Evolutionary game theory and cognition

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Two player games

- A game between two players (Alice and Bob) is represented by a matrix G of pairs.

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- If Alice plays strategy i and Bob plays strategy j then $(a,b) := G_{ij}$ is the outcome, where a corresponds to the change in Alice's utility and b to Bob's.


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- Zero-sum games are the epitome of competition. Any gain for Alice is a loss for Bob, and vice-versa.

Coordination games

Definition

A two-strategy game G is a coordination game if we have

$$G = \begin{pmatrix} (a_1, b_1) & (c_2, d_1) \\ (c_1, d_2) & (a_2, b_2) \end{pmatrix}$$

And $a_1 > c_1$, $a_2 > c_2$, $b_1 > d_1$, $b_2 > d_2$.

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- The diagonals are always better for both players, they just have to figure out how to pick the same strategy.
- Captures the idea of win-win, lose-lose situations.

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- Being non-zero-sum does not ensure cooperation.

Prisoner's dilemma

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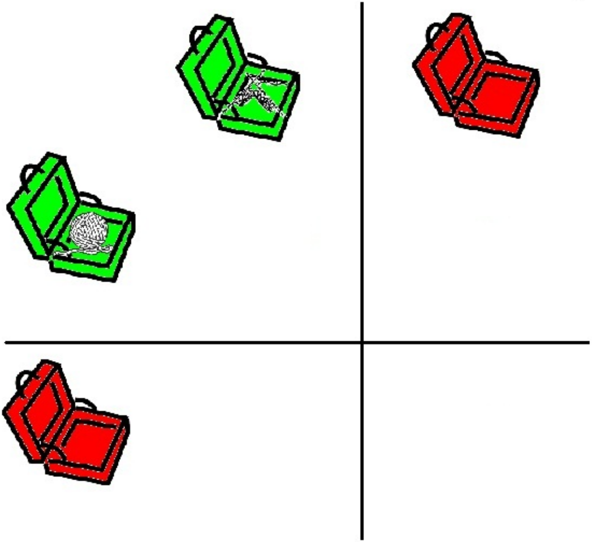

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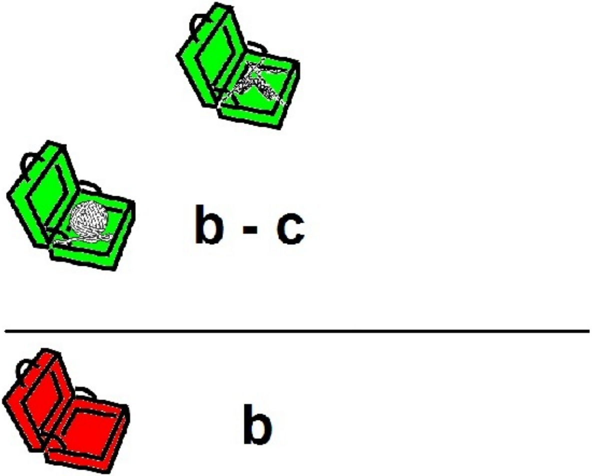

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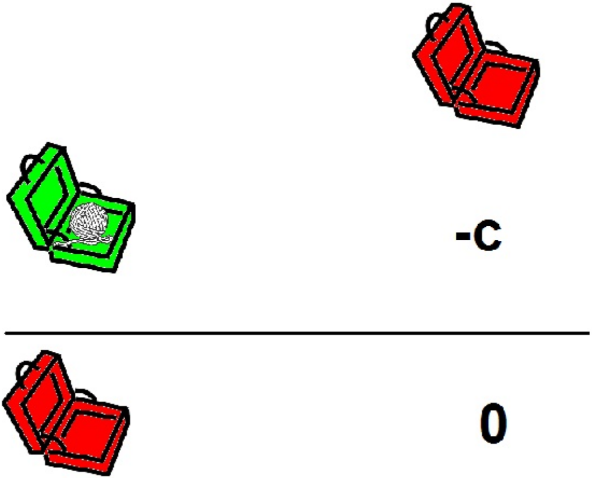

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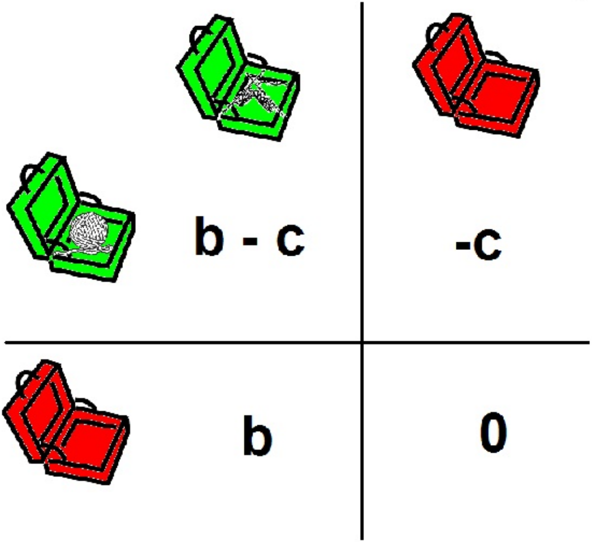

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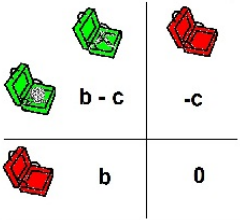

$$\begin{pmatrix} (b-c, b-c) & (-c, b) \\ (b, -c) & (0, 0) \end{pmatrix}$$

- b is the benefit of receiving and c is the cost of giving.
- Strategy 1 is called cooperate or C and strategy 2 is called defect or D.

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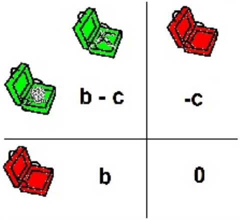

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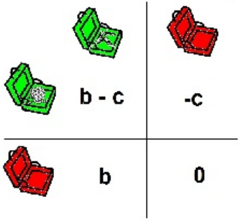

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- Strategy 1 is called cooperate or C and strategy 2 is called defect or D.
- The rational strategy (or Nash equilibrium) is mutual defection.
- The best for the players taken together (or Pareto optimum) is mutual cooperation.

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- Neither player is preferred or treated differently: every player is identical with respect to the game rules.
- The representation of a game does not require writing those confusing pairs
- For example, all 2 player 2 strategy symmetric games can be written in the form:

$$\begin{pmatrix} (R,R) & (S,T) \\ (T,S) & (P,P) \end{pmatrix} \to \begin{pmatrix} R & S \\ T & P \end{pmatrix}$$

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- This captures the interesting two player games.
- Allows us to reduce the general game to two parameters by removing constant offset and picking our units:

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} \to \begin{pmatrix} 1 & U \\ V & 0 \end{pmatrix}$$

U-V plane

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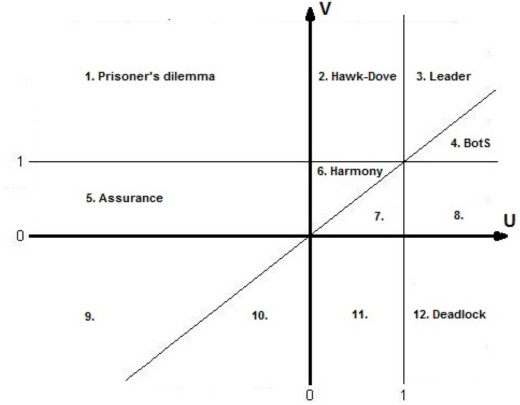

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- If we only allow pure strategies then replace $G(i, j)$ by G_{ij}
- If we allow mixed strategies, then every game has at least one Nash equilibrium

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Example

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- Let's bound rationality and see what happens!

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- What happens to rationality?

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A strategy s is an evolutionary stable strategy for a game G if for all other strategies r we have (a) $fst(G(s, s)) > fst(G(r, s))$, or (b) $fst(G(s, s)) = fst(G(r, s))$ and $fst(G(s, r)) > fst(G(r, r))$.

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- Compare this to the Nash equilibrium conditions.
- The conditions are almost identical: we can think of the evolutionary process as a rational process (entity?)!.

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- Various augmentations of the model create fascinating results, among them: cooperation.

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- Tag-based conditional strategies

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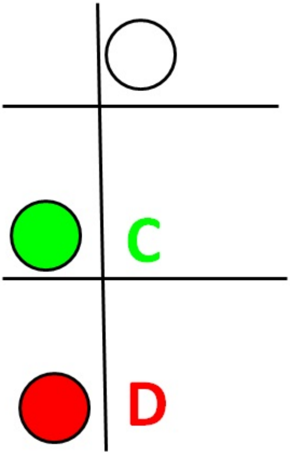

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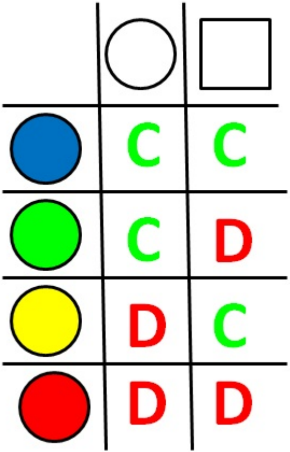

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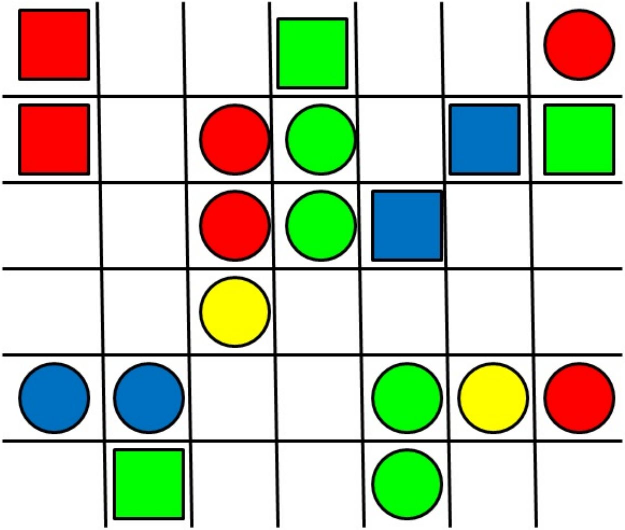

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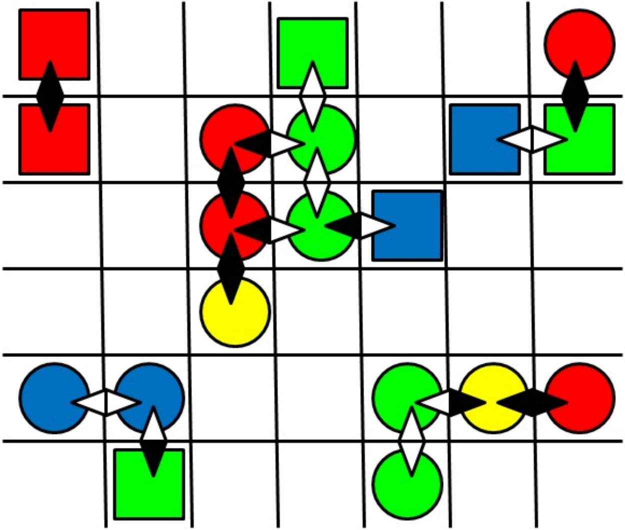

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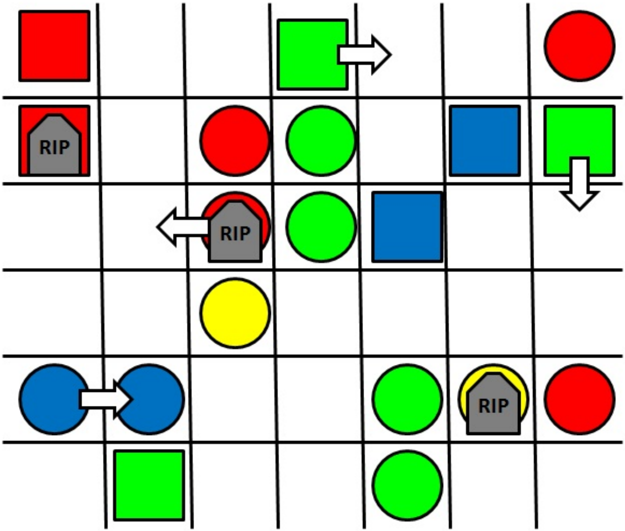

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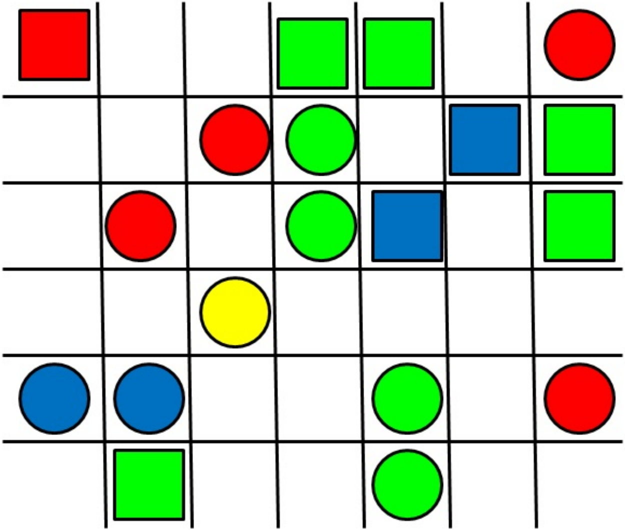

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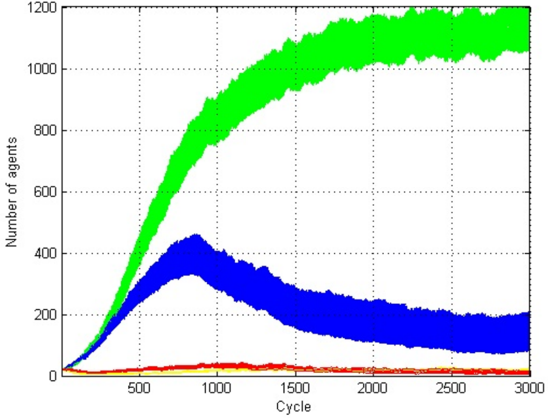

Thank you!

For more info feel free to contact me at:

artem.kaznatcheev@mail.mcgill.ca

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Some fun resources:

1. Robert Wright: "The evolution of compassion"
   http://www.ted.com/talks/lang/eng/robert_wright_the_evolution_of_compassion.html
2. Howard Rheingold: "On collaboration"
   http://www.ted.com/talks/lang/eng/howard_rheingold_on_collaboration.html
3. Jonathan Haidt: "On the moral roots of liberals and conservatives"
   http://www.ted.com/talks/jonathan_haidt_on_the_moral_mind.html
4. Artem Kaznatcheev: "Evolving Cooperation"
   http://www.youtube.com/watch?v=bRuE3oP-JT8
5. Stanford Encyclopedia of Philosophy: "Evolutionary Game Theory"
   http://plato.stanford.edu/entries/game-evolutionary/