Evolutionary game theory and cognition

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▶ If Alice plays strategy i and Bob plays strategy j then $(a, b) := G_{ij}$ is the outcome, where a corresponds to the change in Alice's utility and b to Bob's.

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► Zero-sum games are the epitome of competition. Any gain for Alice is a loss for Bob, and vice-versa.

Coordination games

Definition

A two-strategy game G is a coordination game if we have

$$G = \begin{bmatrix} (a_1, b_1) & (c_2, d_1) \\ (c_1, d_2) & (a_2, b_2) \end{bmatrix}$$

And $a_1 > c_1$, $a_2 > c_2$, $b_1 > d_1$, $b_2 > d_2$.

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- ► The diagonals are always better for both players, they just have to figure out how to pick the same strategy.
- ► Captures the idea of win-win, lose-lose situations.

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- ▶ Upside: zero-sum and coordination provide a good duality between impossibility of cooperation and obvious cooperation.
- ▶ Downside: both types of games are really boring. The most interesting games (from a mathematical and modeling point of view) are neither zero-sum nor coordination.
- ▶ Being non-zero-sum does not ensure cooperation.

Prisoner's dilemma

$$\begin{bmatrix} (b-c,b-c) & (-c,b) \\ (b,-c) & (0,0) \end{bmatrix}$$

- b is the benefit of receiving and c is the cost of giving.
- Strategy 1 is called cooperate or C and strategy 2 is called defect or D.

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- ► The rational strategy (or Nash equilibrium) is mutual defection.
- ► The best for the players taken together (or Pareto optimum) is mutual cooperation.

Nash equilibrium

Definition

A strategy pair (p, q) is a Nash equilibrium of a game G if for all other strategies r we have:

$$fst(G(p,q)) \geq fst(G(r,q))$$

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- ▶ If we only allow pure strategies then replace G(i,j) by G_{ij}
- ► If we allow mixed strategies, then every game has at least one Nash equilibrium

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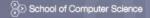
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- ► To find a Nash equilibrium Alice needs to be able to simulate the game and she must be able to place yourself in Bob's shoes.
- ▶ Do we even expect humans to be able to do all of this?
- Let's bound rationality and see what happens!

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- Simplest model of biological evolution.
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- What happens to rationality?

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A strategy s is an evolutionary stable strategy for a game G if for all other strategies r we have (a) fst(G(s,s)) > fst(G(r,s)), or (b) fst(G(s,s)) = fst(G(r,s)) and fst(G(s,r)) > fst(G(r,r)).

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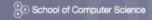
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- Compare this to the Nash equilibrium conditions.
- The conditions are almost identical: we can think of the evolutionary process as a rational process (entity?)!.



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- ▶ The assumptions of the ESS:
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- ▶ Various augmentations of the model create fascinating results, among them: cooperation.

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- ▶ Direct reciprocity: the ability to remember previous interactions
- ► Indirect reciprocity: the ability to track social constructs like reputation