Computeational Thinking:
What is the science in computer science?

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Computer science is no more about computers than astronomy is about telescopes.

– E. Dijkstra
“Computational Thinking will be a fundamental skill used by everyone in the world in the middle of the 21st century”

A grand vision

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- In many areas, it is already a reality: engineers, biologists, chemists, physicist... anthropologist, psychologists, economist, criminologist
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What is computational thinking?

Computational thinking is the key
• for solving problems
• for achieving what one human alone cannot do
• for understanding the power and limits of human intelligence and capabilities of machines

What it is and what it is not
• Conceptual – not just about writing programs
• A way humans think – not computers
• Fundamental – not rote, mechanical skill

Computational thinking is drawing fundamentally
on concepts from computer science!
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The two A’s of computational thinking

- Abstraction
  - Helps solve problems
  - Transfer and solve multiple problems
  - Multiple layers of abstraction - Different view points

- Automation
  - Goal is to execute, simulate, observe behavior of abstract problem descriptions
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Computer Science is a science of abstraction – creating the right model for a problem and devising the appropriate mechanizable techniques to solve it.

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**Example, please!**
Example

Search for me in the phone book
Search for me in the phone book

Lessons learned

• We need a way to describe what we did
• We can apply our solution to other search problems
  Search for an artist on your ipod; search for a participant in a list
• There are different solutions; which is the best one?
• What are the requirements we exploited?
Four axes of computational thinking

1. Document solutions (to be able to revisit later)
2. Communicate solutions (to your friend, co-worker, etc.)
3. Analyze and study solutions (Which one is better? Why? Equally powerful?)
4. Implement solutions
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This talk: Computer science has diverse roots

- From flow charts to abstract machines (Engineering)
  Focus on 1 and 2
- Reasoning in ancient and modern times (Philosophy, Mathematics)
  Focus on 2, 3 and 4
- Thinking about the limits and power of computation
From flow charts to abstract machines
When did flow charts originate?

- 1921: F. Gilberth introduces it to the American Society of Mechanical Engineers (ASME)
- 1930s: Industrial engineering curricula
- 1940s: Reaches industry
- 1950s: Model computer programs

What is their use today?

- Unified Modelling Language (UML)
- Used pervasively in software engineering.
Flow charts in other sciences

Used commonly in other science

- Medicine
- Chemistry
- Biology
- Economics
- ...
Flow charts can be complex!

Advantages

- Organize the thoughts
- High-level view; abstract over details;
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- Interactions can be complex!
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**Disadvantages**

- Interactions can be complex!
- No rigorous analysis possible
  What is the shortest path? What components are connected?
- Hard to communicate the meaning.
  What is the meaning of each line? What is the meaning of the colors?
- Deriving an implementation not always obvious
  How can we implement and mechanize the process?
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Analyzing flow charts = Study graphs

- Abstract representation of a set of objects where some pairs of the objects are connected by links.

Undirected graph

Directed graph

Weighted graph

- Some questions about graphs:
  - What is the shortest path?
  - Can we partition graphs in strongly connected components?
  - Optimal route through a graph connecting two nodes?
  - Can we color the nodes such that directly connected nodes will have different colors?
Implementing flow charts = Finite state machine

- Composed of a finite number of states, transitions between those states, and actions - Special case of a graph
- More precise, computational model than flow charts
- Allows us to execute, simulate and observe behavior
• **Abstract machines** are a theoretical model of a computer and allow us to describe our problem computationally
  
  – Finite State Machines / Automata
  – Turing Machines
  – Post Machines
  – Register machines
  – ...  

⇒ Amazing fact: These are all formally equivalent!

⇒ Whether you have a Mac or a PC they can compute the same things (theoretically) and are subject to the same limitations.

Church-Turing thesis:

Abstract machines capture the informal notion of computability.
Reasoning in ancient and modern times
Long before there were computers, people were interested in describing computations and thinking computationally.

- Gottfried Wilhelm Leibniz (1646–1716): Philosopher, mathematician (inventor of calculus), built calculating machine, binary arithmetic
- Leibniz’s dream: Compile an *encyclopedia* of all human knowledge.

‘The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate [calculemus], without further ado, to see who is right.’ (The Art of Discovery 1685, W 51)
Propositional and predicate logic

- George Boole (1815–1864) : Propositional logic
  Logical reasoning ↔ algebraic reasoning! Formulate the laws of reasoning about classes (concepts) in analogy to the rules of algebra

- Gottlob Frege (1848–1925) : Predicate logic
  - Boole’s logic is not expressive enough.
  - We need to express (quantifiers):
    ‘For all x, …’ and ‘There exists an x, such that …’

- Notation that allows to make proofs gap-free: mechanical, without recourse to intuition
  E. g., ‘All amounts a, if a > 2 and I buy a sandwich then the remaining amount is a − 2.

  Modern notation:
  \[ \forall a. \left( \text{amount}(a) \land a > 2 \land \text{buy\_sandwich} \rightarrow \text{amount}(a - 2) \right). \]

- Modus ponens with axioms of arithmetic
Modelling Computation using Logic

- Unifying foundational framework
- Powerful tool for modeling and reasoning about aspects of computation i.e. correctness
- Computation $\equiv$ Constructing a proof
  $\implies$ Logic programming
- Translate state transition systems into logic!

"I expect that digital computing machines will eventually stimulate a considerable interest in symbolic logic ... The language in which one communicates with these machines ... forms a sort of symbolic logic."

A. Turing
Modelling Computation using Functions

- $\mu$-recursive functions (Gödel 1930s): precisely describe what is computable
- The $\lambda$-calculus (Church 1936): a calculus of anonymous functions. e.g., $(\lambda x. 2x)$ instead of $f(x) = 2x$
- Modelling Vending Machine:

\[
\begin{align*}
buy\_sandwich(a) &= \begin{cases} 
a - 2 & a \geq 2 \\
\bot & \text{otherwise}
\end{cases} \\
buy\_coke(a) &= \begin{cases} 
a - 1 & a \geq 1 \\
\bot & \text{otherwise}
\end{cases}
\end{align*}
\]

- Computation = Evaluating functions $\implies$ Functional Programming
**Programs = Proofs**

- Gentzen (1935): Calculus of Natural Deduction
  
  Calculus to capture reasoning practice

- Curry (1958): Observed a connection between logic and functions

- Howard (1969): Observed there is an isomorphism between
  
  proofs $\iff$ functions
  
  proposition $\iff$ types
  
  proof transformations $\iff$ function evaluation

- Highly influential to programming and language design

“For my money, Gentzen’s natural deduction and Church’s lambda calculus
are on a par with Einstein’s relativity and Dirac’s quantum physics for
elegance and insight. And the maths are a lot simpler. “

P. Wadler
Programming language paradigms

- **Abstract machines** → **imperative programming**
  - Series of instructions and commands; loops
  - Maintain state implicitly or explicitly
  - Examples: Assembler, C, Basic, C++, Java

- **Logic** → **logic programming**
  - No state
  - Driving force: Subset of first-order logic
  - Computation = proof search
  - Examples: Prolog, Datalog, logical frameworks

- **λ-calculus** → **functional programming**
  - Avoid state
  - Driving force: Recursion, functions, data-types
  - Computation = application of functions to arguments
  - Examples: Lisp, ML, Haskell, F#

“A language that doesn’t affect the way you think about programming, is not worth knowing.”

- Alan Perlis
Thinking about the limits and power of computation
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- What are the limitations of computations?
- Can we solve any problem?
- Can we solve any problem \textit{efficiently}, i.e. in practice?
Limitations of computing

• Does there exist a yes-or-no answer for every problem?
  ⇒ Turing (1936) Halting problem
  ⇒ Church (1936) Decision problem

• Are there problems which cannot be computed?
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• Suppose that solutions to a problem can be verified quickly. Then, can the solutions themselves also be computed quickly?
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- Suppose that solutions to a problem can be verified quickly. Then, can the solutions themselves also be computed quickly?
  - Stipulated by S. Cook in 1971 – as of today: unsolved
  - 1 Million Dollar prize for a solution offered by the Clay Institute of Mathematics!
$1 \text{ M Question: } P = \text{NP?}$

Example NP-Problem:

Suppose that you are organizing housing accommodations for a group of 400 students. Space is limited and only 100 students will receive places in the dormitory. To complicate matters, the Dean has provided you with a list of pairs of incompatible students, and requested that no pair from this list appear in your final choice.

- Easy to check if solution is satisfactory
- Generating a solution is hard – it is completely impractical!
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  \( N = \text{combinations of choosing 100 students from 400 applicants} \)
  \( N > \text{number of atoms in the universe} \)
- The problem cannot be solved by brute force – but can we compute a solution quickly by some procedure?
Consequences of $P = NP$

Negative consequences:

- Cryptography: We rely on certain problems being difficult; A constructive, efficient solution could break many existing cryptosystems such as Public-key cryptography which is used in transactions with banks, online shopping sites, etc.

Positive consequences (enormous!):

- Rendering tractable many currently mathematically intractable problems.
- Some NP problems: Travelling salesman problem, logistics, protein structure prediction, ... .

“...it would transform mathematics by allowing a computer to find a formal proof of any theorem which has a proof of a reasonable length, since formal proofs can easily be recognized in polynomial time. Example problems may well include all of the CMI prize problems.”

Stephen Cook
Beyond the computer as a machine

What is computable when we consider the computer as the combination of human and machine? How can we exploit the limitations of computing?
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What is computable when we consider the computer as the combination of human and machine? How can we exploit the limitations of computing?

**Captcha** : Completely Automated Public Turing test to tell Computers and Humans Apart

- Challenge-response test used in computing to ensure that the response is not generated by a computer.
Take home message

- Computational thinking is a fundamental skill – it is learning to think at different layers of abstractions. It is as fundamental as knowing some basics about probability theory or discrete algebra.

- It has fascinated human beings in the past – it continues to fascinate us today.

- Computer science has deep philosophical, mathematical and engineering challenges.

- Computers (machine and human!) allow us
  - to go beyond solving problems on paper
  - to go beyond what one human being could achieve
  - to explore and understand our surrounding world

- Computer science is about computational thinking – it is challenging and tests the limits of our creativity and intelligence.
That’s it!
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Thank you.