_i\|_2^2, \quad i = 1, \ldots, m.$$

where p_i is the parity vector for the original linear model with i-th equation (observation) removed.

- **Fault identification.**

$$\delta_{ii}^2 = \|p\|_2^2 - \|p_i\|_2^2, \quad i = 1, \ldots, m.$$

or

$$\delta_{ij}^2 = \|p_i\|_2^2 - \|p_{ij}\|_2^2, \quad i, j = 1, \ldots, m, \; i \neq j.$$

where p_{ij} is the parity vector for the original linear model with both i-th and j-th equations removed.

For more details about fault detection and identification using the above statistics, see [4] and references therein.

No matter whether we use δ^2 or δ_{ii}^2 $(i = 1, \ldots, m)$ for detection, the minimum satellite requirements are the same. It is also true that no matter we use δ_{ii}^2 $(i = 1, \ldots, m)$ or δ_{ij}^2 $(i, j = 1, \ldots, m, \; i \neq j)$ for identification, the minimum satellite requirements are the

same. In the next section we will discuss the determination of minimum satellite requirements.

In addition to fault detection and identification, an FDE algorithm has another important part: the calculation of the horizontal protection level (HPL). HPL is used to determine if a detection function is available. There are several approaches to computing HPL based on a linear model (18), see [4], [5] and the references therein.

Remark 3.1 *The typical approach to fault detection and identification for the code based problem only requires that we have a model of the form (18). The carrier phase based models we will establish later have the same form as (18), but where there can only be a fault in any one of the first $m_s \leq m$ observations, and not in the last $m - m_s$ equations. In (18) each of the m_s ($= m$ in the code based case here) equations that we might consider deleting corresponds to a single satellite, and the noise covariance matrix corresponding to these particular equations is diagonal, and these noises are independent of the noises for the remaining $m - m_s$ ($= 0$ in the code based case here) equations. This means the effect of a given satellite (with its possible fault) at a given time can be removed simply by deleting an equation, and this ensures the δ, δ_{ii} and δ_{ij} have their required meanings.*

These conditions are sufficient for these results to be applicable to the carrier phase based problem here, as well as for our generalization of a typical approach to HPL for the code based problem to be applicable for computing HPL for the carrier phase based problem.

4 DETERMINING SATELLITE REQUIREMENTS

We outline how to obtain the minimal satellite requirements for position estimation, fault detection, and fault identification for the linear model

$$y = Gz + v + b, \qquad v \sim N(0, \sigma^2 I),$$

where G is $m \times n$ of any rank. Remember an equation removal method will remove only one equation at a time, but a satellite removal method may remove more than one equation at a time. The linear least squares estimate of z is $\hat{z} = G^{\dagger} y$, with residual $r = y - G\hat{z}$. The norm of the residual $||r||_2$ cannot be increased by eliminating equations.

For a unique position estimate we require that G have full column rank n, and so we require that

$$m \geq n = \text{ rank}(G).$$

For fault detection to be meaningful we only require the possibility of a nonzero residual for the original model, so that the norm of the residual can decrease if any equations are removed. That is, we simply require that

$$m > \text{ rank}(G).$$

Note that these first two conditions depend solely on the original model, and so will give the same satellite requirements for whatever equation removal or satellite removal method is used.

To analyze the case of fault identification, let us call the model with one or more equations removed for fault detection, the fault detection model. The equation and satellite removal methods may lead to different fault detection models. We will see that satellite removal methods may lead to zero columns, and so if these are removed as well as rows for the fault detection step, the fault detection model will have the form

$$\bar{y} = \bar{G}\bar{z} + \bar{v} + \bar{b}, \qquad \bar{v} \sim N(0, \sigma^2 I),$$

where $\bar{G}$ is $\bar{m} \times \bar{n}$, say. To have meaningful fault identification statistics we only require the possibility of a nonzero residual for the fault detection model, so that the norm of the residual can decrease if any further equations are removed. That is, for fault identification we simply require that

$$\bar{m} > \text{ rank}(\bar{G}).$$

5 SOME INTEGRITY TEST DIFFICULTIES

There are a few difficulties in developing models for integrity tests for the carrier phase based GPS solution:

1. **Double differencing**. Many carrier phase based position algorithms, including the van Graas and Lee algorithm [8], use double differenced measurements. Each equation in the double differenced measurement equations involves inputs from two satellites (see (2)) , making it difficult to detect which satellite, if any, is faulty, just by dropping individual equations.

2. **Satellite requirements**. Usually more satellites are needed for carrier phase based positioning algorithms than for code based positioning algorithms, because the carrier phase case involves unknown integer ambiguities. We want to keep the satellite requirements reasonable, while still obtaining reliable results.

3. **Nonlinearity**. Remark 2.1 emphasized the nonlinearity inherent in the problem. This is particularly difficult for integrity testing, for if there is a faulty measurement, then this will usually lead to a poor estimate of the position, and so to an inaccurate matrix in the model.

4. **Ramp faults**. Step faults should be reasonably easy to detect immediately after they happen. But ramp faults that are slowly increasing may lie below the detection threshold for several time steps. During that time they could be altering the position estimates, and then the nonlinearity just mentioned could lead to the resulting inaccurate model somewhat disguising the presence of the faults. It is important to detect such faults, see [7].

The above difficulties (except nonlinearity, which can not be eliminated) can be ameliorated to some extent once we realize we can do our tests as a separate, but parallel computation to the position estimation. That is, we are not restrained by the form of the position estimation model, and so can develop and work with a very different model for testing, which might however use the results from the position estimation computation.

6 THE 2-STEP MODEL FOR CARRIER PHASE INTEGRITY TESTS

It was shown in [1] that careful use of single differences is in general preferable to using double differences. Here we will show how the difficulty caused by each double difference measurement involving two satellites can be resolved by unwinding the double difference measurement equation back to a single difference measurement equation of the form (1). This reintroduces the unknown $\Delta\beta_k$, but we will show how to handle that in the development. Note how any computation using triple differences, see (4), is using information from two time steps. We will show how it is possible to combine the same two time steps in a very different way to attain our integrity test models.

The 2-step model for carrier phase based GPS integrity tests is obtained by stacking (1) on top of (1) for the previous time step,

$$y_k^{SD} = \lambda^{-1}E_k^{SD}x_k + a^{SD} + e\Delta\beta_k + v_k^{SD},$$
$$y_{k-1}^{SD} = \lambda^{-1}E_{k-1}^{SD}x_{k-1} + a^{SD} + e\Delta\beta_{k-1} + v_{k-1}^{SD}. \quad (21)$$

Here we assume there is no relationship between $\Delta\beta_{k-1}$ and $\Delta\beta_k$, so this approach can even handle the case of a step error in the clock in either the roving or the stationary receiver. By this we mean that any change from $\Delta\beta_{k-1}$ to $\Delta\beta_k$ will appear as an *unknown* in the model here, and will not lead to a significant residual signalling a fault. This is exactly what double differencing wants, so *this approach will maintain all the advantages of double differencing.*

To emphasize this, we note that (2) and (4) follow from (21), so everything we can learn from (2) and (4) is already contained in (21). Double differencing is just one way of eliminating $\Delta\beta_{k-1}$ and $\Delta\beta_k$, but any results we can compute from (2) and (4) can necessarily be found (apart from rounding errors) from (21).

The equations (21) then give with $s_k \equiv a^{SD} + e\Delta\beta_k$ and $\Delta_{2,k} \equiv \Delta\beta_{k-1} - \Delta\beta_k$,

$$\begin{bmatrix} y_k^{SD} \\ y_{k-1}^{SD} \end{bmatrix} = \begin{bmatrix} \lambda^{-1}E_k^{SD} & 0 & 0\ I_{m_s} \\ 0 & \lambda^{-1}E_{k-1}^{SD} & e\ I_{m_s} \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-1} \\ \Delta_{2,k} \\ s_k \end{bmatrix} + \begin{bmatrix} v_k^{SD} \\ v_{k-1}^{SD} \end{bmatrix}. \quad (22)$$

Now $v_k^{SD} \sim N(0, \sigma^2 I_{m_s})$, while v_k^{SD} and v_{k-1}^{SD} are independent, and with obvious notation this becomes (where we drop the time step indices k for simplicity)

$$y = Gz + v, \qquad v \sim N(0, \sigma^2 I_{2m_s}). \quad (23)$$

Remark 6.1 *We see from the covariance matrix that the noise elements are independent of each other, and any one of the equations can be dropped without affecting the others. Clearly each of the first m_s equations corresponds to a distinct satellite, and this independence avoids the first difficulty mentioned in Section 5, so that the typical code based approaches to fault detection, identification, and HPL are immediately applicable to (23), see Remark 3.1.*

The 2-step model (22)–(23) gives us a basis to work from. There are $2m_s$ equations in $m_s + 7$ unknowns, so

$$G \text{ full column rank} \Rightarrow m_s \geq 7.$$

We discuss this rank again later. If the matrix G has full column rank, we have the following analysis of satellite requirements using this model alone. If there are $m_s = 7$ satellites we have 14 equations in 14 unknowns, and so we could estimate the position (and s_k and $\Delta_{2,k}$), but the residual and parity vectors will be zero, so there can be no fault detection. If there are $m_s = 8$ satellites we have 16 equations in 15 unknowns, so we can also do fault detection, but not identification. If there are $m_s = 9$ satellites we have 18 equations in 16 unknowns, and we can do both fault detection and identification. Summarizing for this 2-step equation removal method:

Purpose	Minimum # of satellites
Position estimation	7
Fault detection	8
Fault identification	9

Because the model (22)–(23) is essentially independent of the position estimates in [8], it does not give very useful HPL for those estimates, see Remark 6.4 later. However it does deal directly with the measurements from the satellites, and so can be used, at least in theory, to detect and identify faults in these measurements. A possible algorithm would be (when sufficient satellites are available):

- Use the position estimates from [8] to approximate the submatrices in G, $\lambda^{-1}E_k^{SD} := \lambda^{-1}E(\hat{x}_k)^{SD}$, $\lambda^{-1}E_{k-1}^{SD} := \lambda^{-1}E(\hat{x}_{k-1})^{SD}$.
- Apply a typical approach to code based fault detection to (22), to detect a possible fault in one of the first m_s equations.
- If a fault is detected, apply a typical approach to code based fault identification to (22).

Note that this is a nonlinear model, and a faulty measurement could give a faulty $\hat{x}_k$, which could give an inaccurate G. We have seen that the typical approaches to fault detection and identification correspond to eliminating certain measurement equations, leading to $\bar{G}$ say in (23). When we do this, we could use the remaining equations to re-estimate $\hat{x}_{k-1}$ and $\hat{x}_k$ and update $\bar{G}$, perhaps iteratively.

A subtle difficulty that this model can handle is the following. The above algorithm is fine if there is no fault at time $k-1$, but a clear fault appears at time k. However if there is a very slowly increasing (or ramp) fault in the i-th satellite say, it may not be detected when it first starts, so that in later steps it will be present in both the i-th and (m_s+i)-th equations in (22). Thus a satellite removal method of fault detection and identification could look at deleting both equations i and $i+m_s$, in a series of tests for $i = 1, \ldots, m_s$. Remember we refer to this as the satellite removal method, to distinguish it from the equation removal method which deletes single equations, rather than all equations corresponding to a given satellite.

The handling of the satellite removal method is also subtle — when equations i and $i+m_s$ are both deleted from (22) we see that the i-th element of s_k no longer appears in the remaining equations, and so it can be dropped too. This leaves $2m_s - 2$ equations in $m_s + 6$ unknowns. Thus when $m_s = 8$, deleting the *two* equations takes (22) from a 16×15 system to a 14×14 system, and again fault detection can be carried out with only 8 satellites. Similarly if $m_s = 9$, deleting equations i and $i+m_s$ takes it from an 18×16 system to a 16×15 system, then deleting equations j and $j+m_s$ takes it from a 16×15 system to a 14×14 system. It follows that any of the typical code based approaches for fault identification could be used, leading to the *same* satellite requirements for this *satellite removal* 2-step method:

Purpose	Minimum # of satellites
Position estimation	7
Fault detection	8
Fault identification	9

For this simple 2-step model, if there is a step fault, then both methods should find it, but if there is a slow ramp fault, the satellite removal method could well pick it up sooner than the equation removal method. If only one is to be used, then we recommend using the satellite removal method.

Let us now examine the rank of G in (22)–(23). We assumed for (2) that E_k^{DD} had full column rank. If it did not have, then the least squares estimate for x_k in (5), see (6), would not be uniquely defined, and we could not have integrity. Since from (1)–(2)

$$E_k^{SD} = \begin{bmatrix} (e_k^1)^T \\ \cdot \\ (e_k^{m_s})^T \end{bmatrix}, \quad E_k^{DD} = JE_k^{SD} = \begin{bmatrix} (e_k^1 - e_k^2)^T \\ \cdot \\ (e_k^1 - e_k^{m_s})^T \end{bmatrix}, \tag{24}$$

it is clear that E_k^{SD} must have full column rank too. Suppose G does *not* have full column rank. Consider nonzero w such that

$$Gw = \begin{bmatrix} \lambda^{-1}E_k^{SD} & 0 & 0 & I_{m_s} \\ 0 & \lambda^{-1}E_{k-1}^{SD} & e & I_{m_s} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \omega \\ w_3 \end{bmatrix} = 0,$$

$$\text{so} \qquad -\lambda w_3 = E_k^{SD}w_1 = E_{k-1}^{SD}w_2 + e\omega\lambda.$$

If $w_3 = 0$ then $w_1 = 0$, and

$$E_{k-1}^{DD}w_2 = JE_{k-1}^{SD}w_2 = -Je\omega\lambda = 0,$$

so $w_2 = 0$, showing that $\omega = 0$ and so $w = 0$, a contradiction. Thus $w_3 \neq 0$ and

$$-\lambda w_3 = E_k^{SD}w_1 = E_{k-1}^{SD}w_2 + e\omega\lambda \neq 0.$$

It follows that $w_1 \neq 0$, and so $w_2 \neq 0$, since

$$0 \neq E_k^{DD}w_1 = E_{k-1}^{DD}w_2.$$

Remark 6.2 *This shows if the column spaces of E_k^{DD} and E_{k-1}^{DD} do* not *intersect, G has full column rank. Since E_k^{DD} has full column rank and is $(m_s - 1) \times 3$ we must have $m_s \geq 4$ for G in (22)-(23) to have full column rank, so from now on assume $m_s \geq 4$. Our*

computations suggest full column rank will be the usual case for $x_{k-1} \neq x_k$, but since x_{k-1} and x_k are fairly close, G in (22) can be poorly conditioned, where we have often found condition numbers as bad as 10^6 for the 2-step matrix.

Remark 6.3 *This leads to a possible disadvantage of this 2-step model. We showed in Section 3 that the statistics used depend on Q_2, where $G = Q_1 R$ is the QR factorization of G, and $[Q_1, Q_2]$ is an orthogonal matrix. Q_2 always has condition number unity, and all the ill-condition of G is contained in R, however the* sensitivity *of Q_2 depends on the elements of R, see for example [2]. Our initial theoretical analysis shows it is possible that small errors in the model could lead to significant errors in the computed statistics.*

Remark 6.4 *Estimating x_k from (22)-(23) will usually be quite inferior to the approach in [8], since (22)-(23) would only use these two steps, while [8] use information from all previous steps as well. When the [8] estimate has converged, our computations clearly confirm this. Thus the resulting error covariance matrix from (22)-(23) is also likely to be too large in norm, and although we could obtain HPL for the estimates from (22)-(23), they would almost certainly be too large to be useful. A poorly conditioned G (see Remark 6.2) corresponds to an error covariance matrix of large norm, which confirms this suspicion. In fact computations showed the 2-norms of the* position *error covariance matrices from this approach can be 3 orders of magnitude larger than those from the positioning algorithm in [8].*

7 THE j-STEP MODEL FOR CARRIER PHASE INTEGRITY TESTS

An ineradicable difficulty of the 2-step model for carrier phase based GPS integrity tests is that it requires at least 8 satellites for fault detection. One possible way of overcoming this difficulty is to expand the 2-step model (21)–(23) to a j-step model, for not too large $j = 2, 3, \ldots$, and we now examine this. With $s_k \equiv a^{SD} + e\Delta\beta_k$ as before, and $\Delta_{i,k} \equiv \Delta\beta_{k-i+1} - \Delta\beta_k$ the last j steps of (1) can be written

$$\begin{aligned} y_k^{SD} &= \lambda^{-1} E_k^{SD} x_k + s_k + v_k^{SD}, \\ y_{k-i+1}^{SD} &= \lambda^{-1} E_{k-i+1}^{SD} x_{k-i+1} + s_k + e\Delta_{i,k} + v_{k-i+1}^{SD}, \\ & \qquad i = 2, 3, \ldots, j, \end{aligned} \tag{25}$$

which for $j = 2$ is equivalent to (21)–(23). In matrix form

$$\begin{bmatrix} y_k^{SD} \\ y_{k-1}^{SD} \\ \cdot \\ y_{k-j+2}^{SD} \\ y_{k-j+1}^{SD} \end{bmatrix} = \begin{bmatrix} \lambda^{-1}E_k^{SD} & 0 & 0 & \cdot & 0 & 0 & 0 & I_{m_s} \\ 0 & \lambda^{-1}E_{k-1}^{SD} & e & \cdot & 0 & 0 & 0 & I_{m_s} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & e & 0 & 0 & I_{m_s} \\ 0 & 0 & 0 & \cdot & 0 & \lambda^{-1}E_{k-j+1}^{SD} & e & I_{m_s} \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-1} \\ \Delta_{2,k} \\ \cdot \\ \Delta_{j-1,k} \\ x_{k-j+1} \\ \Delta_{j,k} \\ s_k \end{bmatrix} + \begin{bmatrix} v_k^{SD} \\ v_{k-1}^{SD} \\ \cdot \\ v_{k-j+1}^{SD} \\ v_{k-j+1}^{SD} \end{bmatrix}. \tag{26}$$

Now $v_k^{SD} \sim N(0, \sigma^2 I_{m_s})$, while $v_{k-j+1}^{SD}, \ldots, v_k^{SD}$ are independent, and with obvious notation this becomes (where we drop the time step indices k for simplicity)

$$y = Gz + v, \qquad v \sim N(0, \sigma^2 I_{jm_s}). \tag{27}$$

Using a similar argument to that which led to Remark 6.2, it is straightforward to show that for this j-step G to have less than full column rank, the column spaces of $E_{k-j+1}^{DD}, \ldots, E_k^{DD}$ must *all* have a *common* nonzero subspace. Again we will assume $E_{k-j+1}^{DD}, \ldots, E_k^{DD}$ all have full column rank, and $m_s \geq 4$. Since the above criterion becomes more restrictive as j increases, this suggests the condition of j-step G will not get worse (the condition number will not increase) as j increases.

It is clear that Remark 6.1 also applies here, so that the typical code based approaches to fault detection, identification, and HPL are immediately applicable to (27).

The j-step equation removal method will eliminate just one equation to test for a possible fault, and two to identify a faulty satellite. A key property of this j-step model (of which the 2-step model is a special case) is the existence of the satellite removal method, which for fault detection will eliminate the i-th satellite entirely from the model, along with one unknown, the i-th element of s_k. As we mentioned earlier for the 2-step model, this may be useful for handling slowly increasing (ramp) faults.

We now analyze the satellite requirements. The j-step model has $m_s j$ equations in $m_s + 4j - 1$ unknowns. Thus to solve the system we need $m_s \geq 4 + 3/(j-1)$ satellites. For the equation removal method we eliminate just one equation at a time, so (assuming our

matrices still have full column rank) our requirements are: for fault detection $m_s \geq 4 + 4/(j-1)$, and for fault identification $m_s \geq 4 + 5/(j-1)$.

	Minimum # of satellites				
Using j-steps:	2	3	4	5	≥ 6
Position estimation	7	6	5	5	5
Fault detection	8	6	6	5	5
Fault identification	9	7	6	6	5

This at first seems attractive with $j = 4$, as we can obtain statistics for fault identification with only 6 satellites, and with $j = 5$ we can obtain statistics for fault detection with only 5 satellites. However this may not be good at detecting and identifying slowly increasing faults.

If we use the satellite removal j-step method which eliminates all appearances of the i-th satellite to obtain fault detection statistics (in the hope of superior ramp fault detection), we have the same satellite requirements for position estimation and fault detection, see Section 4. But now the original $m_s j$ equations in $m_s + 4j - 1$ unknowns becomes $(m_s - 1)j$ equations in $m_s + 4j - 2$ unknowns for the fault detection model. Assuming this has full column rank, we need $m_s > 5 + 3/(j-1)$ satellites for this to possibly have a nonzero residual, and so to have fault identification. Our requirements for the satellite removal j-step method are then:

	Minimum # of satellites			
Using j-steps:	2	3	4	≥ 5
Position estimation	7	6	5	5
Fault detection	8	6	6	5
Fault identification	9	7	7	6

Suppose there is a very slowly increasing fault, and it is not detected even after several steps. Now suppose we have 6 satellites and have decided to use the 5-step model. If we eliminate just one equation at a time, then since eliminating either 1 or 2 equations will still leave faulty equations in the set, this will probably leave significant $\|p_i\|_2$ and $\|p_{i,l}\|_2$ for all i and l, and this approach will not be ideal. On the other hand eliminating all appearances of the faulty satellite *will* (at least in theory) give us useful statistics. Thus, *if* we use this approach, we recommend using the *satellite* removal 5-step method to obtain statistics for fault detection and identification if at least 6 satellites are available (or for just fault detection if only 5 satellites are available).

The use of this j-step method with $j > 2$ will probably give smaller error covariance matrices and more useful HPL than for $j = 2$, but because of the ill-condition of G, these radius will still probably be too large until we use an impractically large j. However if we restrict ourselves to using the van Graas and Lee [8] approach for estimation, the satellite removal method applied to these j-step models is the only approach we have found so far that (at least in theory) provides a way of detecting and identifying slowly increasing faults as quickly as possible. Thus some such approach may be needed to meet the requirements of [7]. If this is so, then for fault detection and identification we could consider using the satellite removal 5-step model, but in conjunction with a different method for computing HPL. So we need to provide such a method.

Unfortunately for small j (so the computational load is acceptable) the ill-condition mentioned in Remark 6.2 will be present. This could also limit the reliability of the fault detection and identification tests, see Remark 6.3. For this reason alone it again seems necessary to consider another model based on single difference measurements.

8 THE PRIOR INFORMATION MODEL FOR CARRIER PHASE INTEGRITY TESTS

Here we produce a more efficient model by including the results of the van Graas and Lee [8] algorithm in our integrity testing. First we look at what seems possible.

Since a^{SD} is constant, the equation (1) corresponds to m_s new single difference measurements (one for each satellite) and only 4 *new* unknowns ($\Delta\beta_k$ and the 3 elements of x_k) at each time step. This supports the comment in [8] that once a good estimate has been attained, only 4 satellites are needed to maintain accurate position estimates. One way of thinking of this is that once we have an accurate estimate of a^{SD}, then $m_s = 4$ equations are sufficient to accurately estimate the 4 new unknowns each step.

The total number of equations and unknowns shows we would not expect to obtain *convergence* with only 4 satellites for the general problem, but in some cases such as very slowly varying $\Delta\beta_k$, this may be possible. However we could hope to eventually obtain convergence of our estimates with signals from only $m_s = 5$ satellites, from the following argument. If the system was linear we could argue that at the k-th time step there is a total of $m_s k$ equations and $m_s + 4k$ unknowns, so with $m_s = 5$ there would be the same number of equations as unknowns at $k = 5$ steps, and more equations than unknowns when $k > 5$. Five is not a large number, and the GPS carrier phase system is not highly nonlinear, so without further analysis it at least seems plausible to obtain convergence with only

5 satellites. And if convergence is obtained, we could theoretically test for faults using just 5 satellites, since at each time step we would have $m_s = 5$ equations in only 4 new unknowns.

We now show that this last hypothetical conclusion is in fact true.

Assume we have unbiased estimates $\hat{x}_k$ of x_k, $\hat{x}_{k-1}$ of x_{k-1}, and $\hat{a}^{DD}_{k-1}$ of a^{DD}, obtained from the algorithm of van Graas and Lee [8], see (17), such that

$$\begin{bmatrix} \tilde{x}_{k-1} \\ \tilde{a}^{DD}_{k-1} \end{bmatrix} \equiv \begin{bmatrix} \hat{x}_{k-1} - x_{k-1} \\ \hat{a}^{DD}_{k-1} - a^{DD} \end{bmatrix} \sim N\left(0, H_{k-1} \equiv \begin{bmatrix} H^{(11)}_{k-1} & (H^{(21)}_{k-1})^T \\ H^{(21)}_{k-1} & H^{(22)}_{k-1} \end{bmatrix}\right). \quad (28)$$

Writing $s_k \equiv e\Delta\beta_k + a^{SD}$ as before, so that

$$Js_k = a^{DD}$$

for the J given in (2.1), and computing $E^{SD}_k := E(\hat{x}_k)^{SD}$, we see we effectively have with (1)

$$\begin{bmatrix} y^{SD}_k \\ \hat{a}^{DD}_{k-1} \end{bmatrix} = \begin{bmatrix} \lambda^{-1}E^{SD}_k & I_{m_s} \\ 0 & J \end{bmatrix} \begin{bmatrix} x_k \\ s_k \end{bmatrix} + v_k, \quad (29)$$

$$v_k \equiv \begin{bmatrix} v^{SD}_k \\ \tilde{a}^{DD}_{k-1} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma^2 I_{m_s} & 0 \\ 0 & H^{(22)}_{k-1} \end{bmatrix}\right),$$

since v^{SD}_k is independent of $\tilde{a}^{DD}_{k-1}$. $H^{(22)}_{k-1}$ is positive definite, so this model can be transformed to one with noise covariance I.

We have assumed E^{DD}_k has full column rank, and we now show this implies the measurement matrix in (29) has full column rank. Remember $E^{DD}_k \equiv JE^{SD}_k$ is $(m_s - 1) \times 3$, requiring $m_s \geq 4$ for full column rank, and $Je = 0$ (see (2.1) *et seq.*), so

$$\begin{bmatrix} \lambda^{-1}E^{SD}_k & I_{m_s} \\ 0 & J \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = 0$$
$$\Rightarrow \quad Jz_2 = 0, \quad \lambda^{-1}E^{SD}_k z_1 + z_2 = 0$$
$$\Rightarrow \quad \lambda^{-1}E^{DD}_k z_1 = 0,$$

leading to $z_1 = 0$ and $z_2 = 0$, thus the measurement matrix has full column rank.

Thus (29) describes $2m_s - 1$ equations in $m_s + 3$ unknowns. With $H^{(22)}_{k-1}$ positive definite, (29) can be converted to a standard linear model, which will then have a unique least squares solution when $m_s \geq 4$. Any of the first m_s equations can be deleted without affecting the others, and the typical code based methods for fault detection, identification, and computation of HPL can be applied here. This gives the following satellite requirements (we assume deleting the row leaves the matrix with full column rank):

Purpose	Minimum # of satellites
(Position estimation	4)
Fault detection	5
Fault identification	6

The parentheses are used because this approach already uses the van Graas and Lee [8] estimate of x_k to compute E^{SD}_k, and the estimate $\hat{a}^{DD}_{k-1}$ in the left hand side of (29). We do not need to estimate x_k again. However it would be interesting to do so and compare the two. Our computations showed the two had almost the same accuracy in a run starting with an initial position error of 1 metre.

The minimum number of 4 satellites required for positioning here (when we have convergence) at least supports the comment by van Graas and Lee [8].

Since $\hat{a}^{DD}_{k-1}$ has been estimated from all previous measurements, this approach does not have the ability to throw out the i-th satellite measurement at any but the present time step — unlike the model in Section 6. It is possible to add the information $\hat{a}^{DD}_{k-2}$ to (1) at times $k-1$ and k, and use the vector of 'knowns'

$$\begin{bmatrix} y^{SD}_k \\ y^{SD}_{k-1} \\ \hat{a}^{DD}_{k-2} \end{bmatrix}$$

in the model, and then we could throw out the i-th satellite measurement from time steps $k-1$ and k, and so on. It can be shown that such extended models will have the same satellite requirements as (29), but this probably also increases the ill-condition of the matrix in the model.

9 SUMMARY AND CONCLUSIONS

Carrier phase based integrity monitoring is crucial to some GPS applications like aircraft landing. We have provided essentially two classes of models that could be used for checking the integrity of position estimates for this carrier phase problem, and we called these:

- *The j-step model.* The 2-step model is a special case of this. This is based solely on single difference measurement equations, and can be used for fault detection and identification. It could also be used for computing HPL, but unless j is so large as to make efficient computations impractical, the HPL will almost certainly be too large to be useful.

  If only two epochs are used, this approach requires at least 7 satellites to estimate position, 8 to detect a fault, and 9 to identify the faulty satellite. However if more epochs are used, fewer

satellites are needed. With 4 epochs the satellite removal method designed to be sensitive to ramp faults requires 5, 6 and 7 satellites respectively. With 6 epochs an equation removal method (which might be less effective for ramp faults) requires only 5, 5, and 5 satellites respectively. This matches the requirements suggested by the comment in [8, p.610].

- *The prior information model.* This is based on single difference measurements and the unbiased estimates $\hat{a}^{DD}_{k-1}$ (obtained from the van Graas and Lee algorithm [8]) of a^{DD}, the vector of double differences of the integer ambiguities. This appears practical for fault detection and identification, and for computing HPL.

  For fault detection and identification, at least 5 and 6 satellites are required, respectively.

Our initial simulations suggest both approaches perform very well for detection and identification, while the second approach gives better HPL.

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