

Proofs of Nineteen More 1.0 Rated Theorems  
in the TPTP Problem Library

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## Abstract

In July 2003, the automated theorem proving system called Octopus obtained proofs of 24 1.0 rated theorems in the TPTP Problem Library [1,2]. The 1.0 rating is assigned to theorems that no ATP system has been given credit as proving. Since that time other ATP systems have found proofs of five of these 24 theorems [3-7] while Octopus gone on to obtain proofs of an additional 19 1.0 rated theorems. This technical report provides their proofs.

## General Background

This technical report is an extension of an earlier technical report [1] in which proofs found by Octopus of 24 1.0 rated theorems from the TPTP Problem Library (Version 2.5.0) [2] are given. These 1.0 rated theorems are theorems for which no proof, to that time, had been obtained and reported to the authors of the TPTP Problem Library. Subsequently, proofs of five of these theorems have been obtained by other ATP systems. Now, since then, proofs of an additional 19 1.0 rated theorems have been obtained by Octopus. These proofs constitute this report.

The July report contained the results of experiments carried out on a 51-processor PC-based system. In the autumn months of 2003, the computing facilities in McGill's School of Computer Science were upgraded. The new facilities increased the strength of Octopus in three ways: more computers, faster computers, and more memory. Rather than 51 processors, the improved system used 151 processors. Moreover, the processors were considerably faster, consisting mostly of Pentium 4 and AMD 2000 processors. The system was executing approximately 400 billion instructions per second. Lastly, the amount of high-speed memory on each processor was increased; a large hash table in the program was doubled in size and now stores information on as many as 16 million clauses ( $2^{24}$ ) in 192 megabytes.

With the improved system, proofs of an additional 19 1.0 rated theorems were found, bringing the total to 42. To date, no other ATP has reported a proof to any of these most recent 18. The lengths of two proofs in this group are approximately 600 inferences in length.

It might be pointed out that while Octopus proved an additional 19 theorems, two of the theorems turned out to be identical though named differently in the TPTP Problem Library (FLD041-1 and FLD042-1). It was only when proofs of the two theorems were found to be essentially identical that it was realized that they were the same theorem (with a small permutation in the ordering of the given clauses). TPTP Problem Library author Geoff Sutcliffe confirmed the finding when informed.

The proof found by Octopus of FLD047-3.p was an oddity. Proofs of all the other theorems were found using the same version of Octopus and using identical default setting for all search parameters. For FLD047-3.p, however, a proof was found when the size of the clause hash table was set to 8 million entries ( $2^{23}$ ), but not found when the usual 16 million entry hash table was used. Exactly why this happened is beyond the scope of this report, but it is related to the saving of clauses encountered during the search.

There are approximately 5000 theorems in 30 different categories in the TPTP Problem Library. Of these, approximately 1000 are rated 1.0. While we attempted to prove many of them with Octopus, there was no systematic approach to this effort. Certain categories yielded proofs reported in the first technical report, and these categories were the focus of attention. Only one proof (NUM044-1) came from a new category. Octopus is weakest when proving theorems in several categories, and there was little effort made to solve theorems in these categories even when using the learning capabilities of Octopus. Attempts were limited to a maximum of 30 minutes per theorem. Many theorems, however, were given far less time as the demands on the computing facilities limited their use for this effort.

## The Proofs

Proofs of the 18 theorems are given in the Appendix. The reader is referred to the earlier technical report to understand the printout. Unlike the first technical report, square parentheses are included in the printout of each clause. This greatly increases the number of characters in the printout though making it easier to understand. In addition, information about the search as it was carried out is not included.

## Observations

There is a high likelihood that proofs of other 1.0-rated theorems will be obtained in the coming months. The learning strategy used to obtain these proofs has considerable room for improvement.

## References

- [1] Sutcliffe, G., & Suttner, C.B., The TPTP Problem Library, *JAR*, v. 21( 2), pp. 177-203, 1998.
- [2] Newborn, M., Proofs of Twenty-Four 1.0-Rated Theorems in the TPTP Problem Library, McGill University School of Computer Science Technical Report SOCS-03.6, July 2003.
- [3] [http://www.cs.miami.edu/~tptp/cgi-bin/DVTPTP2WWW/view\\_file.pl?Category=Solutions&Domain=ANA&File=ANA004-5](http://www.cs.miami.edu/~tptp/cgi-bin/DVTPTP2WWW/view_file.pl?Category=Solutions&Domain=ANA&File=ANA004-5)
- [4] [http://www.cs.miami.edu/~tptp/cgi-bin/DVTPTP2WWW/view\\_file.pl?Category=Solutions&Domain=FLD&File=FLD007-1](http://www.cs.miami.edu/~tptp/cgi-bin/DVTPTP2WWW/view_file.pl?Category=Solutions&Domain=FLD&File=FLD007-1)
- [5] [http://www.cs.miami.edu/~tptp/cgi-bin/DVTPTP2WWW/view\\_file.pl?Category=Solutions&Domain=FLD&File=FLD057-3](http://www.cs.miami.edu/~tptp/cgi-bin/DVTPTP2WWW/view_file.pl?Category=Solutions&Domain=FLD&File=FLD057-3)
- [6] [http://www.cs.miami.edu/~tptp/cgi-bin/DVTPTP2WWW/view\\_file.pl?Category=Solutions&Domain=SET&File=SET018-3](http://www.cs.miami.edu/~tptp/cgi-bin/DVTPTP2WWW/view_file.pl?Category=Solutions&Domain=SET&File=SET018-3)
- [7] [http://www.cs.miami.edu/~tptp/cgi-bin/DVTPTP2WWW/view\\_file.pl?Category=Solutions&Domain=SET&File=SET097-7](http://www.cs.miami.edu/~tptp/cgi-bin/DVTPTP2WWW/view_file.pl?Category=Solutions&Domain=SET&File=SET097-7)

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## List of Theorems

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## ANA004-3

Predicates: less\_or\_equal equal

Functions: epsilon\_0 a\_real\_number 0 : . minimum absolute delta\_1 delta\_2 negate xs f g add half

Renamed predicates: LE E.

Renamed functions: EPO ARN 0 : . MIN ABS delta\_1 delta\_2 N xs f g A half

C50: xvl 53.8.2 t1 W25 46S LE[x,0] LE[y,x] ... ?

Axioms:

```
1: ~LE[x, half[y]] ~LE[z, half[y]] LE[A[x,z], y]
2: ~LE[x, y] ~LE[z, u] LE[A[x,z], A[y, u]]
3: ~LE[x, y] ~LE[y, z] LE[x, z]
4: LE[x, 0] LE[y, 0] ~LE[MIN[x, y], 0]
5: ~E.[x, y] ~LE[x, z] LE[y, z]
6: ~E.[x, y] ~LE[z, x] LE[z, y]
7: ~E.[x, y] ~E.[y, z] E.[x, z]
8: ~LE[A[ABS[x], ABS[y]], z] LE[ABS[A[x, y]], z]
9: ~LE[x, y] LE[A[x, z], A[y, z]]
10: LE[x, y] LE[y, x]
11: ~E.[x, y] E.[MIN[x, z], MIN[y, z]]
12: ~E.[x, y] E.[MIN[z, x], MIN[z, y]]
13: ~E.[x, y] E.[ABS[x], ABS[y]]
14: ~E.[x, y] E.[delta_1[x], delta_1[y]]
15: ~E.[x, y] E.[delta_2[x], delta_2[y]]
16: >LE[x, 0] ~LE[half[x], 0]
17: ~E.[x, y] E.[N[x], N[y]]
18: ~LE[x, MIN[y, z]] LE[x, y]
19: ~LE[x, y] LE[x, MIN[x, y]]
20: ~LE[x, MIN[y, z]] LE[x, z]
21: ~LE[x, y] LE[x, MIN[y, x]]
22: ~E.[x, y] LE[x, y]
23: ~E.[x, y] E.[A[x, z], A[y, z]]
24: ~E.[x, y] E.[A[z, x], A[z, y]]
25: ~E.[x, y] E.[half[x], half[y]]
26: ~LE[x, y] E.[MIN[x, y], x]
27: ~LE[x, y] E.[MIN[y, x], x]
28: ~E.[x, y] E.[xs[x], xs[y]]
29: ~E.[x, y] E.[f[x], f[y]]
30: ~E.[x, y] E.[g[x], g[y]]
31: ~E.[x, y] E.[y, x]
32: LE[ABS[A[x, y]], A[ABS[x], ABS[y]]]
33: LE[A[A[x, y], z], A[x, A[y, z]]]
34: LE[A[x, A[y, z]], A[A[x, y], z]]
35: LE[A[half[x], half[x]], x]
36: LE[x, A[half[x], half[x]]]
37: LE[A[x, y], A[y, x]]
38: E.[A[A[x, y], z], A[x, A[y, z]]]
39: E.[A[half[x], half[x]], x]
40: E.[A[x, y], A[y, x]]
41: LE[MIN[x, y], x]
42: LE[MIN[x, y], y]
43: E.[x, x]
```

Negated conclusion:

```
44S LE[x, 0] ~LE[ABS[A[y, N[ARN]]], delta_1[x]] LE[ABS[A[f[y], N[f[ARN]]], x]
45S LE[x, 0] ~LE[ABS[A[y, N[ARN]]], delta_2[x]] LE[ABS[A[g[y], N[g[ARN]]], x]
46S>LE[x, 0] LE[y, x] ... ?
47S LE[x, 0] ~LE[delta_1[x], 0]
48S LE[x, 0] ~LE[delta_2[x], 0]
49S>~LE[EPO, 0]
50S>LE[x, 0] ~LE[ABS[A[A[f[xs[x]], N[f[ARN]]], A[g[xs[x]], N[g[ARN]]]], EPO]
```

Phase 0 clauses used in proof:

```
54S>(49a*46a) LE[x, EPO] ... ?
59S>(49a*16a) ~LE[half[EPO], 0]
```

Phases 1 and 2 clauses used in proof:

```
71S>(59a, 16a) ~LE[half[half[EPO]], 0]
72S>(71a, 50a) ~LE[ABS[A[A[f[xs[half[half[EPO]]]], N[f[ARN]]], A[g[xs[half[half[EPO]]],
N[g[ARN]]]], EPO]
73S>(72a, 54a) [] ... ?
```

Inferences derived from proof before weakened clause repaired

59 72

Inferences derived when weakened clause replaced with given clause

54

PHASE 0: 0 s PHASE 1: 0 s PHASE 2: -NA- Total Time: 0 s

NOD: 103 RES: 876 FAC: 2 T: 53 V: 32 L: 16  
CTE: 173 CTH: 0 CSZ: 16777216  
UTE: 10923 UTH: 9735 UTF: 0 SBA: 10  
BAS: 50 RED: 70 LEN: 2+3=5

Proof found to theorem by slave 50!

Axioms:

```

1 >~LE[x, half[y]] ~LE[z, half[y]] LE[A[x,z], y]
2: ~LE[x,y] ~LE[z,u] LE[A[x,z], A[y,u]]
3: ~LE[x,y] ~LE[y,z] LE[x,z]
4 >LE[x,0] LE[y,0] ~LE[MIN[x,y],0]
5: ~E.[x,y] ~LE[x,z] LE[y,z]
6: ~E.[x,y] ~LE[z,x] LE[z,y]
7: ~E.[x,y] ~E.[y,z] E.[x,z]
8 >~LE[A[ABS[x], ABS[y]], z] LE[ABS[A[x,y]], z]
9: ~LE[x,y] LE[A[x,z], A[y,z]]
10: LE[x,y] LE[y,x]
11: ~E.[x,y] E.[MIN[x,z], MIN[y,z]]
12: ~E.[x,y] E.[MIN[z,x], MIN[z,y]]
13: ~E.[x,y] E.[ABS[x], ABS[y]]
14: ~E.[x,y] E.[delta_1[x], delta_1[y]]
15: ~E.[x,y] E.[delta_2[x], delta_2[y]]
16S LE[x,0] ~LE[half[x],0]
17: ~E.[x,y] E.[N[x], N[y]]
18 >~LE[x, MIN[y,z]] LE[x,y]
19: ~LE[x,y] LE[x, MIN[x,y]]
20 >~LE[x, MIN[y,z]] LE[x,z]
21: ~LE[x,y] LE[x, MIN[y,x]]
22: ~E.[x,y] LE[x,y]
23: ~E.[x,y] E.[A[x,z], A[y,z]]
24: ~E.[x,y] E.[A[z,x], A[z,y]]
25: ~E.[x,y] E.[half[x], half[y]]
26: ~LE[x,y] E.[MIN[x,y], x]
27: ~LE[x,y] E.[MIN[y,x], x]
28: ~E.[x,y] E.[xs[x], xs[y]]
29: ~E.[x,y] E.[f[x], f[y]]
30: ~E.[x,y] E.[g[x], g[y]]
31: ~E.[x,y] E.[y,x]
32: LE[ABS[A[x,y]], A[ABS[x], ABS[y]]]
33: LE[A[A[x,y], z], A[x, A[y,z]]]
34: LE[A[x, A[y,z]], A[A[x,y], z]]
35: LE[A[half[x], half[x]], x]
36: LE[x, A[half[x], half[x]]]
37: LE[A[x,y], A[y,x]]
38: E.[A[A[x,y], z], A[x, A[y,z]]]
39: E.[A[half[x], half[x]], x]
40: E.[A[x,y], A[y,x]]
41: LE[MIN[x,y], x]
42: LE[MIN[x,y], y]
43: E.[x,x]

```

Negated conclusion:

```

44S>LE[x,0] ~LE[ABS[A[y, N[ARN]]], delta_1[x]] LE[ABS[A[f[y], N[f[ARN]]], x]
45S>LE[x,0] ~LE[ABS[A[y, N[ARN]]], delta_2[x]] LE[ABS[A[g[y], N[g[ARN]]], x]
46S>LE[x,0] LE[ABS[A[xs[x], N[ARN]]], x]
47S>LE[x,0] ~LE[delta_1[x], 0]
48S>LE[x,0] ~LE[delta_2[x], 0]
49S ~LE[EPO, 0]
50S>LE[x,0] ~LE[ABS[A[A[f[xs[x]], N[f[ARN]]], A[g[xs[x]], N[g[ARN]]]], EPO]
-----

```

Phase 0 clauses used in proof:

```

51S>~LE[half[EPO], 0]
65S>(51a*48a) ~LE[delta_2[half[EPO]], 0]
66S>(51a*47a) ~LE[delta_1[half[EPO]], 0]

```

Phases 1 and 2 clauses used in proof:

```

71S>(66a, 4b) LE[x,0] ~LE[MIN[x, delta_1[half[EPO]]], 0]
72S>(71a, 65a) ~LE[MIN[delta_2[half[EPO]], delta_1[half[EPO]]], 0]
73S>(72a, 46a) LE[ABS[A[xs[MIN[delta_2[half[EPO]], delta_1[half[EPO]]], N[ARN]]],
MIN[delta_2[half[EPO]], delta_1[half[EPO]]]]
74S>(73a, 20a) LE[ABS[A[xs[MIN[delta_2[half[EPO]], delta_1[half[EPO]]], N[ARN]]], delta_1[half[EPO]]]
75S>(74a, 44b) LE[half[EPO], 0]
LE[ABS[A[f[xs[MIN[delta_2[half[EPO]], delta_1[half[EPO]]], N[f[ARN]]], half[EPO]]]
76S>(75a, 51a) LE[ABS[A[f[xs[MIN[delta_2[half[EPO]], delta_1[half[EPO]]], N[f[ARN]]], half[EPO]]]

77S>(66a, 4b) LE[x,0] ~LE[MIN[x, delta_1[half[EPO]]], 0]
78S>(77a, 65a) ~LE[MIN[delta_2[half[EPO]], delta_1[half[EPO]]], 0]
79S>(78a, 46a) LE[ABS[A[xs[MIN[delta_2[half[EPO]], delta_1[half[EPO]]], N[ARN]]],
MIN[delta_2[half[EPO]], delta_1[half[EPO]]]]
80S>(79a, 18a) LE[ABS[A[xs[MIN[delta_2[half[EPO]], delta_1[half[EPO]]], N[ARN]]], delta_2[half[EPO]]]

```

```

81S>(80a,45b) LE[half[EP0],0]
LE[ABS[A[g[xs[MIN[delta_2[half[EP0]],delta_1[half[EP0]]]]],N[g[ARN]]]],half[EP0]]
82S>(81a,51a) LE[ABS[A[g[xs[MIN[delta_2[half[EP0]],delta_1[half[EP0]]]]],N[g[ARN]]]],half[EP0]]

83S>(66a,4b) LE[x,0] ~LE[MIN[x,delta_1[half[EP0]]],0]
84S>(83a,65a) ~LE[MIN[delta_2[half[EP0]],delta_1[half[EP0]]],0]
85S>(84a,50a) ~LE[ABS[A[A[f[xs[MIN[delta_2[half[EP0]],delta_1[half[EP0]]]]],N[f[ARN]]],
A[g[xs[MIN[delta_2[half[EP0]],delta_1[half[EP0]]]]],N[g[ARN]]]],EP0]
86S>(85a,8b) ~LE[A[ABS[A[f[xs[MIN[delta_2[half[EP0]],delta_1[half[EP0]]]]],N[f[ARN]]],
ABS[A[g[xs[MIN[delta_2[half[EP0]],delta_1[half[EP0]]]]],N[g[ARN]]]],EP0]
87S>(86a,1c) ~LE[ABS[A[f[xs[MIN[delta_2[half[EP0]],delta_1[half[EP0]]]]],N[f[ARN]]],half[EP0]]
~LE[ABS[A[g[xs[MIN[delta_2[half[EP0]],delta_1[half[EP0]]]]],N[g[ARN]]],half[EP0]]
88S>(87a,76a) ~LE[ABS[A[g[xs[MIN[delta_2[half[EP0]],delta_1[half[EP0]]]]],N[g[ARN]]],half[EP0]]
89S>(88a,82a) []

```

```

PHASE 0: 0 s PHASE 1: 184 s PHASE 2: 70 s Total Time: 254 s
NOD: 696269 RES: 4282416 FAC: 646 T: 53 V: 32 L: 16
CTE: 706756 CTH: 125096 CTF: 10202 CSZ: 16777216
UTE: 13496361 UTH: 22677374 UTF: 128768 SBA: 7131
BAS: 50 RED: 70 LEN: 3+19=22

```

## FLD008-2

Predicates: equalish defined less\_or\_equal  
Functions: c (S) d (S) b (S) a (S) . multiplicative\_identity additive\_identity : additive\_inverse  
multiplicative\_inverse add multiply  
Renamed predicates: equalish defined LE  
Renamed functions: c d b a . 1 0 : AI MI A M

C35: Spawned: 150 xvl 12.10.3 W17 28S equalish[A[x,AI[b]],d] ... ?

Axioms:

```
1: equalish[M[x,M[y,z]],M[M[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
2: equalish[A[M[x,y],M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y]
3: equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
4: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
5: LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
6: equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]
7: defined[MI[x]] ~defined[x] equalish[x,0]
8: LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]
9: equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[x,z]
10: LE[x,y] ~LE[z,y] ~equalish[z,x]
11 >equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]
12: equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: equalish[x,y] ~LE[x,y] ~LE[y,x]
15: defined[M[x,y]] ~defined[x] ~defined[y]
16 >equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]
17: defined[A[x,y]] ~defined[x] ~defined[y]
18 >equalish[x,y] ~equalish[x,z] ~equalish[z,y]
19 >equalish[A[x,AI[x]],0] ~defined[x]
20 >equalish[M[1,x],x] ~defined[x]
21 >equalish[A[0,x],x] ~defined[x]
22 >defined[AI[x]] ~defined[x]
23: equalish[x,x] ~defined[x]
24 >equalish[x,y] ~equalish[y,x]
25: defined[1]
26: ~equalish[0,1]
27: defined[0]
```

Negated conclusion:

```
28S>equalish[A[x,AI[b]],d] ... ?
29S>~equalish[AI[c],d]
30S equalish[A[a,b],c]
31S defined[a]
32S>defined[b]
33S>defined[c]
34S>defined[d]
```

Phase 0 clauses used in proof:

```
38S>(34a*20b) equalish[M[1,d],d]
39S>(34a*19b) equalish[A[d,AI[d]],0]
41S>(33a*22b) defined[AI[c]]
42S>(33a*21b) equalish[A[0,c],c]
44S>(33a*19b) equalish[A[c,AI[c]],0]
46S>(32a*22b) defined[AI[b]]
49S>(32a*19b) equalish[A[b,AI[b]],0]
```

Phases 1 and 2 clauses used in proof:

```
52S>(49a,24b) equalish[0,A[b,AI[b]]]
53S>(52a,18b) equalish[0,x] ~equalish[A[b,AI[b]],x]
54S>(53b,28a) equalish[0,d] ... ?

55S>(44a,18c) equalish[x,0] ~equalish[x,A[c,AI[c]]]
56S>(55b,24a) equalish[x,0] ~equalish[A[c,AI[c]],x]
57S>(56a,18b) ~equalish[A[c,AI[c]],x] equalish[x,y] ~equalish[0,y]
58S>(57b,29a) ~equalish[A[c,AI[c]],AI[c]] ~equalish[0,d]
59S>(58b,54a) ~equalish[A[c,AI[c]],AI[c]] ... ?

60S>(41a,21b) equalish[A[0,AI[c]],AI[c]]
61S>(60a,18c) equalish[x,AI[c]] ~equalish[x,A[0,AI[c]]]
62S>(61b,11a) equalish[A[x,AI[c]],AI[c]] ~defined[AI[c]] ~equalish[x,0]
63S>(62a,59a) ~defined[AI[c]] ~equalish[c,0] ... ?
64S>(63a,41a) ~equalish[c,0] ... ?

65S>(39a,18c) equalish[x,0] ~equalish[x,A[d,AI[d]]]
66S>(65b,24a) equalish[x,0] ~equalish[A[d,AI[d]],x]
67S>(66a,64a) ~equalish[A[d,AI[d]],c] ... ?
```

68S>(39a,18b) equalish[A[d,AI[d]],x] ~equalish[0,x]  
69S>(68b,24a) equalish[A[d,AI[d]],x] ~equalish[x,0]  
70S>(69a,18b) ~equalish[x,0] equalish[A[d,AI[d]],y] ~equalish[x,y]  
71S>(70a,49a) equalish[A[d,AI[d]],x] ~equalish[A[b,AI[b]],x]  
72S>(71a,67a) ~equalish[A[b,AI[b]],c] ... ?  
73S>(38a,24b) equalish[d,M[1,d]]  
74S>(73a,18c) equalish[x,M[1,d]] ~equalish[x,d]  
75S>(74a,18b) ~equalish[x,d] equalish[x,y] ~equalish[M[1,d],y]  
76S>(75a,28a) equalish[A[x,AI[b]],y] ~equalish[M[1,d],y] ... ?  
77: 76|{b/x,c/y} equalish[A[b,AI[b]],c] ~equalish[M[1,d],c] ... ?  
78S>(77a,72a) ~equalish[M[1,d],c] ... ?  
79S>(38a,18b) equalish[M[1,d],x] ~equalish[d,x]  
80S>(79b,24a) equalish[M[1,d],x] ~equalish[x,d]  
81S>(80a,18b) ~equalish[x,d] equalish[M[1,d],y] ~equalish[x,y]  
82S>(81a,28a) equalish[M[1,d],x] ~equalish[A[y,AI[b]],x] ... ?  
83: 82|{c/x} equalish[M[1,d],c] ~equalish[A[x,AI[b]],c] ... ?  
84S>(83a,78a) ~equalish[A[x,AI[b]],c] ... ?  
85S>(49a,18c) equalish[x,0] ~equalish[x,A[b,AI[b]]]  
86S>(85b,24a) equalish[x,0] ~equalish[A[b,AI[b]],x]  
87S>(86b,28a) equalish[d,0] ... ?  
88S>(33a,16c) equalish[A[x,c],A[c,x]] ~defined[x]  
89S>(88a,24b) ~defined[x] equalish[A[c,x],A[x,c]]  
90S>(89b,18b) ~defined[x] equalish[A[c,x],y] ~equalish[A[x,c],y]  
91S>(90a,46a) equalish[A[c,AI[b]],x] ~equalish[A[AI[b],c],x]  
92S>(91a,84a) ~equalish[A[AI[b],c],c] ... ?  
93S>(42a,18c) equalish[x,c] ~equalish[x,A[0,c]]  
94S>(93b,11a) equalish[A[x,c],c] ~defined[c] ~equalish[x,0]  
95S>(94b,33a) equalish[A[x,c],c] ~equalish[x,0]  
96S>(95b,18a) equalish[A[x,c],c] ~equalish[x,y] ~equalish[y,0]  
97S>(96b,28a) equalish[A[A[x,AI[b]],c],c] ~equalish[d,0] ... ?  
98S>(97b,87a) equalish[A[A[x,AI[b]],c],c] ... ?  
99S>(46a,21b) equalish[A[0,AI[b]],AI[b]]  
100S>(99a,24b) equalish[AI[b],A[0,AI[b]]]  
101S>(100a,11c) equalish[A[AI[b],x],A[A[0,AI[b]],x]] ~defined[x]  
102S>(101a,18b) ~defined[x] equalish[A[AI[b],x],y] ~equalish[A[A[0,AI[b]],x],y]  
103S>(102a,33a) equalish[A[AI[b],c],x] ~equalish[A[A[0,AI[b]],c],x]  
104S>(103a,92a) ~equalish[A[A[0,AI[b]],c],c] ... ?  
105S>(104a,98a) [] ... ?

Inferences derived from proof before weakened clause repaired  
38 39 41 42 44 46 49 53 58 62 66 71 86 91 103

Inferences derived when weakened clause replaced with given clause  
76 82 97

PHASE 0: 0 s, PHASE 1: 10 s, PHASE 2: 24 s, Total Time: 34 s  
NOD: 178595 RES: 1139774 FAC: 20220 T: 12 V: 32 L: 16  
CTE: 108347 CTH: 31555 CTF: 0 CSZ: 16777216  
UTE: 212597 UTH: 316570 UTF: 0 SBA: 0  
BAS: 34 RED: 51 LEN: 7+54=61  
Proof found to theorem by slave 35!

Axioms:

1: equalish[M[x,M[y,z]],M[M[x,y],z]] ~defined[x] ~defined[y] ~defined[z]  
2: equalish[A[M[x,y],M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y]  
3 >equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]  
4: LE[x,y] LE[y,x] ~defined[x] ~defined[y]  
5: LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]  
6: equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]  
7: defined[MI[x]] ~defined[x] equalish[x,0]  
8: LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]  
9: equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[x,z]  
10: LE[x,y] ~LE[z,y] ~equalish[z,x]  
11S>equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]  
12: equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]  
13: LE[x,y] ~LE[x,z] ~LE[z,y]  
14: equalish[x,y] ~LE[x,y] ~LE[y,x]  
15: defined[M[x,y]] ~defined[x] ~defined[y]  
16S>equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]  
17 >defined[A[x,y]] ~defined[x] ~defined[y]  
18S>equalish[x,y] ~equalish[x,z] ~equalish[z,y]  
19S>equalish[A[x,AI[x]],0] ~defined[x]  
20S equalish[M[1,x],x] ~defined[x]  
21S>equalish[A[0,x],x] ~defined[x]

```

22S>defined[AI[x]] ~defined[x]
23 >equalish[x,x] ~defined[x]
24S>equalish[x,y] ~equalish[y,x]
25: defined[1]
26: ~equalish[0,1]
27: defined[0]

```

Negated conclusion:

```

28S>equalish[A[AI[a],AI[b]],d]
29S>~equalish[AI[c],d]
30S>equalish[A[a,b],c]
31S>defined[a]
32S>defined[b]
33S>defined[c]
34S>defined[d]

```

-----

Phase 0 clauses used in proof:

```

35S>equalish[M[1,d],d]
37S>defined[AI[c]]
39S>equalish[A[c,AI[c]],0]
40S>defined[AI[b]]
42S>equalish[0,x] ~equalish[A[b,AI[b]],x]
44S>equalish[A[x,AI[c]],AI[c]] ~defined[AI[c]] ~equalish[x,0]
48S>equalish[A[c,AI[b]],x] ~equalish[A[AI[b],c],x]
49S>equalish[A[AI[b],c],x] ~equalish[A[A[0,AI[b]],c],x]
53S>(44b*37a) equalish[A[x,AI[c]],AI[c]] ~equalish[x,0]

```

Phases 1 and 2 clauses used in proof:

```

54S>(31a,21b) equalish[A[0,a],a]
55S>(54a,24b) equalish[a,A[0,a]]

```

```

56S>(40a,17c) defined[A[x,AI[b]]] ~defined[x]
57S>(56b,32a) defined[A[b,AI[b]]]

```

```

58S>(42b,23a) equalish[0,A[b,AI[b]]] ~defined[A[b,AI[b]]]
59S>(58b,57a) equalish[0,A[b,AI[b]]]
60S>(59a,11c) equalish[A[0,x],A[A[b,AI[b]],x]] ~defined[x]
61S>(60a,18c) ~defined[x] equalish[y,A[A[b,AI[b]],x]] ~equalish[y,A[0,x]]
62S>(61a,31a) equalish[x,A[A[b,AI[b]],a]] ~equalish[x,A[0,a]]
63: 62|{a/x} equalish[a,A[A[b,AI[b]],a]] ~equalish[a,A[0,a]]
64S>(63b,55a) equalish[a,A[A[b,AI[b]],a]]

```

```

65S>(34a,21b) equalish[A[0,d],d]
66S>(65a,24b) equalish[d,A[0,d]]

```

```

67S>(40a,17c) defined[A[x,AI[b]]] ~defined[x]
68S>(67b,32a) defined[A[b,AI[b]]]
69S>(68a,16b) equalish[A[A[b,AI[b]],x],A[x,A[b,AI[b]]] ~defined[x]
70S>(69a,18c) ~defined[x] equalish[y,A[x,A[b,AI[b]]] ~equalish[y,A[A[b,AI[b]],x]]
71S>(70a,31a) equalish[x,A[a,A[b,AI[b]]] ~equalish[x,A[A[b,AI[b]],a]]
72: 71|{a/x} equalish[a,A[a,A[b,AI[b]]] ~equalish[a,A[A[b,AI[b]],a]]
73S>(72b,64a) equalish[a,A[a,A[b,AI[b]]]

```

```

74S>(39a,11c) equalish[A[A[c,AI[c]],x],A[0,x]] ~defined[x]
75S>(74a,24b) ~defined[x] equalish[A[0,x],A[A[c,AI[c]],x]]
76S>(75b,18c) ~defined[x] equalish[y,A[A[c,AI[c]],x]] ~equalish[y,A[0,x]]
77S>(76a,34a) equalish[x,A[A[c,AI[c]],d]] ~equalish[x,A[0,d]]
78: 77|{d/x} equalish[d,A[A[c,AI[c]],d]] ~equalish[d,A[0,d]]
79S>(78b,66a) equalish[d,A[A[c,AI[c]],d]]

```

```

80S>(37a,17c) defined[A[x,AI[c]]] ~defined[x]
81S>(80b,33a) defined[A[c,AI[c]]]
82S>(81a,16b) equalish[A[A[c,AI[c]],x],A[x,A[c,AI[c]]] ~defined[x]
83S>(82a,18c) ~defined[x] equalish[y,A[x,A[c,AI[c]]] ~equalish[y,A[A[c,AI[c]],x]]
84S>(83a,34a) equalish[x,A[d,A[c,AI[c]]] ~equalish[x,A[A[c,AI[c]],d]]
85: 84|{d/x} equalish[d,A[d,A[c,AI[c]]] ~equalish[d,A[A[c,AI[c]],d]]
86S>(85b,79a) equalish[d,A[d,A[c,AI[c]]]

```

```

87S>(40a,3d) equalish[A[x,A[y,AI[b]]],A[A[x,y],AI[b]] ~defined[x] ~defined[y]
88S>(87b,31a) equalish[A[a,A[x,AI[b]]],A[A[a,x],AI[b]] ~defined[x]
89S>(88a,18c) ~defined[x] equalish[y,A[A[a,x],AI[b]]] ~equalish[y,A[a,A[x,AI[b]]]
90S>(89b,24b) ~defined[x] ~equalish[y,A[a,A[x,AI[b]]] equalish[A[A[a,x],AI[b]],y]
91S>(90a,32a) ~equalish[x,A[a,A[b,AI[b]]] equalish[A[A[a,b],AI[b]],x]
92S>(91a,73a) equalish[A[A[a,b],AI[b]],a]

```

```

93S>(30a,11c) equalish[A[A[a,b],x],A[c,x]] ~defined[x]
94S>(93a,24b) ~defined[x] equalish[A[c,x],A[A[a,b],x]]
95S>(94b,18b) ~defined[x] equalish[A[c,x],y] ~equalish[A[A[a,b],x],y]
96S>(95b,24b) ~defined[x] ~equalish[A[A[a,b],x],y] equalish[y,A[c,x]]
97S>(96a,40a) ~equalish[A[A[a,b],AI[b]],x] equalish[x,A[c,AI[b]]]

```

98S>(97a,92a) equalish[a,A[c,AI[b]]]

99S>(37a,3d) equalish[A[x,A[y,AI[c]]],A[A[x,y],AI[c]]] ~defined[x] ~defined[y]  
100S>(99b,34a) equalish[A[d,A[x,AI[c]]],A[A[d,x],AI[c]]] ~defined[x]  
101S>(100a,18c) ~defined[x] equalish[y,A[A[d,x],AI[c]]] ~equalish[y,A[d,A[x,AI[c]]]]  
102S>(101b,24b) ~defined[x] ~equalish[y,A[d,A[x,AI[c]]]] equalish[A[A[d,x],AI[c]],y]  
103S>(102a,33a) ~equalish[x,A[d,A[c,AI[c]]]] equalish[A[A[d,c],AI[c]],x]  
104S>(103a,86a) equalish[A[A[d,c],AI[c]],d]

105S>(24b,21a) equalish[x,A[0,x]] ~defined[x]  
106S>(105a,11c) ~defined[x] equalish[A[x,y],A[A[0,x],y]] ~defined[y]  
107S>(106a,40a) equalish[A[AI[b],x],A[A[0,AI[b]],x]] ~defined[x]  
108S>(107b,33a) equalish[A[AI[b],c],A[A[0,AI[b]],c]]]

109S>(37a,3d) equalish[A[x,A[y,AI[c]]],A[A[x,y],AI[c]]] ~defined[x] ~defined[y]  
110S>(109b,34a) equalish[A[d,A[x,AI[c]]],A[A[d,x],AI[c]]] ~defined[x]  
111S>(110b,33a) equalish[A[d,A[c,AI[c]]],A[A[d,c],AI[c]]]  
112S>(111a,18c) equalish[x,A[A[d,c],AI[c]]] ~equalish[x,A[d,A[c,AI[c]]]]  
113S>(112a,18b) ~equalish[x,A[d,A[c,AI[c]]]] equalish[x,y] ~equalish[A[A[d,c],AI[c]],y]  
114S>(113b,29a) ~equalish[AI[c],A[d,A[c,AI[c]]]] ~equalish[A[A[d,c],AI[c]],d]  
115S>(114b,104a) ~equalish[AI[c],A[d,A[c,AI[c]]]]]

116S>(24b,18a) equalish[x,y] ~equalish[y,z] ~equalish[z,x]  
117S>(116a,49b) ~equalish[x,y] ~equalish[y,A[A[0,AI[b]],c]] equalish[A[AI[b],c],x]  
118S>(117b,48a) ~equalish[x,A[c,AI[b]]] equalish[A[AI[b],c],x] ~equalish[A[AI[b],c],A[A[0,AI[b]],c]]  
119S>(118a,98a) equalish[A[AI[b],c],a] ~equalish[A[AI[b],c],A[A[0,AI[b]],c]]  
120S>(119b,108a) equalish[A[AI[b],c],a]

121S>(31a,22b) defined[AI[a]]

122S>(37a,3d) equalish[A[x,A[y,AI[c]]],A[A[x,y],AI[c]]] ~defined[x] ~defined[y]  
123S>(122b,34a) equalish[A[d,A[x,AI[c]]],A[A[d,x],AI[c]]] ~defined[x]  
124S>(123a,18b) ~defined[x] equalish[A[d,A[x,AI[c]]],y] ~equalish[A[A[d,x],AI[c]],y]  
125S>(124b,24b) ~defined[x] ~equalish[A[A[d,x],AI[c]],y] equalish[y,A[d,A[x,AI[c]]]]  
126S>(125b,53a) ~defined[x] equalish[AI[c],A[d,A[x,AI[c]]]] ~equalish[A[d,x],0]  
127S>(126a,33a) equalish[AI[c],A[d,A[c,AI[c]]]] ~equalish[A[d,c],0]  
128S>(127a,115a) ~equalish[A[d,c],0]

129S>(19a,18c) ~defined[x] equalish[y,0] ~equalish[y,A[x,AI[x]]]  
130S>(129a,31a) equalish[x,0] ~equalish[x,A[a,AI[a]]]  
131S>(130b,11a) equalish[A[x,AI[a]],0] ~defined[AI[a]] ~equalish[x,a]  
132S>(131b,121a) equalish[A[x,AI[a]],0] ~equalish[x,a]  
133S>(132b,48a) equalish[A[A[c,AI[b]],AI[a]],0] ~equalish[A[AI[b],c],a]  
134S>(133b,120a) equalish[A[A[c,AI[b]],AI[a]],0]

135S>(40a,17b) defined[A[AI[b],x]] ~defined[x]  
136S>(135b,33a) defined[A[AI[b],c]]]

137S>(35a,11c) equalish[A[M[1,d],x],A[d,x]] ~defined[x]  
138S>(137b,33a) equalish[A[M[1,d],c],A[d,c]]  
139S>(138a,18c) equalish[x,A[d,c]] ~equalish[x,A[M[1,d],c]]  
140S>(139a,24b) ~equalish[x,A[M[1,d],c]] equalish[A[d,c],x]  
141S>(140a,24a) equalish[A[d,c],x] ~equalish[A[M[1,d],c],x]  
142S>(141a,128a) ~equalish[A[M[1,d],c],0]

143S>(48b,23a) equalish[A[c,AI[b]],A[AI[b],c]] ~defined[A[AI[b],c]]  
144S>(143b,136a) equalish[A[c,AI[b]],A[AI[b],c]]  
145S>(144a,11c) equalish[A[A[c,AI[b]],x],A[A[AI[b],c],x]] ~defined[x]  
146S>(145a,18c) ~defined[x] equalish[y,A[A[AI[b],c],x]] ~equalish[y,A[A[c,AI[b]],x]]  
147S>(146a,22a) equalish[x,A[A[AI[b],c],AI[y]]] ~equalish[x,A[A[c,AI[b]],AI[y]]] ~defined[y]  
148S>(147b,24a) equalish[x,A[A[AI[b],c],AI[y]]] ~defined[y] ~equalish[A[A[c,AI[b]],AI[y]],x]  
149S>(148b,31a) equalish[x,A[A[AI[b],c],AI[a]]] ~equalish[A[A[c,AI[b]],AI[a]],x]  
150: 149 {0/x} equalish[0,A[A[AI[b],c],AI[a]]] ~equalish[A[A[c,AI[b]],AI[a]],0]  
151S>(150b,134a) equalish[0,A[A[AI[b],c],AI[a]]]

152S>(35a,11c) equalish[A[M[1,d],x],A[d,x]] ~defined[x]  
153S>(152b,33a) equalish[A[M[1,d],c],A[d,c]]  
154S>(153a,18b) equalish[A[M[1,d],c],x] ~equalish[A[d,c],x]  
155S>(154b,24a) equalish[A[M[1,d],c],x] ~equalish[x,A[d,c]]  
156S>(155a,142a) ~equalish[0,A[d,c]]]

157S>(40a,3b) equalish[A[AI[b],A[x,y]],A[A[AI[b],x],y]] ~defined[x] ~defined[y]  
158S>(157b,33a) equalish[A[AI[b],A[c,x]],A[A[AI[b],c],x]] ~defined[x]  
159S>(158a,18b) ~defined[x] equalish[A[AI[b],A[c,x]],y] ~equalish[A[A[AI[b],c],x],y]  
160S>(159a,22a) equalish[A[AI[b],A[c,AI[x]]],y] ~equalish[A[A[AI[b],c],AI[x]],y] ~defined[x]  
161S>(160b,24a) equalish[A[AI[b],A[c,AI[x]]],y] ~defined[x] ~equalish[y,A[A[AI[b],c],AI[x]]]  
162S>(161b,31a) equalish[A[AI[b],A[c,AI[a]]],x] ~equalish[x,A[A[AI[b],c],AI[a]]]  
163: 162 {0/x} equalish[A[AI[b],A[c,AI[a]],0] ~equalish[0,A[A[AI[b],c],AI[a]]]  
164S>(163b,151a) equalish[A[AI[b],A[c,AI[a]],0]

165S>(40a,17c) defined[A[x,AI[b]]] ~defined[x]

```

166S>(165b,22a) defined[A[AI[x],AI[b]]] ~defined[x]
167S>(166b,31a) defined[A[AI[a],AI[b]]]

168S>(33a,17b) defined[A[c,x]] ~defined[x]
169S>(168b,22a) defined[A[c,AI[x]]] ~defined[x]
170S>(169b,31a) defined[A[c,AI[a]]]
171S>(170a,16b) equalish[A[A[c,AI[a]],x],A[x,A[c,AI[a]]]] ~defined[x]
172S>(171a,18b) ~defined[x] equalish[A[A[c,AI[a]],x],y] ~equalish[A[x,A[c,AI[a]]],y]
173S>(172a,40a) equalish[A[A[c,AI[a]],AI[b]],x] ~equalish[A[AI[b],A[c,AI[a]]],x]
174: 173|{0/x} equalish[A[A[c,AI[a]],AI[b]],0] ~equalish[A[AI[b],A[c,AI[a]]],0]
175S>(174b,164a) equalish[A[A[c,AI[a]],AI[b]],0]

176S>(33a,11b) equalish[A[x,c],A[y,c]] ~equalish[x,y]
177S>(176b,28a) equalish[A[A[AI[a],AI[b]],c],A[d,c]]
178S>(177a,18c) equalish[x,A[d,c]] ~equalish[x,A[A[AI[a],AI[b]],c]]
179S>(178b,16a) equalish[A[c,A[AI[a],AI[b]]],A[d,c]] ~defined[c] ~defined[A[AI[a],AI[b]]]
180S>(179b,33a) equalish[A[c,A[AI[a],AI[b]]],A[d,c]] ~defined[A[AI[a],AI[b]]]
181S>(180b,167a) equalish[A[c,A[AI[a],AI[b]]],A[d,c]]
182S>(181a,18c) equalish[x,A[d,c]] ~equalish[x,A[c,A[AI[a],AI[b]]]]
183S>(182a,156a) ~equalish[0,A[c,A[AI[a],AI[b]]]]

184S>(31a,22b) defined[AI[a]]
185S>(184a,3c) equalish[A[x,A[AI[a],y]],A[A[x,AI[a]],y]] ~defined[x] ~defined[y]
186S>(185b,33a) equalish[A[c,A[AI[a],x]],A[A[c,AI[a]],x]] ~defined[x]
187S>(186a,18b) ~defined[x] equalish[A[c,A[AI[a],x]],y] ~equalish[A[A[c,AI[a]],x],y]
188S>(187b,24b) ~defined[x] ~equalish[A[A[c,AI[a]],x],y] equalish[y,A[c,A[AI[a],x]]]
189S>(188a,40a) ~equalish[A[A[c,AI[a]],AI[b]],x] equalish[x,A[c,A[AI[a],AI[b]]]]
190S>(189a,175a) equalish[0,A[c,A[AI[a],AI[b]]]]
191S>(190a,183a) []

```

```

PHASE 0: 0 s, PHASE 1: 118 s, PHASE 2: 453 s, Total Time: 571 s
NOD: 3220035 RES: 19025412 FAC: 409489 T: 12 V: 32 L: 16
CTE: 1432373 CTH: 675932 CTF: 0 CSZ: 16777216
UTE: 1492858 UTH: 3213207 UTF: 0 SBA: 0
BAS: 34 RED: 52 LEN: 9+138=147

```

## FLD012-3

Predicates: sum product less\_or\_equal defined  
Functions: additive\_identity b multiplicative\_identity a : . add additive\_inverse multiply  
multiplicative\_inverse  
Renamed predicates: S P LE defined  
Renamed functions: 0 b l a : . A AI M MI

C2: Spawned: 150 xvl 13.14.4 W0 31S defined[x] ... ?

Axioms:

```
1 >S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]
2 >P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]
3: LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]
4: LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]
5 >P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]
6 >P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]
7: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
8: S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]
9 >S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]
10 >P[MI[x],x,1] S[0,x,0] ~defined[x]
11 >defined[MI[x]] ~defined[x] S[0,x,0]
12: S[0,x,y] ~LE[x,y] ~LE[y,x]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14 >defined[M[x,y]] ~defined[x] ~defined[y]
15 >P[x,y,M[x,y]] ~defined[x] ~defined[y]
16: defined[A[x,y]] ~defined[x] ~defined[y]
17: S[x,y,A[x,y]] ~defined[x] ~defined[y]
18 >S[AI[x],x,0] ~defined[x]
19 >P[1,x,x] ~defined[x]
20: defined[AI[x]] ~defined[x]
21 >S[0,x,x] ~defined[x]
22 >P[x,y,z] ~P[y,x,z]
23 >S[x,y,z] ~S[y,x,z]
24: ~S[0,0,1]
25 >defined[1]
26 >defined[0]
```

Negated conclusion:

```
27S>~P[1,MI[M[a,b]],M[MI[a],MI[b]]]
28S>~S[0,a,0]
29S>~S[0,b,0]
30S>defined[a]
31S>defined[x] ... ?
```

Phase 0 clauses used in proof:

```
32S>(31a*21b) S[0,x,x] ... ?
34S>(31a*19b) P[1,x,x] ... ?
35S>(31a*18b) S[AI[x],x,0] ... ?
38S>(31a*15b) P[x,y,M[x,y]] ~defined[y] ... ?
39S>(31a*14b) defined[M[x,y]] ~defined[y] ... ?
40S>(31a*11b) defined[MI[x]] S[0,x,0] ... ?
41S>(31a*10c) P[MI[x],x,1] S[0,x,0] ... ?
44S>(39b*31a) defined[M[x,y]] ... ?
45S>(38b*31a) P[x,y,M[x,y]] ... ?
```

Phases 1 and 2 clauses used in proof:

```
48S>(35a,9b) S[0,x,y] ~S[z,x,u] ~S[AI[z],u,y] ... ?
49S>(48b,23a) S[0,x,y] ~S[AI[z],u,y] ~S[x,z,u] ... ?
50S>(49a,29a) ~S[AI[x],y,0] ~S[b,x,y] ... ?
51S>(50a,35a) ~S[b,x,x] ... ?

52S>(45a,1e) S[x,M[y,z],u] ~S[v,y,w] ~P[w,z,u] ~P[v,z,x] ... ?
53S>(52b,21a) S[x,M[y,z],u] ~P[y,z,u] ~P[0,z,x] ~defined[y] ... ?
54S>(53d,31a) S[x,M[y,z],u] ~P[y,z,u] ~P[0,z,x] ... ?
55S>(54b,45a) S[x,M[y,z],M[y,z]] ~P[0,z,x] ... ?
56S>(55b,45a) S[M[0,x],M[y,x],M[y,x]] ... ?

57S>(44a,18b) S[AI[M[x,y]],M[x,y],0] ... ?
58S>(57a,9d) S[x,y,0] ~S[AI[M[z,u]],v,x] ~S[v,y,M[z,u]] ... ?
59S>(58b,35a) S[0,x,0] ~S[M[y,z],x,M[y,z]] ... ?
60S>(59a,9d) ~S[M[x,y],z,M[x,y]] S[u,v,0] ~S[0,w,u] ~S[w,v,z] ... ?
61S>(60ad) S[x,M[y,z],0] ~S[0,M[y,z],x] ~S[M[y,z],M[y,z],M[y,z]] ... ?
62S>(61b,32a) S[M[x,y],M[x,y],0] ~S[M[x,y],M[x,y],M[x,y]] ... ?
63: 62|{0/x} S[M[0,x],M[0,x],0] ~S[M[0,x],M[0,x],M[0,x]] ... ?
64S>(63b,56a) S[M[0,x],M[0,x],0] ... ?

65S>(41b,28a) P[MI[a],a,1] ... ?
66S>(65a,22b) P[a,MI[a],1] ... ?
```

67S>(66a,1e) S[x,1,y] ~S[z,a,u] ~P[u,MI[a],y] ~P[z,MI[a],x] ... ?  
68S>(67b,32a) S[x,1,y] ~P[a,MI[a],y] ~P[0,MI[a],x] ... ?  
69S>(68b,66a) S[x,1,1] ~P[0,MI[a],x] ... ?  
70S>(69a,51a) ~P[0,MI[a],b] ... ?  
71: (22a,5c) ~P[x,y,z] P[u,z,v] ~P[u,y,w] ~P[w,x,v]  
72: (71b,22b) ~P[x,y,z] ~P[u,y,v] ~P[v,x,w] P[z,u,w]  
73S>(72b,41a) ~P[x,y,z] ~P[1,x,u] P[z,MI[y],u] S[0,y,0] ... ?  
74S>(73a,22a) ~P[1,x,y] P[z,MI[u],y] S[0,u,0] ~P[u,x,z] ... ?  
75S>(74a,34a) P[x,MI[y],z] S[0,y,0] ~P[y,z,x] ... ?  
76S>(75b,28a) P[x,MI[a],y] ~P[a,y,x] ... ?  
77: 76|{0/x,b/y} P[0,MI[a],b] ~P[a,b,0] ... ?  
78S>(77a,70a) ~P[a,b,0] ... ?  
79S>(32a,2b) P[x,y,z] ~P[0,y,u] ~P[x,y,v] ~S[u,v,z] ... ?  
80S>(79bc) P[0,x,y] ~P[0,x,z] ~S[z,z,y] ... ?  
81S>(80b,15a) P[0,x,y] ~S[M[0,x],M[0,x],y] ~defined[0] ~defined[x] ... ?  
82S>(81c,26a) P[0,x,y] ~S[M[0,x],M[0,x],y] ~defined[x] ... ?  
83S>(82c,31a) P[0,x,y] ~S[M[0,x],M[0,x],y] ... ?  
84S>(83a,22b) ~S[M[0,x],M[0,x],y] P[x,0,y] ... ?  
85: 84|{0/y} ~S[M[0,x],M[0,x],0] P[x,0,0] ... ?  
86S>(85a,64a) P[x,0,0] ... ?  
87S>(30a,15b) P[a,x,M[a,x]] ~defined[x]  
88S>(87b,31a) P[a,x,M[a,x]] ... ?  
89S>(88a,2d) P[x,y,z] ~S[u,a,x] ~P[u,y,v] ~S[v,M[a,y],z] ... ?  
90S>(89b,21a) P[a,x,y] ~P[0,x,z] ~S[z,M[a,x],y] ~defined[a] ... ?  
91S>(90d,30a) P[a,x,y] ~P[0,x,z] ~S[z,M[a,x],y] ... ?  
92S>(91b,22a) P[a,x,y] ~S[z,M[a,x],y] ~P[x,0,z] ... ?  
93S>(92b,40b) P[a,x,0] ~P[x,0,0] defined[MI[M[a,x]]] ... ?  
94S>(93a,78a) ~P[b,0,0] defined[MI[M[a,b]]] ... ?  
95S>(94a,86a) defined[MI[M[a,b]]] ... ?  
96S>(41a,22b) S[0,x,0] P[x,MI[x],1] ... ?  
97S>(96a,29a) P[b,MI[b],1] ... ?  
98: (22b,5a) P[x,y,z] ~P[y,u,v] ~P[u,w,x] ~P[v,w,z]  
99S>(98a,27a) ~P[MI[M[a,b]],x,y] ~P[x,z,1] ~P[y,z,M[MI[a],MI[b]]]  
100S>(99a,15a) ~P[x,y,1] ~P[M[MI[M[a,b]],x],y,M[MI[a],MI[b]]] ~defined[MI[M[a,b]]] ~defined[x]  
101S>(100c,95a) ~P[x,y,1] ~P[M[MI[M[a,b]],x],y,M[MI[a],MI[b]]] ~defined[x] ... ?  
102S>(101c,31a) ~P[x,y,1] ~P[M[MI[M[a,b]],x],y,M[MI[a],MI[b]]] ... ?  
103S>(102b,6a) ~P[x,y,1] ~P[z,u,M[MI[M[a,b]],x]] ~P[u,y,v] ~P[z,v,M[MI[a],MI[b]]] ... ?  
104S>(103b,22a) ~P[x,y,1] ~P[z,y,u] ~P[v,u,M[MI[a],MI[b]]] ~P[z,v,M[MI[M[a,b]],x]] ... ?  
105S>(104b,34a) ~P[x,y,1] ~P[z,y,M[MI[a],MI[b]]] ~P[1,z,M[MI[M[a,b]],x]] ... ?  
106S>(105b,45a) ~P[x,MI[b],1] ~P[1,MI[a],M[MI[M[a,b]],x]] ... ?  
107S>(106a,97a) ~P[1,MI[a],M[MI[M[a,b]],b]] ... ?  
108S>(32a,2b) P[x,y,z] ~P[0,y,u] ~P[x,y,v] ~S[u,v,z] ... ?  
109S>(108bc) P[0,x,y] ~P[0,x,z] ~S[z,z,y] ... ?  
110S>(109b,22a) P[0,x,y] ~S[z,z,y] ~P[x,0,z] ... ?  
111S>(110b,32a) P[0,x,0] ~P[x,0,0] ... ?  
112S>(111b,86a) P[0,x,0] ... ?  
113: (26a,15b) P[0,x,M[0,x]] ~defined[x]  
114S>(113b,31a) P[0,x,M[0,x]] ... ?  
115S>(114a,5c) P[x,M[0,y],z] ~P[x,0,u] ~P[u,y,z] ... ?  
116S>(115a,22b) ~P[x,0,y] ~P[y,z,u] P[M[0,z],x,u] ... ?  
117S>(116a,34a) ~P[0,x,y] P[M[0,x],1,y] ... ?  
118: 117|{0/y} ~P[0,x,0] P[M[0,x],1,0] ... ?  
119S>(118a,112a) P[M[0,x],1,0] ... ?  
120: (25a,15c) P[x,1,M[x,1]] ~defined[x]  
121S>(120b,31a) P[x,1,M[x,1]] ... ?  
122S>(121a,5c) P[x,M[y,1],z] ~P[x,y,u] ~P[u,1,z] ... ?  
123S>(122a,22b) ~P[x,y,z] ~P[z,1,u] P[M[y,1],x,u] ... ?  
124S>(123a,45a) ~P[M[x,y],1,z] P[M[y,1],x,z] ... ?  
125: 124|{0/x,0/z} ~P[M[0,x],1,0] P[M[x,1],0,0] ... ?  
126S>(125a,119a) P[M[x,1],0,0] ... ?  
127S>(41a,5b) S[0,x,0] P[MI[x],y,z] ~P[x,u,y] ~P[1,u,z] ... ?  
128S>(127a,28a) P[MI[a],x,y] ~P[a,z,x] ~P[1,z,y] ... ?  
129S>(128b,15a) P[MI[a],M[a,x],y] ~P[1,x,y] ~defined[a] ~defined[x] ... ?  
130S>(129c,30a) P[MI[a],M[a,x],y] ~P[1,x,y] ~defined[x] ... ?  
131S>(130c,31a) P[MI[a],M[a,x],y] ~P[1,x,y] ... ?  
132S>(131a,5b) ~P[1,x,y] P[MI[a],z,u] ~P[M[a,x],v,z] ~P[y,v,u] ... ?  
133S>(132b,22b) ~P[1,x,y] ~P[M[a,x],z,u] ~P[y,z,v] P[u,MI[a],v] ... ?  
134S>(133a,34a) ~P[M[a,x],y,z] ~P[x,y,u] P[z,MI[a],u] ... ?  
135S>(134b,34a) ~P[M[a,1],x,y] P[y,MI[a],x] ... ?  
136: 135|{0/x,0/y} ~P[M[a,1],0,0] P[0,MI[a],0] ... ?  
137S>(136a,126a) P[0,MI[a],0] ... ?

```

138S>(45a,5c) P[x,M[y,z],u] ~P[x,y,v] ~P[v,z,u] ... ?
139S>(138b,10a) P[MI[x],M[x,y],z] ~P[l,y,z] S[0,x,0] ~defined[x] ... ?
140S>(139d,31a) P[MI[x],M[x,y],z] ~P[l,y,z] S[0,x,0] ... ?
141S>(140b,19a) P[MI[x],M[x,y],y] S[0,x,0] ~defined[y] ... ?
142S>(141c,31a) P[MI[x],M[x,y],y] S[0,x,0] ... ?
143S>(142b,28a) P[MI[a],M[a,x],x] ... ?
144S>(143a,22b) P[M[a,x],MI[a],x] ... ?
145S>(144a,1e) S[x,y,z] ~S[u,M[a,y],v] ~P[v,MI[a],z] ~P[u,MI[a],x] ... ?
146S>(145a,29a) ~S[x,M[a,b],y] ~P[y,MI[a],0] ~P[x,MI[a],0] ... ?
147S>(146bc) ~S[x,M[a,b],x] ~P[x,MI[a],0] ... ?
148: 147|{0/x} ~S[0,M[a,b],0] ~P[0,MI[a],0] ... ?
149S>(148b,137a) ~S[0,M[a,b],0] ... ?

150S>(45a,5c) P[x,M[y,z],u] ~P[x,y,v] ~P[v,z,u] ... ?
151S>(150a,22b) ~P[x,y,z] ~P[z,u,v] P[M[y,u],x,v] ... ?
152S>(151b,34a) ~P[x,y,1] P[M[y,z],x,z] ... ?
153S>(152b,6c) ~P[x,y,1] P[z,x,u] ~P[v,M[y,w],z] ~P[v,w,u] ... ?
154S>(153a,41a) P[x,MI[y],z] ~P[u,M[y,v],x] ~P[u,v,z] S[0,y,0] ... ?
155S>(154b,41a) P[l,MI[x],y] ~P[MI[M[x,z]],z,y] S[0,x,0] S[0,M[x,z],0] ... ?
156S>(155b,45a) P[l,MI[x],M[MI[M[x,y]],y]] S[0,x,0] S[0,M[x,y],0] ... ?
157S>(156b,28a) P[l,MI[a],M[MI[M[a,x]],x]] S[0,M[a,x],0] ... ?
158S>(157a,107a) S[0,M[a,b],0] ... ?
159S>(158a,149a) [] ... ?

```

Inferences derived from proof before variable replaced by constant  
72 87 100 113 120

Inferences derived when variable is replaced by original constant  
32 34 35 38 39 40 41 44 45 48 51 52 55 57 58 73 75 79 88 89 91 92 97 108 114 115 116 121  
122 124 127 138 140 142 150

```

PHASE 0: 0 s PHASE 1: 74 s PHASE 2: 135 s Total Time: 209 s
NOD: 5404468 RES: 14995377 FAC: 1400815 T: 13 V: 32 L: 16
CTE: 3827759 CTH: 1893594 CTF: 44 CSZ: 16777216
UTE: 4779523 UTH: 40037213 UTF: 25 SBA: 0
BAS: 31 RED: 23 LEN: 9+112=121

```

Proof found to theorem by slave 2!

Axioms:

```

1S>S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]
2S P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]
3: LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]
4: LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]
5S>P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]
6S>P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]
7: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
8: S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]
9S>S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]
10S>P[MI[x],x,1] S[0,x,0] ~defined[x]
11S>defined[MI[x]] ~defined[x] S[0,x,0]
12: S[0,x,y] ~LE[x,y] ~LE[y,x]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14S>defined[M[x,y]] ~defined[x] ~defined[y]
15S>P[x,y,M[x,y]] ~defined[x] ~defined[y]
16 >defined[A[x,y]] ~defined[x] ~defined[y]
17 >S[x,y,A[x,y]] ~defined[x] ~defined[y]
18S>S[AI[x],x,0] ~defined[x]
19S>P[l,x,x] ~defined[x]
20 >defined[AI[x]] ~defined[x]
21S>S[0,x,x] ~defined[x]
22S>P[x,y,z] ~P[y,x,z]
23S>S[x,y,z] ~S[y,x,z]
24: ~S[0,0,1]
25S>defined[l]
26S>defined[0]

```

Negated conclusion:

```

27S>~P[l,MI[M[a,b]],M[MI[a],MI[b]]]
28S>~S[0,a,0]
29S>~S[0,b,0]
30S>defined[a]
31S>defined[b]

```

-----

Phase 0 clauses used in proof:

```

38S>defined[MI[b]] S[0,b,0]
48S>~P[x,b,y] ~P[l,x,z] P[y,MI[b],z] S[0,b,0]
51S>P[a,b,M[a,b]]
52S>P[x,b,y] ~S[z,a,x] ~P[z,b,u] ~S[u,M[a,b],y]
55S>P[b,MI[b],1]

```

```

56S>P[0,b,M[0,b]]
70S>(48d*29a) ~P[x,b,y] ~P[1,x,z] P[y,MI[b],z]
72S>(38b*29a) defined[MI[b]]

Phases 1 and 2 clauses used in proof:
77S>(70a,56a) ~P[1,0,x] P[M[0,b],MI[b],x]
78S>(77a,19a) P[M[0,b],MI[b],0] ~defined[0]
79S>(78b,26a) P[M[0,b],MI[b],0]

80S>(22b,19a) P[x,1,x] ~defined[x]
81S>(80b,20a) P[AI[x],1,AI[x]] ~defined[x]
82S>(81b,72a) P[AI[MI[b]],1,AI[MI[b]]]

83S>(56a,5b) P[0,x,y] ~P[b,z,x] ~P[M[0,b],z,y]
84S>(83b,55a) P[0,1,x] ~P[M[0,b],MI[b],x]
85: 84|{0/x} P[0,1,0] ~P[M[0,b],MI[b],0]
86S>(85b,79a) P[0,1,0]

87S>(72a,18b) S[AI[MI[b]],MI[b],0]
88S>(87a,9b) S[0,x,y] ~S[MI[b],x,z] ~S[AI[MI[b]],z,y]
89S>(88a,28a) ~S[MI[b],a,x] ~S[AI[MI[b]],x,0]
90S>(89a,17a) ~S[AI[MI[b]],A[MI[b],a],0] ~defined[MI[b]] ~defined[a]
91S>(90b,72a) ~S[AI[MI[b]],A[MI[b],a],0] ~defined[a]
92S>(91b,30a) ~S[AI[MI[b]],A[MI[b],a],0]
93S>(92a,1a) ~S[x,y,z] ~P[z,u,0] ~P[x,u,AI[MI[b]]] ~P[y,u,A[MI[b],a]]
94S>(93a,87a) ~P[0,x,0] ~P[AI[MI[b]],x,AI[MI[b]]] ~P[MI[b],x,A[MI[b],a]]
95S>(94a,86a) ~P[AI[MI[b]],1,AI[MI[b]]] ~P[MI[b],1,A[MI[b],a]]
96S>(95a,82a) ~P[MI[b],1,A[MI[b],a]]

97S>(72a,16b) defined[A[MI[b],x]] ~defined[x]
98S>(97b,30a) defined[A[MI[b],a]]
99S>(98a,19b) P[1,A[MI[b],a],A[MI[b],a]]
100S>(99a,48b) ~P[A[MI[b],a],b,x] P[x,MI[b],A[MI[b],a]] S[0,b,0]
101S>(100c,29a) ~P[A[MI[b],a],b,x] P[x,MI[b],A[MI[b],a]]
102S>(101b,22b) ~P[A[MI[b],a],b,x] P[MI[b],x,A[MI[b],a]]
103: 102|{1/x} ~P[A[MI[b],a],b,1] P[MI[b],1,A[MI[b],a]]
104S>(103b,96a) ~P[A[MI[b],a],b,1]

105S>(25a,21b) S[0,1,1]
106S>(105a,23b) S[1,0,1]

107S>(30a,10c) P[MI[a],a,1] S[0,a,0]
108S>(107b,28a) P[MI[a],a,1]

109S>(30a,11b) defined[MI[a]] S[0,a,0]
110S>(109b,28a) defined[MI[a]]

111S>(72a,17b) S[MI[b],x,A[MI[b],x]] ~defined[x]
112S>(111b,30a) S[MI[b],a,A[MI[b],a]]
113S>(112a,52b) P[A[MI[b],a],b,x] ~P[MI[b],b,y] ~S[y,M[a,b],x]
114S>(113b,10a) P[A[MI[b],a],b,x] ~S[1,M[a,b],x] S[0,b,0] ~defined[b]
115S>(114c,29a) P[A[MI[b],a],b,x] ~S[1,M[a,b],x] ~defined[b]
116S>(115c,31a) P[A[MI[b],a],b,x] ~S[1,M[a,b],x]
117S>(116b,9a) P[A[MI[b],a],b,x] ~S[y,z,1] ~S[z,M[a,b],u] ~S[y,u,x]
118S>(117bd) P[A[MI[b],a],b,1] ~S[x,M[a,b],x] ~S[y,x,1]
119S>(118a,104a) ~S[x,M[a,b],x] ~S[y,x,1]
120: 119|{0/x,1/y} ~S[0,M[a,b],0] ~S[1,0,1]
121S>(120b,106a) ~S[0,M[a,b],0]

122S>(27a,6a) ~P[x,y,1] ~P[y,MI[M[a,b]],z] ~P[x,z,M[MI[a],MI[b]]]
123S>(122b,22a) ~P[x,y,1] ~P[x,z,M[MI[a],MI[b]]] ~P[MI[M[a,b]],y,z]
124S>(123b,15a) ~P[MI[a],x,1] ~P[MI[M[a,b]],x,MI[b]] ~defined[MI[a]] ~defined[MI[b]]
125S>(124a,108a) ~P[MI[M[a,b]],a,MI[b]] ~defined[MI[a]] ~defined[MI[b]]
126S>(125b,110a) ~P[MI[M[a,b]],a,MI[b]] ~defined[MI[b]]
127S>(126b,72a) ~P[MI[M[a,b]],a,MI[b]]

128S>(31a,14c) defined[M[x,b]] ~defined[x]
129S>(128b,30a) defined[M[a,b]]

130S>(70a,51a) ~P[1,a,x] P[M[a,b],MI[b],x]
131S>(130a,19a) P[M[a,b],MI[b],a] ~defined[a]
132S>(131b,30a) P[M[a,b],MI[b],a]
133S>(132a,5c) P[x,a,y] ~P[x,M[a,b],z] ~P[z,MI[b],y]
134S>(133b,10a) P[MI[M[a,b]],a,x] ~P[1,MI[b],x] S[0,M[a,b],0] ~defined[M[a,b]]
135S>(134c,121a) P[MI[M[a,b]],a,x] ~P[1,MI[b],x] ~defined[M[a,b]]
136S>(135c,129a) P[MI[M[a,b]],a,x] ~P[1,MI[b],x]
137S>(136b,19a) P[MI[M[a,b]],a,MI[b]] ~defined[MI[b]]
138S>(137a,127a) ~defined[MI[b]]
139S>(138a,72a) []

PHASE 0: 0 s PHASE 1: 62 s PHASE 2: 145 s Total Time: 416 s

```

NOD: 4980527	RES: 20950969	FAC: 1721458	T: 13 V: 32 L: 16
CTE: 2588546	CTH: 987899	CTF: 0	CSZ: 16777216
UTE: 2767264	UTH: 5791244	UTF: 0	SBA: 0
BAS: 31	RED: 67	LEN: 8+63=71	

# FLD041-1

Predicates: equalish defined less\_or\_equal  
Functions: b a additive\_identity . multiplicative\_identity : multiplicative\_inverse additive\_inverse  
add multiply  
Renamed predicates: equalish defined LE  
Renamed functions: b a 0 . 1 : MI AI A M

C100: Spawned: 150 xv1 26.10.3 W49 2S equalish[A[u,M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z]  
~defined[y] ... ?

## Axioms:

```
1: equalish[M[x,M[y,z]],M[M[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
2S>equalish[A[u,M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y] ... ?
3: equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
4: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
5: LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
6: equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]
7: defined[MI[x]] ~defined[x] equalish[x,0]
8: LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]
9 >equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[x,z]
10: LE[x,y] ~LE[z,y] ~equalish[z,x]
11 >equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]
12 >equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: equalish[x,y] ~LE[x,y] ~LE[y,x]
15: defined[M[x,y]] ~defined[x] ~defined[y]
16 >equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]
17 >defined[A[x,y]] ~defined[x] ~defined[y]
18 >equalish[x,y] ~equalish[x,z] ~equalish[z,y]
19 >equalish[A[x,AI[x]],0] ~defined[x]
20 >equalish[M[1,x],x] ~defined[x]
21 >equalish[A[0,x],x] ~defined[x]
22 >defined[AI[x]] ~defined[x]
23: equalish[x,x] ~defined[x]
24 >equalish[x,y] ~equalish[y,x]
25 >defined[1]
26: ~equalish[0,1]
27 >defined[0]
```

## Negated conclusion:

```
28S equalish[M[a,b],0]
29S>~equalish[a,0]
30S ~equalish[b,0]
31S>defined[a]
32S defined[b]
```

## Phase 0 clauses used in proof:

```
39S>(31a*22b) defined[AI[a]]
40S>(31a*21b) equalish[A[0,a],a]
42S>(31a*19b) equalish[A[a,AI[a]],0]
44S>(29a*24a) ~equalish[0,a]
```

## Phases 1 and 2 clauses used in proof:

```
49S>(44a,18a) ~equalish[0,x] ~equalish[x,a]
50S>(49a,24a) ~equalish[x,a] ~equalish[x,0]
51S>(50a,40a) ~equalish[A[0,a],0]

52S>(42a,18c) equalish[x,0] ~equalish[x,A[a,AI[a]]]
53S>(52b,16a) equalish[A[AI[a],a],0] ~defined[AI[a]] ~defined[a]
54S>(53b,39a) equalish[A[AI[a],a],0] ~defined[a]
55S>(54b,31a) equalish[A[AI[a],a],0]
56S>(55a,18c) equalish[x,0] ~equalish[x,A[AI[a],a]]
57S>(56b,11a) equalish[A[x,a],0] ~defined[a] ~equalish[x,AI[a]]
58S>(57a,51a) ~defined[a] ~equalish[0,AI[a]]
59S>(58a,31a) ~equalish[0,AI[a]]

60S>(39a,20b) equalish[M[1,AI[a]],AI[a]]
61S>(60a,18c) equalish[x,AI[a]] ~equalish[x,M[1,AI[a]]]
62S>(61b,12a) equalish[M[AI[a],1],AI[a]] ~defined[AI[a]] ~defined[1]
63S>(62b,39a) equalish[M[AI[a],1],AI[a]] ~defined[1]
64S>(63b,25a) equalish[M[AI[a],1],AI[a]]
65S>(64a,18c) equalish[x,AI[a]] ~equalish[x,M[AI[a],1]]
66S>(65a,59a) ~equalish[0,M[AI[a],1]]

67S>(39a,21b) equalish[A[0,AI[a]],AI[a]]
68S>(67a,9c) equalish[M[A[0,AI[a]],x],M[AI[a],x]] ~defined[x]
69S>(68b,25a) equalish[M[A[0,AI[a]],1],M[AI[a],1]]
```

```

70S>(69a,18c) equalish[x,M[AI[a],1]] ~equalish[x,M[A[0,AI[a]],1]]
71S>(70a,66a) ~equalish[0,M[A[0,AI[a]],1]]

72S>(39a,17c) defined[A[x,AI[a]]] ~defined[x]
73S>(72b,31a) defined[A[a,AI[a]]]

74S>(39a,2c) equalish[A[x,M[AI[a],y]],M[A[z,AI[a]],y]] ~defined[z] ~defined[y] ... ?
75S>(74b,27a) equalish[A[x,M[AI[a],y]],M[A[0,AI[a]],y]] ~defined[y] ... ?
76S>(75b,25a) equalish[A[x,M[AI[a],1]],M[A[0,AI[a]],1]] ... ?
77S>(76a,18c) equalish[x,M[A[0,AI[a]],1]] ~equalish[x,A[y,M[AI[a],1]]] ... ?
78: 77|{0/x} equalish[0,M[A[0,AI[a]],1]] ~equalish[0,A[x,M[AI[a],1]]] ... ?
79S>(78a,71a) ~equalish[0,A[x,M[AI[a],1]]] ... ?

80S>(42a,18c) equalish[x,0] ~equalish[x,A[a,AI[a]]]
81S>(80b,20a) equalish[M[1,A[a,AI[a]],0] ~defined[A[a,AI[a]]]
82S>(81b,73a) equalish[M[1,A[a,AI[a]],0]
83S>(82a,18c) equalish[x,0] ~equalish[x,M[1,A[a,AI[a]]]
84S>(83b,12a) equalish[M[A[a,AI[a]],1],0] ~defined[A[a,AI[a]]] ~defined[1]
85S>(84b,73a) equalish[M[A[a,AI[a]],1],0] ~defined[1]
86S>(85b,25a) equalish[M[A[a,AI[a]],1],0]

87S>(39a,2c) equalish[A[x,M[AI[a],y]],M[A[z,AI[a]],y]] ~defined[z] ~defined[y] ... ?
88S>(87b,31a) equalish[A[x,M[AI[a],y]],M[A[a,AI[a]],y]] ~defined[y] ... ?
89S>(88a,18b) ~defined[x] equalish[A[y,M[AI[a],x]],z] ~equalish[M[A[a,AI[a]],x],z] ... ?
90S>(89b,24b) ~defined[x] ~equalish[M[A[a,AI[a]],x],y] equalish[y,A[z,M[AI[a],x]]] ... ?
91S>(90a,25a) ~equalish[M[A[a,AI[a]],1],x] equalish[x,A[y,M[AI[a],1]]] ... ?
92: 91|{0/x} ~equalish[M[A[a,AI[a]],1],0] equalish[0,A[x,M[AI[a],1]]] ... ?
93S>(92a,86a) equalish[0,A[x,M[AI[a],1]]] ... ?
94S>(93a,79a) [] ... ?

```

Inferences derived from proof before variable replaced by constant  
40 42 44 51 59 66 71 73 86

Inferences derived when variable is replaced by original constant  
76 77 88 89 91

```

PHASE 0: 0 s PHASE 1: 5 s PHASE 2: 17 s Total Time: 22 s
NOD: 589710 RES: 3624739 FAC: 67360 T: 26 V: 32 L: 16
CTE: 214345 CTH: 43277 CTF: 0 CSZ: 16777216
UTE: 1059532 UTH: 1038642 UTF: 0 SBA: 0
BAS: 32 RED: 48 LEN: 4+46=50

```

Proof found to theorem by slave 100!

Axioms:

```

1 >equalish[M[x,M[y,z]],M[M[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
2S>equalish[A[M[x,y],M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y]
3: equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
4: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
5: LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
6 >equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]
7 >defined[MI[x]] ~defined[x] equalish[x,0]
8: LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]
9S>equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[x,z]
10: LE[x,y] ~LE[z,y] ~equalish[z,x]
11S>equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]
12S>equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: equalish[x,y] ~LE[x,y] ~LE[y,x]
15 >defined[M[x,y]] ~defined[x] ~defined[y]
16S>equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]
17S>defined[A[x,y]] ~defined[x] ~defined[y]
18S>equalish[x,y] ~equalish[x,z] ~equalish[z,y]
19S>equalish[A[x,AI[x]],0] ~defined[x]
20S>equalish[M[1,x],x] ~defined[x]
21S>equalish[A[0,x],x] ~defined[x]
22S defined[AI[x]] ~defined[x]
23 >equalish[x,x] ~defined[x]
24S>equalish[x,y] ~equalish[y,x]
25S>defined[1]
26: ~equalish[0,1]
27S>defined[0]

```

Negated conclusion:

```

28S>equalish[M[a,b],0]
29S>~equalish[a,0]
30S>~equalish[b,0]
31S>defined[a]
32S>defined[b]

```

-----

Phase 0 clauses used in proof:

```
33S>equalish[A[0,a],a]
34S>equalish[A[a,AI[a]],0]
36S>~equalish[A[0,a],0]
40S>defined[A[a,AI[a]]]
41S>equalish[M[A[a,AI[a]],1],0]
44S>equalish[A[M[a,x],M[AI[a],x]],M[A[a,AI[a]],x]] ~defined[x]
45S>~defined[x] equalish[A[M[a,x],M[AI[a],x]],y] ~equalish[M[A[a,AI[a]],x],y]
46S>~equalish[M[A[a,AI[a]],1],x] equalish[x,A[M[a,1],M[AI[a],1]]]
47S>(46a*41a) equalish[0,A[M[a,1],M[AI[a],1]]]
48S>(44b*40a) equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],M[A[a,AI[a]],A[a,AI[a]]]]
```

Phases 1 and 2 clauses used in proof:

```
49S>(47a,24b) equalish[A[M[a,1],M[AI[a],1]],0]

50S>(46a,12a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[A[a,AI[a]]] ~defined[1]
51S>(50b,40a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[1]
52S>(51b,25a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]
53S>(52a,18b) equalish[M[1,A[a,AI[a]]],x] ~equalish[A[M[a,1],M[AI[a],1]],x]
54: 53 {0/x} equalish[M[1,A[a,AI[a]]],0] ~equalish[A[M[a,1],M[AI[a],1]],0]
55S>(54b,49a) equalish[M[1,A[a,AI[a]]],0]

56S>(47a,24b) equalish[A[M[a,1],M[AI[a],1]],0]

57S>(40a,9b) equalish[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]] ~equalish[x,y]
58S>(57b,21a) equalish[M[A[0,x],A[a,AI[a]]],M[x,A[a,AI[a]]] ~defined[x]
59S>(58a,18b) ~defined[x] equalish[M[A[0,x],A[a,AI[a]]],y] ~equalish[M[x,A[a,AI[a]]],y]
60S>(59a,25a) equalish[M[A[0,1],A[a,AI[a]]],x] ~equalish[M[1,A[a,AI[a]]],x]
61: 60 {0/x} equalish[M[A[0,1],A[a,AI[a]]],0] ~equalish[M[1,A[a,AI[a]]],0]
62S>(61b,55a) equalish[M[A[0,1],A[a,AI[a]]],0]

63S>(46a,12a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[A[a,AI[a]]] ~defined[1]
64S>(63b,40a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[1]
65S>(64b,25a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]
66S>(65a,18b) equalish[M[1,A[a,AI[a]]],x] ~equalish[A[M[a,1],M[AI[a],1]],x]
67: 66 {0/x} equalish[M[1,A[a,AI[a]]],0] ~equalish[A[M[a,1],M[AI[a],1]],0]
68S>(67b,56a) equalish[M[1,A[a,AI[a]]],0]

69S>(40a,9b) equalish[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]] ~equalish[x,y]
70S>(69b,16a) equalish[M[A[x,y],A[a,AI[a]]],M[A[y,x],A[a,AI[a]]] ~defined[x] ~defined[y]
71S>(70b,25a) equalish[M[A[1,x],A[a,AI[a]]],M[A[x,1],A[a,AI[a]]] ~defined[x]
72S>(71a,18b) ~defined[x] equalish[M[A[1,x],A[a,AI[a]]],y] ~equalish[M[A[x,1],A[a,AI[a]]],y]
73S>(72a,27a) equalish[M[A[1,0],A[a,AI[a]]],x] ~equalish[M[A[0,1],A[a,AI[a]]],x]
74: 73 {0/x} equalish[M[A[1,0],A[a,AI[a]]],0] ~equalish[M[A[0,1],A[a,AI[a]]],0]
75S>(74b,62a) equalish[M[A[1,0],A[a,AI[a]]],0]

76S>(40a,9b) equalish[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]] ~equalish[x,y]
77S>(76b,21a) equalish[M[A[0,x],A[a,AI[a]]],M[x,A[a,AI[a]]] ~defined[x]
78S>(77a,18b) ~defined[x] equalish[M[A[0,x],A[a,AI[a]]],y] ~equalish[M[x,A[a,AI[a]]],y]
79S>(78a,25a) equalish[M[A[0,1],A[a,AI[a]]],x] ~equalish[M[1,A[a,AI[a]]],x]
80: 79 {0/x} equalish[M[A[0,1],A[a,AI[a]]],0] ~equalish[M[1,A[a,AI[a]]],0]
81S>(80b,68a) equalish[M[A[0,1],A[a,AI[a]]],0]

82S>(33a,18c) equalish[x,a] ~equalish[x,A[0,a]]
83S>(82b,16a) equalish[A[a,0],a] ~defined[a] ~defined[0]
84S>(83b,31a) equalish[A[a,0],a] ~defined[0]
85S>(84b,27a) equalish[A[a,0],a]
86S>(85a,9c) equalish[M[A[a,0],x],M[a,x]] ~defined[x]
87S>(86a,18b) ~defined[x] equalish[M[A[a,0],x],y] ~equalish[M[a,x],y]
88S>(87a,32a) equalish[M[A[a,0],b],x] ~equalish[M[a,b],x]
89S>(88b,28a) equalish[M[A[a,0],b],0]

90S>(40a,2d) equalish[A[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]],M[A[x,y],A[a,AI[a]]] ~defined[x]
~defined[y]
91S>(90b,25a) equalish[A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]],M[A[1,x],A[a,AI[a]]] ~defined[x]
92S>(91a,18b) ~defined[x] equalish[A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]],y]
~equalish[M[A[1,x],A[a,AI[a]]],y]
93S>(92b,24b) ~defined[x] ~equalish[M[A[1,x],A[a,AI[a]]],y]
equalish[y,A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]]]
94S>(93a,27a) ~equalish[M[A[1,0],A[a,AI[a]]],x] equalish[x,A[M[1,A[a,AI[a]],M[0,A[a,AI[a]]]]]
95S>(94a,75a) equalish[0,A[M[1,A[a,AI[a]]],M[0,A[a,AI[a]]]]

96S>(40a,9b) equalish[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]] ~equalish[x,y]
97S>(96b,11a) equalish[M[A[x,y],A[a,AI[a]]],M[A[z,y],A[a,AI[a]]] ~defined[y] ~equalish[x,z]
98S>(97b,25a) equalish[M[A[x,1],A[a,AI[a]]],M[A[y,1],A[a,AI[a]]] ~equalish[x,y]
99S>(98a,18b) ~equalish[x,y] equalish[M[A[x,1],A[a,AI[a]]],z] ~equalish[M[A[y,1],A[a,AI[a]]],z]
100S>(99a,34a) equalish[M[A[A[a,AI[a]],1],A[a,AI[a]],x] ~equalish[M[A[0,1],A[a,AI[a]]],x]
101: 100 {0/x} equalish[M[A[A[a,AI[a]],1],A[a,AI[a]],0] ~equalish[M[A[0,1],A[a,AI[a]]],0]
102S>(101b,81a) equalish[M[A[A[a,AI[a]],1],A[a,AI[a]],0]

103S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]
```

```

104S>(103b,27a) defined[M[0,A[a,AI[a]]]]
105S>(104a,11b) equalish[A[x,M[0,A[a,AI[a]]]],A[y,M[0,A[a,AI[a]]]] ~equalish[x,y]
106S>(105b,20a) equalish[A[M[1,x]],M[0,A[a,AI[a]]]],A[x,M[0,A[a,AI[a]]]] ~defined[x]
107S>(106a,18c) ~defined[x] equalish[y,A[x,M[0,A[a,AI[a]]]]
~equalish[y,A[M[1,x]],M[0,A[a,AI[a]]]]
108S>(107a,40a) equalish[x,A[A[a,AI[a]],M[0,A[a,AI[a]]]]
~equalish[x,A[M[1,A[a,AI[a]]],M[0,A[a,AI[a]]]]
109: 108|{0/x} equalish[0,A[A[a,AI[a]],M[0,A[a,AI[a]]]]
~equalish[0,A[M[1,A[a,AI[a]]],M[0,A[a,AI[a]]]]
110S>(109b,95a) equalish[0,A[A[a,AI[a]],M[0,A[a,AI[a]]]]

111S>(32a,2d) equalish[A[M[x,b],M[y,b]],M[A[x,y],b]] ~defined[x] ~defined[y]
112S>(111b,31a) equalish[A[M[a,b],M[x,b]],M[A[a,x],b]] ~defined[x]
113S>(112a,18b) ~defined[x] equalish[A[M[a,b],M[x,b]],y] ~equalish[M[A[a,x],b],y]
114S>(113b,24b) ~defined[x] ~equalish[M[A[a,x],b],y] equalish[y,A[M[a,b],M[x,b]]]
115S>(114a,27a) ~equalish[M[A[a,0],b],x] equalish[x,A[M[a,b],M[0,b]]]
116S>(115a,89a) equalish[0,A[M[a,b],M[0,b]]]

117S>(40a,9b) equalish[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]] ~equalish[x,y]
118S>(117b,16a) equalish[M[A[x,y],A[a,AI[a]]],M[A[y,x],A[a,AI[a]]] ~defined[x] ~defined[y]
119S>(118b,25a) equalish[M[A[1,x],A[a,AI[a]]],M[A[x,1],A[a,AI[a]]] ~defined[x]
120S>(119a,18b) ~defined[x] equalish[M[A[1,x],A[a,AI[a]]],y] ~equalish[M[A[x,1],A[a,AI[a]]],y]
121S>(120a,40a) equalish[M[A[1,A[a,AI[a]]],A[a,AI[a]]],x]
~equalish[M[A[A[a,AI[a]],1],A[a,AI[a]]],x]
122: 121|{0/x} equalish[M[A[1,A[a,AI[a]]],A[a,AI[a]],0]
~equalish[M[A[A[a,AI[a]],1],A[a,AI[a]],0]
123S>(122b,102a) equalish[M[A[1,A[a,AI[a]]],A[a,AI[a]],0]

124S>(32a,15c) defined[M[x,b]] ~defined[x]
125S>(124b,27a) defined[M[0,b]]
126S>(125a,11b) equalish[A[x,M[0,b]],A[y,M[0,b]]] ~equalish[x,y]
127S>(126a,18c) ~equalish[x,y] equalish[z,A[y,M[0,b]]] ~equalish[z,A[x,M[0,b]]]
128S>(127b,24b) ~equalish[x,y] ~equalish[z,A[x,M[0,b]]] equalish[A[y,M[0,b]],z]
129S>(128a,28a) ~equalish[x,A[M[a,b],M[0,b]]] equalish[A[0,M[0,b]],x]
130S>(129a,116a) equalish[A[0,M[0,b]],0]

131S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]
132S>(131b,27a) defined[M[0,A[a,AI[a]]]
133S>(132a,11b) equalish[A[x,M[0,A[a,AI[a]]]],A[y,M[0,A[a,AI[a]]]] ~equalish[x,y]
134S>(133a,18c) ~equalish[x,y] equalish[z,A[y,M[0,A[a,AI[a]]]] ~equalish[z,A[x,M[0,A[a,AI[a]]]]
135S>(134b,24b) ~equalish[x,y] ~equalish[z,A[x,M[0,A[a,AI[a]]]] equalish[A[y,M[0,A[a,AI[a]]],z]
136S>(135a,34a) ~equalish[x,A[A[a,AI[a]],M[0,A[a,AI[a]]]] equalish[A[0,M[0,A[a,AI[a]]],x]
137S>(136a,110a) equalish[A[0,M[0,A[a,AI[a]]],0]

138S>(40a,2d) equalish[A[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]],M[A[x,y],A[a,AI[a]]] ~defined[x]
~defined[y]
139S>(138b,25a) equalish[A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]],M[A[1,x],A[a,AI[a]]] ~defined[x]
140S>(139a,18b) ~defined[x] equalish[A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]],y]
~equalish[M[A[1,x],A[a,AI[a]]],y]
141S>(140b,24b) ~defined[x] ~equalish[M[A[1,x],A[a,AI[a]]],y]
equalish[y,A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]]]
142S>(141a,40a) ~equalish[M[A[1,A[a,AI[a]]],A[a,AI[a]],x]
equalish[x,A[M[1,A[a,AI[a]]],M[A[a,AI[a]],A[a,AI[a]]]]
143S>(142a,123a) equalish[0,A[M[1,A[a,AI[a]]],M[A[a,AI[a]],A[a,AI[a]]]]

144S>(32a,15c) defined[M[x,b]] ~defined[x]
145S>(144a,21b) ~defined[x] equalish[A[0,M[x,b]],M[x,b]]
146S>(145b,24b) ~defined[x] equalish[M[x,b],A[0,M[x,b]]]
147S>(146b,18b) ~defined[x] equalish[M[x,b],y] ~equalish[A[0,M[x,b]],y]
148S>(147b,24b) ~defined[x] ~equalish[A[0,M[x,b]],y] equalish[y,M[x,b]]
149S>(148a,27a) ~equalish[A[0,M[0,b]],x] equalish[x,M[0,b]]
150S>(149a,130a) equalish[0,M[0,b]]

151S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]
152S>(151a,21b) ~defined[x] equalish[A[0,M[x,A[a,AI[a]]],M[x,A[a,AI[a]]]
153S>(152b,24b) ~defined[x] equalish[M[x,A[a,AI[a]]],A[0,M[x,A[a,AI[a]]]]
154S>(153b,18b) ~defined[x] equalish[M[x,A[a,AI[a]]],y] ~equalish[A[0,M[x,A[a,AI[a]]],y]
155S>(154b,24b) ~defined[x] ~equalish[A[0,M[x,A[a,AI[a]]],y] equalish[y,M[x,A[a,AI[a]]]
156S>(155a,27a) ~equalish[A[0,M[0,A[a,AI[a]]],x] equalish[x,M[0,A[a,AI[a]]]
157S>(156a,137a) equalish[0,M[0,A[a,AI[a]]]

158S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]
159S>(158b,40a) defined[M[A[a,AI[a]],A[a,AI[a]]]
160S>(159a,11b) equalish[A[x,M[A[a,AI[a]],A[a,AI[a]]],A[y,M[A[a,AI[a]],A[a,AI[a]]]]
~equalish[x,y]
161S>(160b,20a) equalish[A[M[1,x],M[A[a,AI[a]],A[a,AI[a]]],A[x,M[A[a,AI[a]],A[a,AI[a]]]]
~defined[x]
162S>(161a,18c) ~defined[x] equalish[y,A[x,M[A[a,AI[a]],A[a,AI[a]]]]
~equalish[y,A[M[1,x],M[A[a,AI[a]],A[a,AI[a]]]]
163S>(162a,40a) equalish[x,A[A[a,AI[a]],M[A[a,AI[a]],A[a,AI[a]]]]
~equalish[x,A[M[1,A[a,AI[a]],M[A[a,AI[a]],A[a,AI[a]]]]

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164: 163 {0/x} equalish[0,A[A[a,AI[a]],M[A[a,AI[a]],A[a,AI[a]]]]  
~equalish[0,A[M[1,A[a,AI[a]]],M[A[a,AI[a]],A[a,AI[a]]]]  
165S>(164b,143a) equalish[0,A[A[a,AI[a]],M[A[a,AI[a]],A[a,AI[a]]]]  
  
166S>(24b,9a) equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[z,x]  
167S>(166b,32a) equalish[M[x,b],M[y,b]] ~equalish[y,x]  
168S>(167a,18c) ~equalish[x,y] equalish[z,M[x,b]] ~equalish[z,M[y,b]]  
169S>(168a,34a) equalish[x,M[A[a,AI[a]],b]] ~equalish[x,M[0,b]]  
170S>(169a,24b) ~equalish[x,M[0,b]] equalish[M[A[a,AI[a]],b],x]  
171S>(170a,150a) equalish[M[A[a,AI[a]],b],0]  
  
172S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]  
173S>(172b,40a) defined[M[A[a,AI[a]],A[a,AI[a]]]  
174S>(173a,11b) equalish[A[x,M[A[a,AI[a]],A[a,AI[a]]],A[y,M[A[a,AI[a]],A[a,AI[a]]]]  
~equalish[x,y]  
175S>(174a,18c) ~equalish[x,y] equalish[z,A[y,M[A[a,AI[a]],A[a,AI[a]]]]  
~equalish[z,A[x,M[A[a,AI[a]],A[a,AI[a]]]]  
176S>(175b,24b) ~equalish[x,y] ~equalish[z,A[x,M[A[a,AI[a]],A[a,AI[a]]]]  
equalish[A[y,M[A[a,AI[a]],A[a,AI[a]]],z]  
177S>(176a,34a) ~equalish[x,A[A[a,AI[a]],M[A[a,AI[a]],A[a,AI[a]]]]  
equalish[A[0,M[A[a,AI[a]],A[a,AI[a]]],x]  
178S>(177a,165a) equalish[A[0,M[A[a,AI[a]],A[a,AI[a]]],0]  
  
179S>(31a,17c) defined[A[x,a]] ~defined[x]  
180S>(179b,27a) defined[A[0,a]]  
  
181S>(24b,18a) equalish[x,y] ~equalish[y,z] ~equalish[z,x]  
182S>(181b,41a) equalish[x,M[A[a,AI[a]],1]] ~equalish[0,x]  
183S>(182b,23a) equalish[0,M[A[a,AI[a]],1]] ~defined[0]  
184S>(183b,27a) equalish[0,M[A[a,AI[a]],1]]  
185S>(184a,18c) equalish[x,M[A[a,AI[a]],1]] ~equalish[x,0]  
186S>(185a,45c) ~equalish[M[A[a,AI[a]],x],0] ~defined[x]  
equalish[A[M[a,x],M[AI[a],x]],M[A[a,AI[a]],1]]  
187S>(186a,171a) ~defined[b] equalish[A[M[a,b],M[AI[a],b]],M[A[a,AI[a]],1]]  
188S>(187a,32a) equalish[A[M[a,b],M[AI[a],b]],M[A[a,AI[a]],1]]  
  
189S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]  
190S>(189a,21b) ~defined[x] equalish[A[0,M[x,A[a,AI[a]]],M[x,A[a,AI[a]]]  
191S>(190b,24b) ~defined[x] equalish[M[x,A[a,AI[a]],A[0,M[x,A[a,AI[a]]]]  
192S>(191b,18b) ~defined[x] equalish[M[x,A[a,AI[a]],y] ~equalish[A[0,M[x,A[a,AI[a]]],y]  
193S>(192b,24b) ~defined[x] ~equalish[A[0,M[x,A[a,AI[a]]],y] equalish[y,M[x,A[a,AI[a]]]]  
194S>(193a,40a) ~equalish[A[0,M[A[a,AI[a]],A[a,AI[a]]],x] equalish[x,M[A[a,AI[a]],A[a,AI[a]]]  
195S>(194a,178a) equalish[0,M[A[a,AI[a]],A[a,AI[a]]]  
  
196S>(24b,20a) equalish[x,M[1,x]] ~defined[x]  
197S>(196a,18b) ~defined[x] equalish[x,y] ~equalish[M[1,x],y]  
198S>(197a,31a) equalish[a,x] ~equalish[M[1,a],x]  
199S>(198a,29a) ~equalish[M[1,a],0]  
  
200S>(24a,18c) ~equalish[x,y] equalish[z,x] ~equalish[z,y]  
201S>(200a,48a) equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
~equalish[x,M[A[a,AI[a]],A[a,AI[a]]]  
202S>(201b,18a) equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]] ~equalish[x,y]  
~equalish[y,M[A[a,AI[a]],A[a,AI[a]]]  
203S>(202b,41a) equalish[M[A[a,AI[a]],1],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
~equalish[0,M[A[a,AI[a]],A[a,AI[a]]]  
204S>(203b,195a) equalish[M[A[a,AI[a]],1],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
205S>(204a,18b) equalish[M[A[a,AI[a]],1],x] ~equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],x]  
206S>(205b,48a) equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],A[a,AI[a]]]  
  
207S>(24b,18a) equalish[x,y] ~equalish[y,z] ~equalish[z,x]  
208S>(207a,24b) ~equalish[x,y] ~equalish[y,z] equalish[x,z]  
209S>(208a,24a) ~equalish[x,y] equalish[z,y] ~equalish[x,z]  
210S>(209a,44a) equalish[x,M[A[a,AI[a]],y]] ~equalish[A[M[a,y],M[AI[a],y]],x] ~defined[y]  
211S>(210a,46a) ~equalish[A[M[a,x],M[AI[a],x]],M[A[a,AI[a]],1]] ~defined[x]  
equalish[M[A[a,AI[a]],x],A[M[a,1],M[AI[a],1]]  
212S>(211a,188a) ~defined[b] equalish[M[A[a,AI[a]],b],A[M[a,1],M[AI[a],1]]  
213S>(212a,32a) equalish[M[A[a,AI[a]],b],A[M[a,1],M[AI[a],1]]  
  
214S>(32a,6b) equalish[M[b,MI[b]],1] equalish[b,0]  
215S>(214b,30a) equalish[M[b,MI[b]],1]  
216S>(215a,24b) equalish[1,M[b,MI[b]]]  
217S>(216a,9c) equalish[M[1,x],M[M[b,MI[b]],x]] ~defined[x]  
218S>(217a,18b) ~defined[x] equalish[M[1,x],y] ~equalish[M[M[b,MI[b]],x],y]  
219S>(218a,31a) equalish[M[1,a],x] ~equalish[M[M[b,MI[b]],a],x]  
220S>(219a,199a) ~equalish[M[M[b,MI[b]],a],0]  
  
221S>(48a,24b) equalish[M[A[a,AI[a]],A[a,AI[a]]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
222S>(221a,18c) equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
~equalish[x,M[A[a,AI[a]],A[a,AI[a]]]

223S>(222a,46a) ~equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],A[a,AI[a]]]  
 equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]  
 224S>(223a,206a) equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]  
 225S>(224a,24b) equalish[A[M[a,1],M[AI[a],1],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]]  
 226S>(225a,18c) equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 ~equalish[x,A[M[a,1],M[AI[a],1]]]  
 227S>(226a,45c) ~equalish[M[A[a,AI[a]],x],A[M[a,1],M[AI[a],1]]] ~defined[x]  
 equalish[A[M[a,x],M[AI[a],x]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 228S>(227a,213a) ~defined[b]  
 equalish[A[M[a,b],M[AI[a],b]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 229S>(228a,32a) equalish[A[M[a,b],M[AI[a],b]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
  
 230S>(32a,7b) defined[MI[b]] equalish[b,0]  
 231S>(230b,30a) defined[MI[b]]  
 232S>(231a,15c) defined[M[x,MI[b]]] ~defined[x]  
 233S>(232b,32a) defined[M[b,MI[b]]]  
 234S>(233a,12b) equalish[M[M[b,MI[b]],x],M[x,M[b,MI[b]]] ~defined[x]  
 235S>(234a,18b) ~defined[x] equalish[M[M[b,MI[b]],x],y] ~equalish[M[x,M[b,MI[b]]],y]  
 236S>(235a,31a) equalish[M[M[b,MI[b]],a],x] ~equalish[M[a,M[b,MI[b]]],x]  
 237S>(236a,220a) ~equalish[M[a,M[b,MI[b]]],0]  
  
 238S>(47a,24b) equalish[A[M[a,1],M[AI[a],1]],0]  
  
 239S>(48a,24b) equalish[M[A[a,AI[a]],A[a,AI[a]]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 240S>(239a,18b) equalish[M[A[a,AI[a]],A[a,AI[a]]],x]  
 ~equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],x]  
 241S>(240b,18a) equalish[M[A[a,AI[a]],A[a,AI[a]]],x]  
 ~equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],y] ~equalish[y,x]  
 242S>(241b,24a) equalish[M[A[a,AI[a]],A[a,AI[a]]],x] ~equalish[y,x]  
 ~equalish[y,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 243S>(242b,44a) equalish[M[A[a,AI[a]],A[a,AI[a]]],M[A[a,AI[a]],x]  
 ~equalish[A[M[a,x],M[AI[a],x]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]] ~defined[x]  
 244S>(243a,24b) ~equalish[A[M[a,x],M[AI[a],x]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 ~defined[x] equalish[M[A[a,AI[a]],x],M[A[a,AI[a]],A[a,AI[a]]]  
 245S>(244a,229a) ~defined[b] equalish[M[A[a,AI[a]],b],M[A[a,AI[a]],A[a,AI[a]]]  
 246S>(245a,32a) equalish[M[A[a,AI[a]],b],M[A[a,AI[a]],A[a,AI[a]]]  
  
 247S>(32a,7b) defined[MI[b]] equalish[b,0]  
 248S>(247b,30a) defined[MI[b]]  
 249S>(248a,1d) equalish[M[x,M[y,MI[b]]],M[M[x,y],MI[b]] ~defined[x] ~defined[y]  
 250S>(249b,31a) equalish[M[a,M[x,MI[b]]],M[M[a,x],MI[b]] ~defined[x]  
 251S>(250b,32a) equalish[M[a,M[b,MI[b]]],M[M[a,b],MI[b]]  
 252S>(251a,18b) equalish[M[a,M[b,MI[b]]],x] ~equalish[M[M[a,b],MI[b]],x]  
 253S>(252a,237a) ~equalish[M[M[a,b],MI[b]],0]  
  
 254S>(46a,12a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[A[a,AI[a]]] ~defined[1]  
 255S>(254b,40a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[1]  
 256S>(255b,25a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]  
 257S>(256a,18b) equalish[M[1,A[a,AI[a]]],x] ~equalish[A[M[a,1],M[AI[a],1]],x]  
 258: 257{0/x} equalish[M[1,A[a,AI[a]]],0] ~equalish[A[M[a,1],M[AI[a],1]],0]  
 259S>(258b,238a) equalish[M[1,A[a,AI[a]]],0]  
  
 260S>(30a,7c) defined[MI[b]] ~defined[b]  
 261S>(260b,32a) defined[MI[b]]  
 262S>(261a,9b) equalish[M[x,MI[b]],M[y,MI[b]]] ~equalish[x,y]  
 263S>(262a,18b) ~equalish[x,y] equalish[M[x,MI[b]],z] ~equalish[M[y,MI[b]],z]  
 264S>(263a,28a) equalish[M[M[a,b],MI[b]],x] ~equalish[M[0,MI[b]],x]  
 265S>(264b,24a) equalish[M[M[a,b],MI[b]],x] ~equalish[x,M[0,MI[b]]]  
 266S>(265a,253a) ~equalish[0,M[0,MI[b]]]  
  
 267S>(32a,6b) equalish[M[b,MI[b]],1] equalish[b,0]  
 268S>(267b,30a) equalish[M[b,MI[b]],1]  
 269S>(268a,9c) equalish[M[M[b,MI[b]],x],M[1,x]] ~defined[x]  
 270S>(269a,18b) ~defined[x] equalish[M[M[b,MI[b]],x],y] ~equalish[M[1,x],y]  
 271S>(270a,40a) equalish[M[M[b,MI[b]],A[a,AI[a]]],x] ~equalish[M[1,A[a,AI[a]]],x]  
 272: 271{0/x} equalish[M[M[b,MI[b]],A[a,AI[a]]],0] ~equalish[M[1,A[a,AI[a]]],0]  
 273S>(272b,259a) equalish[M[M[b,MI[b]],A[a,AI[a]]],0]  
  
 274S>(40a,15b) defined[M[A[a,AI[a]],x]] ~defined[x]  
 275S>(274b,25a) defined[M[A[a,AI[a]],1]]  
  
 276S>(24a,9c) ~equalish[x,y] equalish[M[y,z],M[x,z]] ~defined[z]  
 277S>(276a,47a) equalish[M[A[M[a,1],M[AI[a],1]],x],M[0,x]] ~defined[x]  
 278S>(277b,7a) equalish[M[A[M[a,1],M[AI[a],1]],MI[x]],M[0,MI[x]] ~defined[x] equalish[x,0]  
 279S>(278b,32a) equalish[M[A[M[a,1],M[AI[a],1]],MI[b]],M[0,MI[b]] equalish[b,0]  
 280S>(279b,30a) equalish[M[A[M[a,1],M[AI[a],1]],MI[b]],M[0,MI[b]]  
 281S>(280a,18c) equalish[x,M[0,MI[b]]] ~equalish[x,M[A[M[a,1],M[AI[a],1]],MI[b]]  
 282S>(281a,266a) ~equalish[0,M[A[M[a,1],M[AI[a],1]],MI[b]]  
  
 283S>(32a,7b) defined[MI[b]] equalish[b,0]  
 284S>(283b,30a) defined[MI[b]]

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285S>(284a,15c) defined[M[x,MI[b]]] ~defined[x]
286S>(285b,32a) defined[M[b,MI[b]]]
287S>(286a,12c) equalish[M[x,M[b,MI[b]]],M[M[b,MI[b]],x]] ~defined[x]
288S>(287a,18b) ~defined[x] equalish[M[x,M[b,MI[b]]],y] ~equalish[M[M[b,MI[b]],x],y]
289S>(288a,40a) equalish[M[A[a,AI[a]],M[b,MI[b]]],x] ~equalish[M[M[b,MI[b]],A[a,AI[a]]],x]
290: 289|{0/x} equalish[M[A[a,AI[a]],M[b,MI[b]]],0] ~equalish[M[M[b,MI[b]],A[a,AI[a]]],0]
291S>(290b,273a) equalish[M[A[a,AI[a]],M[b,MI[b]]],0]

292S>(32a,7b) defined[MI[b]] equalish[b,0]
293S>(292b,30a) defined[MI[b]]
294S>(293a,1d) equalish[M[x,M[y,MI[b]]],M[M[x,y],MI[b]]] ~defined[x] ~defined[y]
295S>(294b,40a) equalish[M[A[a,AI[a]],M[x,MI[b]]],M[M[A[a,AI[a]],x],MI[b]]] ~defined[x]
296S>(295a,24b) ~defined[x] equalish[M[M[A[a,AI[a]],x],MI[b]],M[A[a,AI[a]],M[x,MI[b]]]]
297S>(296b,18b) ~defined[x] equalish[M[M[A[a,AI[a]],x],MI[b]],y]
~equalish[M[A[a,AI[a]],M[x,MI[b]]],y]
298S>(297a,32a) equalish[M[M[A[a,AI[a]],b],MI[b]],x] ~equalish[M[A[a,AI[a]],M[b,MI[b]]],x]
299: 298|{0/x} equalish[M[M[A[a,AI[a]],b],MI[b]],0] ~equalish[M[A[a,AI[a]],M[b,MI[b]]],0]
300S>(299b,291a) equalish[M[M[A[a,AI[a]],b],MI[b]],0]

301S>(36a,7c) defined[MI[A[0,a]]] ~defined[A[0,a]]
302S>(301b,180a) defined[MI[A[0,a]]]
303S>(302a,19b) equalish[A[MI[A[0,a]],AI[MI[A[0,a]]]],0]
304S>(303a,24b) equalish[0,A[MI[A[0,a]],AI[MI[A[0,a]]]]
305S>(304a,9c) equalish[M[0,x],M[A[MI[A[0,a]],AI[MI[A[0,a]]]],x]] ~defined[x]
306S>(305a,18c) ~defined[x] equalish[y,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],x]] ~equalish[y,M[0,x]]
307S>(306a,40a) equalish[x,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],A[a,AI[a]]]
~equalish[x,M[0,A[a,AI[a]]]
308: 307|{0/x} equalish[0,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],A[a,AI[a]]]
~equalish[0,M[0,A[a,AI[a]]]
309S>(308b,157a) equalish[0,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],A[a,AI[a]]]

310S>(36a,7c) defined[MI[A[0,a]]] ~defined[A[0,a]]
311S>(310b,180a) defined[MI[A[0,a]]]
312S>(311a,19b) equalish[A[MI[A[0,a]],AI[MI[A[0,a]]]],0]
313S>(312a,9c) equalish[M[A[MI[A[0,a]],AI[MI[A[0,a]]]],x],M[0,x]] ~defined[x]
314S>(313a,18c) ~defined[x] equalish[y,M[0,x]] ~equalish[y,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],x]]
315S>(314b,24b) ~defined[x] ~equalish[y,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],x]] equalish[M[0,x],y]
316S>(315a,40a) ~equalish[x,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],A[a,AI[a]]]
equalish[M[0,A[a,AI[a]],x]
317S>(316a,309a) equalish[M[0,A[a,AI[a]]],0]

318S>(34a,9c) equalish[M[A[a,AI[a]],x],M[0,x]] ~defined[x]
319S>(318a*45c) ~defined[x] equalish[A[M[a,x],M[AI[a],x]],M[0,x]]
320S>(319b,18b) ~defined[x] equalish[A[M[a,x],M[AI[a],x]],y] ~equalish[M[0,x],y]
321S>(320a,40a) equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],x] ~equalish[M[0,A[a,AI[a]]],x]
322S>(321a,24b) ~equalish[M[0,A[a,AI[a]]],x] equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]
323S>(322a,317a) equalish[0,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]

324S>(34a,18c) equalish[x,0] ~equalish[x,A[a,AI[a]]]
325S>(324a,18b) ~equalish[x,A[a,AI[a]]] equalish[x,y] ~equalish[0,y]
326S>(325a,23a) equalish[A[a,AI[a]],x] ~equalish[0,x] ~defined[A[a,AI[a]]]
327S>(326c,40a) equalish[A[a,AI[a]],x] ~equalish[0,x]
328S>(327b,18a) equalish[A[a,AI[a]],x] ~equalish[0,y] ~equalish[y,x]
329S>(328a,24b) ~equalish[0,x] ~equalish[x,y] equalish[y,A[a,AI[a]]]
330S>(329b,48a) ~equalish[0,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]
equalish[M[A[a,AI[a]],A[a,AI[a]]],A[a,AI[a]]]
331S>(330a,323a) equalish[M[A[a,AI[a]],A[a,AI[a]]],A[a,AI[a]]]

332S>(34a,18c) equalish[x,0] ~equalish[x,A[a,AI[a]]]
333S>(332b,45b) equalish[A[M[a,x],M[AI[a],x]],0] ~defined[x] ~equalish[M[A[a,AI[a]],x],A[a,AI[a]]]
334S>(333b,40a) equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]],0]
~equalish[M[A[a,AI[a]],A[a,AI[a]]],A[a,AI[a]]]
335S>(334b,331a) equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]],0]
336S>(335a,24b) equalish[0,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]
337S>(336a,18c) equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]] ~equalish[x,0]
338S>(337b,41a) equalish[M[A[a,AI[a]],1],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]

339S>(48a,18c) equalish[x,M[A[a,AI[a]],A[a,AI[a]]]
~equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]
340S>(339a,46a) ~equalish[M[A[a,AI[a]],1],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]
equalish[M[A[a,AI[a]],A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]
341S>(340a,338a) equalish[M[A[a,AI[a]],A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]
342S>(341a,18c) equalish[x,A[M[a,1],M[AI[a],1]]] ~equalish[x,M[A[a,AI[a]],A[a,AI[a]]]
343S>(342a,24b) ~equalish[x,M[A[a,AI[a]],A[a,AI[a]]] equalish[A[M[a,1],M[AI[a],1]],x]
344S>(343a,45b) equalish[A[M[a,1],M[AI[a],1]],A[M[a,x],M[AI[a],x]]] ~defined[x]
~equalish[M[A[a,AI[a]],x],M[A[a,AI[a]],A[a,AI[a]]]
345S>(344b,32a) equalish[A[M[a,1],M[AI[a],1]],A[M[a,b],M[AI[a],b]]]
~equalish[M[A[a,AI[a]],b],M[A[a,AI[a]],A[a,AI[a]]]
346S>(345b,246a) equalish[A[M[a,1],M[AI[a],1]],A[M[a,b],M[AI[a],b]]]

347S>(46a,23a) equalish[M[A[a,AI[a]],1],A[M[a,1],M[AI[a],1]]] ~defined[M[A[a,AI[a]],1]]

```

```

348S>(347b,275a) equalish[M[A[a,AI[a]],1],A[M[a,1],M[AI[a],1]]]
349S>(348a,18b) equalish[M[A[a,AI[a]],1],x] ~equalish[A[M[a,1],M[AI[a],1]],x]
350S>(349b,18a) equalish[M[A[a,AI[a]],1],x] ~equalish[A[M[a,1],M[AI[a],1]],y] ~equalish[y,x]
351S>(350a,24b) ~equalish[A[M[a,1],M[AI[a],1]],x] ~equalish[x,y] equalish[y,M[A[a,AI[a]],1]]
352S>(351b,44a) ~equalish[A[M[a,1],M[AI[a],1]],A[M[a,x],M[AI[a],x]]]
equalish[M[A[a,AI[a]],x],M[A[a,AI[a]],1]] ~defined[x]
353S>(352b,24b) ~equalish[A[M[a,1],M[AI[a],1]],A[M[a,x],M[AI[a],x]]] ~defined[x]
equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],x]]
354S>(353a,346a) ~defined[b] equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],b]]
355S>(354a,32a) equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],b]]

356S>(45a,32a) equalish[A[M[a,b],M[AI[a],b]],x] ~equalish[M[A[a,AI[a]],b],x]
357S>(356b,46b) equalish[A[M[a,b],M[AI[a],b]],A[M[a,1],M[AI[a],1]]]
~equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],b]]
358S>(357b,355a) equalish[A[M[a,b],M[AI[a],b]],A[M[a,1],M[AI[a],1]]]
359S>(358a,9c) equalish[M[A[M[a,b],M[AI[a],b]],x],M[A[M[a,1],M[AI[a],1]],x]] ~defined[x]
360S>(359b,7a) equalish[M[A[M[a,b],M[AI[a],b]],MI[x]],M[A[M[a,1],M[AI[a],1]],MI[x]]] ~defined[x]
equalish[x,0]
361S>(360b,32a) equalish[M[A[M[a,b],M[AI[a],b]],MI[b]],M[A[M[a,1],M[AI[a],1]],MI[b]]]
equalish[b,0]
362S>(361b,30a) equalish[M[A[M[a,b],M[AI[a],b]],MI[b]],M[A[M[a,1],M[AI[a],1]],MI[b]]]
363S>(362a,18c) equalish[x,M[A[M[a,1],M[AI[a],1]],MI[b]]]
~equalish[x,M[A[M[a,b],M[AI[a],b]],MI[b]]]
364S>(363a,282a) ~equalish[0,M[A[M[a,b],M[AI[a],b]],MI[b]]]

365S>(44a,9c) ~defined[x] equalish[M[A[M[a,x],M[AI[a],x]],y],M[M[A[a,AI[a]],x],y]] ~defined[y]
366S>(365a,32a) equalish[M[A[M[a,b],M[AI[a],b]],x],M[M[A[a,AI[a]],b],x]] ~defined[x]
367S>(366b,7a) equalish[M[A[M[a,b],M[AI[a],b]],MI[x]],M[M[A[a,AI[a]],b],MI[x]]] ~defined[x]
equalish[x,0]
368S>(367b,32a) equalish[M[A[M[a,b],M[AI[a],b]],MI[b]],M[M[A[a,AI[a]],b],MI[b]]] equalish[b,0]
369S>(368b,30a) equalish[M[A[M[a,b],M[AI[a],b]],MI[b]],M[M[A[a,AI[a]],b],MI[b]]]
370S>(369a,18b) equalish[M[A[M[a,b],M[AI[a],b]],MI[b]],x] ~equalish[M[M[A[a,AI[a]],b],MI[b]],x]
371S>(370a,24b) ~equalish[M[M[A[a,AI[a]],b],MI[b]],x] equalish[x,M[A[M[a,b],M[AI[a],b]],MI[b]]]
372S>(371a,300a) equalish[0,M[A[M[a,b],M[AI[a],b]],MI[b]]]
373S>(372a,364a) []

```

```

PHASE 0: 0 s PHASE 1: 59 s PHASE 2: 464 s Total Time: 545 s
NOD: 15367420 RES: 69401354 FAC: 2456274 T: 26 V: 32 L: 16
CTE: 2338615 CTH: 1119783 CTF: 0 CSZ: 16777216
UTE: 3336862 UTH: 6753884 UTF: 1 SBA: 0
BAS: 32 RED: 50 LEN: 10+325=335

```

## FLD041-2

Predicates: equalish defined less\_or\_equal  
Functions: c b additive\_identity a . multiplicative\_identity : additive\_inverse multiplicative\_inverse  
add multiply  
Renamed predicates: equalish defined LE  
Renamed functions: c b 0 a . 1 : AI MI A M

C120: Spawmed: 150 xvl 26.10.3 W59 1S equalish[M[x,M[y,z]],M[u,z]] ~defined[x] ~defined[y]  
~defined[z] ... ?

Axioms:

```
1S>equalish[M[x,M[y,z]],M[u,z]] ~defined[x] ~defined[y] ~defined[z] ... ?
2: equalish[A[M[x,y],M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y]
3: equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
4: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
5: LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
6: equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]
7: defined[MI[x]] ~defined[x] equalish[x,0]
8: LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]
9: equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[x,z]
10: LE[x,y] ~LE[z,y] ~equalish[z,x]
11: equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]
12: equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: equalish[x,y] ~LE[x,y] ~LE[y,x]
15 >defined[M[x,y]] ~defined[x] ~defined[y]
16: equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]
17: defined[A[x,y]] ~defined[x] ~defined[y]
18 >equalish[x,y] ~equalish[x,z] ~equalish[z,y]
19: equalish[A[x,AI[x]],0] ~defined[x]
20 >equalish[M[1,x],x] ~defined[x]
21: equalish[A[0,x],x] ~defined[x]
22: defined[AI[x]] ~defined[x]
23: equalish[x,x] ~defined[x]
24 >equalish[x,y] ~equalish[y,x]
25 >defined[1]
26: ~equalish[0,1]
27: defined[0]
```

Negated conclusion:

```
28S ~equalish[a,0]
29S>~equalish[b,0]
30S>equalish[c,0]
31S>equalish[M[a,b],c]
32S>defined[a]
33S>defined[b]
34S defined[c]
```

Phase 0 clauses used in proof:

```
43S>(33a*20b) equalish[M[1,b],b]
50S>(31a*24b) equalish[c,M[a,b]]
51S>(30a*24b) equalish[0,c]
```

Phases 1 and 2 clauses used in proof:

```
52S>(33a,15c) defined[M[x,b]] ~defined[x]
53S>(52b,32a) defined[M[a,b]]
```

```
54S>(51a,18b) equalish[0,x] ~equalish[c,x]
55S>(54a,24b) ~equalish[c,x] equalish[x,0]
56S>(55b,29a) ~equalish[c,b]
```

```
57S>(50a,18b) equalish[c,x] ~equalish[M[a,b],x]
58S>(57b,24a) equalish[c,x] ~equalish[x,M[a,b]]
59S>(58b,20a) equalish[c,M[1,M[a,b]]] ~defined[M[a,b]]
60S>(59b,53a) equalish[c,M[1,M[a,b]]]
```

```
61S>(43a,18c) equalish[x,b] ~equalish[x,M[1,b]]
62S>(61a,56a) ~equalish[c,M[1,b]]
```

```
63S>(33a,1d) equalish[M[x,M[y,b]],M[z,b]] ~defined[x] ~defined[y] ... ?
64S>(63b,25a) equalish[M[1,M[x,b]],M[y,b]] ~defined[x] ... ?
65S>(64a,18c) ~defined[x] equalish[y,M[z,b]] ~equalish[y,M[1,M[x,b]]] ... ?
66S>(65a,32a) equalish[x,M[y,b]] ~equalish[x,M[1,M[a,b]]] ... ?
67: 66[{c/x,1/y} equalish[c,M[1,b]] ~equalish[c,M[1,M[a,b]]] ... ?
68S>(67a,62a) ~equalish[c,M[1,M[a,b]]] ... ?
69S>(68a,60a) [] ... ?
```

Inferences derived from proof before variable replaced by constant

43 50 51 53 56 60 62

Inferences derived when variable is replaced by original constant  
64 66

PHASE 0: 0 s PHASE 1: 2 s PHASE 2: 2 s Total Time: 4 s  
NOD: 62797 RES: 398132 FAC: 7318 T: 26 V: 32 L: 16  
CTE: 71537 CTH: 14530 CTF: 0 CSZ: 16777216  
UTE: 444822 UTH: 475116 UTF: 0 SBA: 0  
BAS: 34 RED: 51 LEN: 3+18=21

Proof found to theorem by slave 120!

Axioms:

```
1S>equalish[M[x,M[y,z]],M[M[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
2 >equalish[A[M[x,y],M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y]
3: equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
4: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
5: LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
6 >equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]
7 >defined[MI[x]] ~defined[x] equalish[x,0]
8: LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]
9 >equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[x,z]
10: LE[x,y] ~LE[z,y] ~equalish[z,x]
11 >equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]
12 >equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: equalish[x,y] ~LE[x,y] ~LE[y,x]
15S>defined[M[x,y]] ~defined[x] ~defined[y]
16 >equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]
17: defined[A[x,y]] ~defined[x] ~defined[y]
18S>equalish[x,y] ~equalish[x,z] ~equalish[z,y]
19: equalish[A[x,AI[x]],0] ~defined[x]
20S>equalish[M[1,x],x] ~defined[x]
21 >equalish[A[0,x],x] ~defined[x]
22: defined[AI[x]] ~defined[x]
23: equalish[x,x] ~defined[x]
24S>equalish[x,y] ~equalish[y,x]
25S defined[1]
26: ~equalish[0,1]
27 >defined[0]
```

Negated conclusion:

```
28S>~equalish[a,0]
29S>~equalish[b,0]
30S>equalish[c,0]
31S>equalish[M[a,b],c]
32S>defined[a]
33S>defined[b]
34S>defined[c]
```

Phase 0 clauses used in proof:

```
38S>defined[M[a,b]]
39S>~equalish[c,b]
40S>equalish[c,M[1,M[a,b]]]
42S>equalish[M[1,M[x,b]],M[M[1,x],b]] ~defined[x]
43S>equalish[x,M[M[1,a],b]] ~equalish[x,M[1,M[a,b]]]
44S>(43b*40a) equalish[c,M[M[1,a],b]]
45S>(42b*38a) equalish[M[1,M[a,b],b]],M[M[1,M[a,b],b]]
```

Phases 1 and 2 clauses used in proof:

```
52S>(33a,9b) equalish[M[x,b],M[y,b]] ~equalish[x,y]
53S>(52b,21a) equalish[M[A[0,x],b],M[x,b]] ~defined[x]
54S>(53a,18b) ~defined[x] equalish[M[A[0,x],b],y] ~equalish[M[x,b],y]
55S>(54a,32a) equalish[M[A[0,a],b],x] ~equalish[M[a,b],x]
56S>(55b,31a) equalish[M[A[0,a],b],c]

57S>(33a,2d) equalish[A[M[x,b],M[y,b]],M[A[x,y],b]] ~defined[x] ~defined[y]
58S>(57b,27a) equalish[A[M[0,b],M[x,b]],M[A[0,x],b]] ~defined[x]
59S>(58a,18b) ~defined[x] equalish[A[M[0,b],M[x,b]],y] ~equalish[M[A[0,x],b],y]
60S>(59a,32a) equalish[A[M[0,b],M[a,b]],x] ~equalish[M[A[0,a],b],x]
61: 60[{c/x} equalish[A[M[0,b],M[a,b]],c] ~equalish[M[A[0,a],b],c]
62S>(61b,56a) equalish[A[M[0,b],M[a,b]],c]

63S>(38a,11b) equalish[A[x,M[a,b]],A[y,M[a,b]]] ~equalish[x,y]
64S>(63b,9a) equalish[A[M[x,y],M[a,b]],A[M[z,y],M[a,b]]] ~defined[y] ~equalish[x,z]
65S>(64b,33a) equalish[A[M[x,b],M[a,b]],A[M[y,b],M[a,b]]] ~equalish[x,y]
66S>(65a,18b) ~equalish[x,y] equalish[A[M[x,b],M[a,b]],z] ~equalish[A[M[y,b],M[a,b]],z]
67S>(66a,30a) equalish[A[M[c,b],M[a,b]],x] ~equalish[A[M[0,b],M[a,b]],x]
68: 67[{c/x} equalish[A[M[c,b],M[a,b]],c] ~equalish[A[M[0,b],M[a,b]],c]
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69S>(68b,62a) equalish[A[M[c,b],M[a,b]],c]

70S>(38a,11b) equalish[A[x,M[a,b]],A[y,M[a,b]]] ~equalish[x,y]
71S>(70b,9a) equalish[A[M[x,y],M[a,b]],A[M[z,y],M[a,b]]] ~defined[y] ~equalish[x,z]
72S>(71b,33a) equalish[A[M[x,b],M[a,b]],A[M[y,b],M[a,b]]] ~equalish[x,y]
73S>(72a,18b) ~equalish[x,y] equalish[A[M[x,b],M[a,b]],z] ~equalish[A[M[y,b],M[a,b]],z]
74S>(73a,31a) equalish[A[M[M[a,b],b],M[a,b]],x] ~equalish[A[M[c,b],M[a,b]],x]
75: 74|{c/x} equalish[A[M[M[a,b],b],M[a,b]],c] ~equalish[A[M[c,b],M[a,b]],c]
76S>(75b,69a) equalish[A[M[M[a,b],b],M[a,b]],c]

77S>(38a,15b) defined[M[M[a,b],x]] ~defined[x]
78S>(77b,33a) defined[M[M[a,b],b]]
79S>(78a,16c) equalish[A[x,M[M[a,b],b]],A[M[M[a,b],b],x]] ~defined[x]
80S>(79a,18b) ~defined[x] equalish[A[x,M[M[a,b],b]],y] ~equalish[A[M[M[a,b],b],x],y]
81S>(80a,38a) equalish[A[M[a,b],M[M[a,b],b]],x] ~equalish[A[M[M[a,b],b],M[a,b]],x]
82: 81|{c/x} equalish[A[M[a,b],M[M[a,b],b]],c] ~equalish[A[M[M[a,b],b],M[a,b]],c]
83S>(82b,76a) equalish[A[M[a,b],M[M[a,b],b]],c]

84S>(38a,2c) equalish[A[M[x,y],M[M[a,b],y]],M[A[x,M[a,b]],y]] ~defined[x] ~defined[y]
85S>(84b,32a) equalish[A[M[a,x],M[M[a,b],x]],M[A[a,M[a,b]],x]] ~defined[x]
86S>(85a,24b) ~defined[x] equalish[M[A[a,M[a,b]],x],A[M[a,x],M[M[a,b],x]]]
87S>(86b,18b) ~defined[x] equalish[M[A[a,M[a,b]],x],y] ~equalish[A[M[a,x],M[M[a,b],x]],y]
88S>(87a,33a) equalish[M[A[a,M[a,b]],b],x] ~equalish[A[M[a,b],M[M[a,b],b]],x]
89: 88|{c/x} equalish[M[A[a,M[a,b]],b],c] ~equalish[A[M[a,b],M[M[a,b],b]],c]
90S>(89b,83a) equalish[M[A[a,M[a,b]],b],c]

91S>(44a,24b) equalish[M[M[1,a],b],c]
92S>(91a,18b) equalish[M[M[1,a],b],x] ~equalish[c,x]
93S>(92b,30a) equalish[M[M[1,a],b],0]

94S>(38a,2c) equalish[A[M[x,y],M[M[a,b],y]],M[A[x,M[a,b]],y]] ~defined[x] ~defined[y]
95S>(94b,32a) equalish[A[M[a,x],M[M[a,b],x]],M[A[a,M[a,b]],x]] ~defined[x]
96S>(95a,18b) ~defined[x] equalish[A[M[a,x],M[M[a,b],x]],y] ~equalish[M[A[a,M[a,b]],x],y]
97S>(96b,24b) ~defined[x] ~equalish[M[A[a,M[a,b]],x],y] equalish[y,A[M[a,x],M[M[a,b],x]]]
98S>(97a,33a) ~equalish[M[A[a,M[a,b]],b],x] equalish[x,A[M[a,b],M[M[a,b],b]]]
99S>(98a,90a) equalish[c,A[M[a,b],M[M[a,b],b]]]

100S>(44a,18c) equalish[x,M[M[1,a],b]] ~equalish[x,c]
101S>(100a,18b) ~equalish[x,c] equalish[x,y] ~equalish[M[M[1,a],b],y]
102S>(101b,28a) ~equalish[a,c] ~equalish[M[M[1,a],b],0]
103S>(102b,93a) ~equalish[a,c]

104S>(38a,15b) defined[M[M[a,b],x]] ~defined[x]
105S>(104b,33a) defined[M[M[a,b],b]]
106S>(105a,11b) equalish[A[x,M[M[a,b],b]],A[y,M[M[a,b],b]]] ~equalish[x,y]
107S>(106b,31a) equalish[A[M[a,b],M[M[a,b],b]],A[c,M[M[a,b],b]]]
108S>(107a,18c) equalish[x,A[c,M[M[a,b],b]]] ~equalish[x,A[M[a,b],M[M[a,b],b]]]
109: 108|{c/x} equalish[c,A[c,M[M[a,b],b]]] ~equalish[c,A[M[a,b],M[M[a,b],b]]]
110S>(109b,99a) equalish[c,A[c,M[M[a,b],b]]]

111S>(24b,20a) equalish[x,M[1,x]] ~defined[x]
112S>(111a,18b) ~defined[x] equalish[x,y] ~equalish[M[1,x],y]
113S>(112a,32a) equalish[a,x] ~equalish[M[1,a],x]
114S>(113a,103a) ~equalish[M[1,a],c]

115S>(38a,15b) defined[M[M[a,b],x]] ~defined[x]
116S>(115b,33a) defined[M[M[a,b],b]]
117S>(116a,11b) equalish[A[x,M[M[a,b],b]],A[y,M[M[a,b],b]]] ~equalish[x,y]
118S>(117a,18c) ~equalish[x,y] equalish[z,A[y,M[M[a,b],b]]] ~equalish[z,A[x,M[M[a,b],b]]]
119S>(118b,24b) ~equalish[x,y] ~equalish[z,A[x,M[M[a,b],b]]] equalish[A[y,M[M[a,b],b]],z]
120S>(119a,30a) ~equalish[x,A[c,M[M[a,b],b]]] equalish[A[0,M[M[a,b],b]],x]
121S>(120a,110a) equalish[A[0,M[M[a,b],b]],c]

122S>(33a,6b) equalish[M[b,MI[b]],1] equalish[b,0]
123S>(122b,29a) equalish[M[b,MI[b]],1]
124S>(123a,24b) equalish[1,M[b,MI[b]]]
125S>(124a,9c) equalish[M[1,x],M[M[b,MI[b]],x]] ~defined[x]
126S>(125a,18b) ~defined[x] equalish[M[1,x],y] ~equalish[M[M[b,MI[b]],x],y]
127S>(126a,32a) equalish[M[1,a],x] ~equalish[M[M[b,MI[b]],a],x]
128S>(127a,114a) ~equalish[M[M[b,MI[b]],a],c]

129S>(33a,7b) defined[MI[b]] equalish[b,0]
130S>(129b,29a) defined[MI[b]]
131S>(130a,15c) defined[M[x,MI[b]]] ~defined[x]
132S>(131b,33a) defined[M[b,MI[b]]]
133S>(132a,12b) equalish[M[M[b,MI[b]],x],M[x,M[b,MI[b]]] ~defined[x]
134S>(133a,18b) ~defined[x] equalish[M[M[b,MI[b]],x],y] ~equalish[M[x,M[b,MI[b]]],y]
135S>(134a,32a) equalish[M[M[b,MI[b]],a],x] ~equalish[M[a,M[b,MI[b]]],x]
136S>(135a,128a) ~equalish[M[a,M[b,MI[b]]],c]

137S>(38a,15b) defined[M[M[a,b],x]] ~defined[x]

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138S>(137a,21b) ~defined[x] equalish[A[0,M[M[a,b],x]],M[M[a,b],x]]
139S>(138b,24b) ~defined[x] equalish[M[M[a,b],x],A[0,M[M[a,b],x]]]
140S>(139b,18b) ~defined[x] equalish[M[M[a,b],x],y] ~equalish[A[0,M[M[a,b],x]],y]
141S>(140b,24b) ~defined[x] ~equalish[A[0,M[M[a,b],x]],y] equalish[y,M[M[a,b],x]]
142S>(141a,33a) ~equalish[A[0,M[M[a,b],b]],x] equalish[x,M[M[a,b],b]]
143S>(142a,121a) equalish[c,M[M[a,b],b]]

144S>(33a,9b) equalish[M[x,b],M[y,b]] ~equalish[x,y]
145S>(144b,20a) equalish[M[M[1,x],b],M[x,b]] ~defined[x]
146S>(145a,24b) ~defined[x] equalish[M[x,b],M[M[1,x],b]]
147S>(146b,18c) ~defined[x] equalish[y,M[M[1,x],b]] ~equalish[y,M[x,b]]
148S>(147b,24b) ~defined[x] ~equalish[y,M[x,b]] equalish[M[M[1,x],b],y]
149S>(148a,38a) ~equalish[x,M[M[a,b],b]] equalish[M[M[1,M[a,b]],b],x]
150S>(149a,143a) equalish[M[M[1,M[a,b]],b],c]

151S>(33a,7b) defined[MI[b]] equalish[b,0]
152S>(151b,29a) defined[MI[b]]
153S>(152a,1d) equalish[M[x,M[y,MI[b]]],M[M[x,y],MI[b]]] ~defined[x] ~defined[y]
154S>(153b,32a) equalish[M[a,M[x,MI[b]]],M[M[a,x],MI[b]]] ~defined[x]
155S>(154b,33a) equalish[M[a,M[b,MI[b]]],M[M[a,b],MI[b]]]
156S>(155a,18b) equalish[M[a,M[b,MI[b]]],x] ~equalish[M[M[a,b],MI[b]],x]
157S>(156a,136a) ~equalish[M[M[a,b],MI[b]],c]

158S>(30a,18c) equalish[x,0] ~equalish[x,c]
159S>(158a,24b) ~equalish[x,c] equalish[0,x]
160S>(159a,18a) equalish[0,x] ~equalish[x,y] ~equalish[y,c]
161S>(160b,45a) equalish[0,M[1,M[M[a,b],b]]] ~equalish[M[M[1,M[a,b]],b],c]
162S>(161b,150a) equalish[0,M[1,M[M[a,b],b]]]

163S>(31a,9c) equalish[M[M[a,b],x],M[c,x]] ~defined[x]
164S>(163b,7a) equalish[M[M[a,b],MI[x]],M[c,MI[x]]] ~defined[x] equalish[x,0]
165S>(164b,33a) equalish[M[M[a,b],MI[b]],M[c,MI[b]]] equalish[b,0]
166S>(165b,29a) equalish[M[M[a,b],MI[b]],M[c,MI[b]]]
167S>(166a,18b) equalish[M[M[a,b],MI[b]],x] ~equalish[M[c,MI[b]],x]
168S>(167a,157a) ~equalish[M[c,MI[b]],c]

169S>(34a,20b) equalish[M[1,c],c]

170S>(29a,7c) defined[MI[b]] ~defined[b]
171S>(170b,33a) defined[MI[b]]
172S>(171a,9b) equalish[M[x,MI[b]],M[y,MI[b]]] ~equalish[x,y]
173S>(172a,18b) ~equalish[x,y] equalish[M[x,MI[b]],z] ~equalish[M[y,MI[b]],z]
174S>(173a,44a) equalish[M[c,MI[b]],x] ~equalish[M[M[M[1,a],b],MI[b]],x]
175S>(174b,24a) equalish[M[c,MI[b]],x] ~equalish[x,M[M[M[1,a],b],MI[b]]]
176S>(175a,168a) ~equalish[c,M[M[M[1,a],b],MI[b]]]

177S>(30a,18b) equalish[c,x] ~equalish[0,x]
178S>(177b,18a) equalish[c,x] ~equalish[0,y] ~equalish[y,x]
179S>(178a,24b) ~equalish[0,x] ~equalish[x,y] equalish[y,c]
180S>(179b,45a) ~equalish[0,M[1,M[M[a,b],b]]] equalish[M[M[1,M[a,b]],b],c]
181S>(180a,162a) equalish[M[M[1,M[a,b]],b],c]
182S>(181a,18b) equalish[M[M[1,M[a,b]],b],x] ~equalish[c,x]
183S>(182b,40a) equalish[M[M[1,M[a,b]],b],M[1,M[a,b]]]

184S>(33a,6b) equalish[M[b,MI[b]],1] equalish[b,0]
185S>(184b,29a) equalish[M[b,MI[b]],1]
186S>(185a,9c) equalish[M[M[b,MI[b]],x],M[1,x]] ~defined[x]
187S>(186a,18b) ~defined[x] equalish[M[M[b,MI[b]],x],y] ~equalish[M[1,x],y]
188S>(187a,34a) equalish[M[M[b,MI[b]],c],x] ~equalish[M[1,c],x]
189: 188|{c/x} equalish[M[M[b,MI[b]],c],c] ~equalish[M[1,c],c]
190S>(189b,169a) equalish[M[M[b,MI[b]],c],c]

191S>(45a,18b) equalish[M[1,M[M[a,b],b]],x] ~equalish[M[M[1,M[a,b]],b],x]
192S>(191a,9c) ~equalish[M[M[1,M[a,b]],b],x] equalish[M[M[1,M[M[a,b],b]],y],M[x,y]] ~defined[y]
193S>(192a,43a) equalish[M[M[1,M[M[a,b],b]],x],M[M[M[1,a],b],x]] ~defined[x]
~equalish[M[M[1,M[a,b]],b],M[1,M[a,b]]]
194S>(193c,183a) equalish[M[M[1,M[M[a,b],b]],x],M[M[M[1,a],b],x]] ~defined[x]
195S>(194b,7a) equalish[M[M[1,M[M[a,b],b]],MI[x]],M[M[M[1,a],b],MI[x]]] ~defined[x]
equalish[x,0]
196S>(195b,33a) equalish[M[M[1,M[M[a,b],b]],MI[b]],M[M[M[1,a],b],MI[b]]] equalish[b,0]
197S>(196b,29a) equalish[M[M[1,M[M[a,b],b]],MI[b]],M[M[M[1,a],b],MI[b]]]
198S>(197a,18c) equalish[x,M[M[M[1,a],b],MI[b]]] ~equalish[x,M[M[1,M[M[a,b],b]],MI[b]]]
199S>(198a,176a) ~equalish[c,M[M[1,M[M[a,b],b]],MI[b]]]

200S>(33a,7b) defined[MI[b]] equalish[b,0]
201S>(200b,29a) defined[MI[b]]
202S>(201a,15c) defined[M[x,MI[b]]] ~defined[x]
203S>(202b,33a) defined[M[b,MI[b]]]
204S>(203a,12c) equalish[M[x,M[b,MI[b]]],M[M[b,MI[b]],x]] ~defined[x]
205S>(204a,18b) ~defined[x] equalish[M[x,M[b,MI[b]],y] ~equalish[M[M[b,MI[b]],x],y]
206S>(205a,34a) equalish[M[c,M[b,MI[b]],x] ~equalish[M[M[b,MI[b]],c],x]

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207: 206|{c/x} equalish[M[c,M[b,MI[b]]],c] ~equalish[M[M[b,MI[b]],c],c]
208S>(207b,190a) equalish[M[c,M[b,MI[b]]],c]

209S>(42a,24b) ~defined[x] equalish[M[M[1,x],b],M[1,M[x,b]]]
210S>(209b,9c) ~defined[x] equalish[M[M[M[1,x],b],y],M[M[1,M[x,b]],y]] ~defined[y]
211S>(210a,38a) equalish[M[M[M[1,M[a,b]],b],x],M[M[1,M[a,b],b],x]] ~defined[x]
212S>(211b,7a) equalish[M[M[M[1,M[a,b]],b],MI[x]],M[M[1,M[a,b],b],MI[x]]] ~defined[x]
equalish[x,0]
213S>(212b,33a) equalish[M[M[M[1,M[a,b]],b],MI[b]],M[M[1,M[a,b],b],MI[b]]] equalish[b,0]
214S>(213b,29a) equalish[M[M[M[1,M[a,b]],b],MI[b]],M[M[1,M[a,b],b],MI[b]]]
215S>(214a,18c) equalish[x,M[M[1,M[a,b],b],MI[b]]] ~equalish[x,M[M[M[1,M[a,b]],b],MI[b]]]
216S>(215a,199a) ~equalish[c,M[M[M[1,M[a,b]],b],MI[b]]]

217S>(33a,7b) defined[MI[b]] equalish[b,0]
218S>(217b,29a) defined[MI[b]]
219S>(218a,1d) equalish[M[x,M[y,MI[b]]],M[M[x,y],MI[b]]] ~defined[x] ~defined[y]
220S>(219b,34a) equalish[M[c,M[x,MI[b]]],M[M[c,x],MI[b]]] ~defined[x]
221S>(220a,24b) ~defined[x] equalish[M[M[c,x],MI[b]],M[c,M[x,MI[b]]]]
222S>(221b,18b) ~defined[x] equalish[M[M[c,x],MI[b]],y] ~equalish[M[c,M[x,MI[b]]],y]
223S>(222a,33a) equalish[M[M[c,b],MI[b]],x] ~equalish[M[c,M[b,MI[b]]],x]
224: 223|{c/x} equalish[M[M[c,b],MI[b]],c] ~equalish[M[c,M[b,MI[b]]],c]
225S>(224b,208a) equalish[M[M[c,b],MI[b]],c]

226S>(40a,9c) equalish[M[c,x],M[M[1,M[a,b]],x]] ~defined[x]
227S>(226a,9c) ~defined[x] equalish[M[M[c,x],y],M[M[M[1,M[a,b]],x],y]] ~defined[y]
228S>(227a,34a) equalish[M[M[c,b],x],M[M[M[1,M[a,b]],b],x]] ~defined[x]
229S>(228b,7a) equalish[M[M[c,b],MI[x]],M[M[M[1,M[a,b]],b],MI[x]]] ~defined[x] equalish[x,0]
230S>(229b,33a) equalish[M[M[c,b],MI[b]],M[M[M[1,M[a,b]],b],MI[b]]] equalish[b,0]
231S>(230b,29a) equalish[M[M[c,b],MI[b]],M[M[M[1,M[a,b]],b],MI[b]]]
232S>(231a,18c) equalish[x,M[M[M[1,M[a,b]],b],MI[b]]] ~equalish[x,M[M[c,b],MI[b]]]
233S>(232a,216a) ~equalish[c,M[M[c,b],MI[b]]]

234S>(33a,7b) defined[MI[b]] equalish[b,0]
235S>(234b,29a) defined[MI[b]]
236S>(235a,1d) equalish[M[x,M[y,MI[b]]],M[M[x,y],MI[b]]] ~defined[x] ~defined[y]
237S>(236b,34a) equalish[M[c,M[x,MI[b]]],M[M[c,x],MI[b]]] ~defined[x]
238S>(237a,18b) ~defined[x] equalish[M[c,M[x,MI[b]]],y] ~equalish[M[M[c,x],MI[b]],y]
239S>(238b,24b) ~defined[x] ~equalish[M[M[c,x],MI[b]],y] equalish[y,M[c,M[x,MI[b]]]]
240S>(239a,33a) ~equalish[M[M[c,b],MI[b]],x] equalish[x,M[c,M[b,MI[b]]]]
241S>(240a,225a) equalish[c,M[c,M[b,MI[b]]]]

242S>(39a,18a) ~equalish[c,x] ~equalish[x,b]
243S>(242b,24a) ~equalish[c,x] ~equalish[b,x]
244S>(243b,7c) ~equalish[c,0] defined[MI[b]] ~defined[b]
245S>(244a,30a) defined[MI[b]] ~defined[b]
246S>(245b,33a) defined[MI[b]]
247S>(246a,1d) equalish[M[x,M[y,MI[b]]],M[M[x,y],MI[b]]] ~defined[x] ~defined[y]
248S>(247b,34a) equalish[M[c,M[x,MI[b]]],M[M[c,x],MI[b]]] ~defined[x]
249S>(248a,18c) ~defined[x] equalish[y,M[M[c,x],MI[b]]] ~equalish[y,M[c,M[x,MI[b]]]]
250S>(249a,33a) equalish[x,M[M[c,b],MI[b]]] ~equalish[x,M[c,M[b,MI[b]]]]
251S>(250a,233a) ~equalish[c,M[c,M[b,MI[b]]]]
252S>(251a,241a) []

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PHASE 0: 0 s PHASE 1: 137 s PHASE 2: 696 s Total Time: 837 s
NOD: 20564409 RES: 112420749 FAC: 2815327 T: 26 V: 32 L: 16
CTE: 4670475 CTH: 1966401 CTF: 28256 CSZ: 16777216
UTE: 8907599 UTH: 14209038 UTF: 30568 SBA: 0
BAS: 34 RED: 52 LEN: 7+201=208

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## FLD042-1

Predicates: equalish defined less\_or\_equal  
Functions: b a additive\_identity . multiplicative\_identity : multiplicative\_inverse additive\_inverse  
add multiply  
Renamed predicates: equalish defined LE  
Renamed functions: b a 0 . 1 : MI AI A M

C100: Spawned: 150 xv1 26.10.3 W49 2S equalish[A[u,M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z]  
~defined[y] ... ?

Axioms:

```
1: equalish[M[x,M[y,z]],M[M[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
2S>equalish[A[u,M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y] ... ?
3: equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
4: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
5: LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
6: equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]
7: defined[MI[x]] ~defined[x] equalish[x,0]
8: LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]
9 >equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[x,z]
10: LE[x,y] ~LE[z,y] ~equalish[z,x]
11 >equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]
12 >equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: equalish[x,y] ~LE[x,y] ~LE[y,x]
15: defined[M[x,y]] ~defined[x] ~defined[y]
16 >equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]
17 >defined[A[x,y]] ~defined[x] ~defined[y]
18 >equalish[x,y] ~equalish[x,z] ~equalish[z,y]
19 >equalish[A[x,AI[x]],0] ~defined[x]
20 >equalish[M[1,x],x] ~defined[x]
21 >equalish[A[0,x],x] ~defined[x]
22 >defined[AI[x]] ~defined[x]
23: equalish[x,x] ~defined[x]
24 >equalish[x,y] ~equalish[y,x]
25 >defined[1]
26: ~equalish[0,1]
27 >defined[0]
```

Negated conclusion:

```
28S equalish[M[a,b],0]
29S ~equalish[b,0]
30S>~equalish[a,0]
31S>defined[a]
32S defined[b]
```

-----  
Phase 0 clauses used in proof:

```
39S>(31a*22b) defined[AI[a]]
40S>(31a*21b) equalish[A[0,a],a]
42S>(31a*19b) equalish[A[a,AI[a]],0]
43S>(30a*24a) ~equalish[0,a]
```

Phases 1 and 2 clauses used in proof:

```
49S>(43a,18a) ~equalish[0,x] ~equalish[x,a]
50S>(49a,24a) ~equalish[x,a] ~equalish[x,0]
51S>(50a,40a) ~equalish[A[0,a],0]

52S>(42a,18c) equalish[x,0] ~equalish[x,A[a,AI[a]]]
53S>(52b,16a) equalish[A[AI[a],a],0] ~defined[AI[a]] ~defined[a]
54S>(53b,39a) equalish[A[AI[a],a],0] ~defined[a]
55S>(54b,31a) equalish[A[AI[a],a],0]
56S>(55a,18c) equalish[x,0] ~equalish[x,A[AI[a],a]]
57S>(56b,11a) equalish[A[x,a],0] ~defined[a] ~equalish[x,AI[a]]
58S>(57a,51a) ~defined[a] ~equalish[0,AI[a]]
59S>(58a,31a) ~equalish[0,AI[a]]

60S>(39a,20b) equalish[M[1,AI[a]],AI[a]]
61S>(60a,18c) equalish[x,AI[a]] ~equalish[x,M[1,AI[a]]]
62S>(61b,12a) equalish[M[AI[a],1],AI[a]] ~defined[AI[a]] ~defined[1]
63S>(62b,39a) equalish[M[AI[a],1],AI[a]] ~defined[1]
64S>(63b,25a) equalish[M[AI[a],1],AI[a]]
65S>(64a,18c) equalish[x,AI[a]] ~equalish[x,M[AI[a],1]]
66S>(65a,59a) ~equalish[0,M[AI[a],1]]

67S>(39a,21b) equalish[A[0,AI[a]],AI[a]]
68S>(67a,9c) equalish[M[A[0,AI[a]],x],M[AI[a],x]] ~defined[x]
69S>(68b,25a) equalish[M[A[0,AI[a]],1],M[AI[a],1]]
70S>(69a,18c) equalish[x,M[AI[a],1]] ~equalish[x,M[A[0,AI[a]],1]]
```

```

71S>(70a,66a) ~equalish[0,M[A[0,AI[a]],1]]
72S>(39a,17c) defined[A[x,AI[a]]] ~defined[x]
73S>(72b,31a) defined[A[a,AI[a]]]

74S>(39a,2c) equalish[A[x,M[AI[a],y]],M[A[z,AI[a]],y]] ~defined[z] ~defined[y] ... ?
75S>(74b,27a) equalish[A[x,M[AI[a],y]],M[A[0,AI[a]],y]] ~defined[y] ... ?
76S>(75b,25a) equalish[A[x,M[AI[a],1]],M[A[0,AI[a]],1]] ... ?
77S>(76a,18c) equalish[x,M[A[0,AI[a]],1]] ~equalish[x,A[y,M[AI[a],1]]] ... ?
78: 77|{0/x} equalish[0,M[A[0,AI[a]],1]] ~equalish[0,A[x,M[AI[a],1]]] ... ?
79S>(78a,71a) ~equalish[0,A[x,M[AI[a],1]]] ... ?

80S>(42a,18c) equalish[x,0] ~equalish[x,A[a,AI[a]]]
81S>(80b,20a) equalish[M[1,A[a,AI[a]],0] ~defined[A[a,AI[a]]]
82S>(81b,73a) equalish[M[1,A[a,AI[a]],0]
83S>(82a,18c) equalish[x,0] ~equalish[x,M[1,A[a,AI[a]]]
84S>(83b,12a) equalish[M[A[a,AI[a]],1],0] ~defined[A[a,AI[a]]] ~defined[1]
85S>(84b,73a) equalish[M[A[a,AI[a]],1],0] ~defined[1]
86S>(85b,25a) equalish[M[A[a,AI[a]],1],0]

87S>(39a,2c) equalish[A[x,M[AI[a],y]],M[A[z,AI[a]],y]] ~defined[z] ~defined[y] ... ?
88S>(87b,31a) equalish[A[x,M[AI[a],y]],M[A[a,AI[a]],y]] ~defined[y] ... ?
89S>(88a,18b) ~defined[x] equalish[A[y,M[AI[a],x]],z] ~equalish[M[A[a,AI[a]],x],z] ... ?
90S>(89b,24b) ~defined[x] ~equalish[M[A[a,AI[a]],x],y] equalish[y,A[z,M[AI[a],x]]] ... ?
91S>(90a,25a) ~equalish[M[A[a,AI[a]],1],x] equalish[x,A[y,M[AI[a],1]]] ... ?
92: 91|{0/x} ~equalish[M[A[a,AI[a]],1],0] equalish[0,A[x,M[AI[a],1]]] ... ?
93S>(92a,86a) equalish[0,A[x,M[AI[a],1]]] ... ?
94S>(93a,79a) [] ... ?

```

Inferences derived from proof before variable replaced by constant  
40 42 43 51 59 66 71 73 86

Inferences derived when variable is replaced by original constant  
76 77 88 89 91

```

PHASE 0: 0 s PHASE 1: 6 s PHASE 2: 17 s Total Time: 23 s
NOD: 589717 RES: 3624771 FAC: 67360 T: 26 V: 32 L: 16
CTE: 214345 CTH: 43277 CTF: 0 CSZ: 16777216
UTE: 1059532 UTH: 1038642 UTF: 0 SBA: 0
BAS: 32 RED: 48 LEN: 4+46=50

```

Proof found to theorem by slave 100!

Axioms:

```

1 >equalish[M[x,M[y,z]],M[M[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
2S>equalish[A[M[x,y],M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y]
3: equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
4: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
5: LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
6 >equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]
7 >defined[MI[x]] ~defined[x] equalish[x,0]
8: LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]
9S>equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[x,z]
10: LE[x,y] ~LE[z,y] ~equalish[z,x]
11S>equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]
12S>equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: equalish[x,y] ~LE[x,y] ~LE[y,x]
15 >defined[M[x,y]] ~defined[x] ~defined[y]
16S>equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]
17S>defined[A[x,y]] ~defined[x] ~defined[y]
18S>equalish[x,y] ~equalish[x,z] ~equalish[z,y]
19S>equalish[A[x,AI[x]],0] ~defined[x]
20S>equalish[M[1,x],x] ~defined[x]
21S>equalish[A[0,x],x] ~defined[x]
22S defined[AI[x]] ~defined[x]
23 >equalish[x,x] ~defined[x]
24S>equalish[x,y] ~equalish[y,x]
25S>defined[1]
26: ~equalish[0,1]
27S>defined[0]

```

Negated conclusion:

```

28S>equalish[M[a,b],0]
29S>~equalish[b,0]
30S>~equalish[a,0]
31S>defined[a]
32S>defined[b]

```

-----  
Phase 0 clauses used in proof:

```

33S>equalish[A[0,a],a]
34S>equalish[A[a,AI[a]],0]
36S>~equalish[A[0,a],0]
40S>defined[A[a,AI[a]]]
41S>equalish[M[A[a,AI[a]],1],0]
44S>equalish[A[M[a,x],M[AI[a],x]],M[A[a,AI[a]],x]] ~defined[x]
45S>~defined[x] equalish[A[M[a,x],M[AI[a],x]],y] ~equalish[M[A[a,AI[a]],x],y]
46S>~equalish[M[A[a,AI[a]],1],x] equalish[x,A[M[a,1],M[AI[a],1]]]
47S>(46a*41a) equalish[0,A[M[a,1],M[AI[a],1]]]
48S>(44b*40a) equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],M[A[a,AI[a]],A[a,AI[a]]]]

Phases 1 and 2 clauses used in proof:
49S>(47a,24b) equalish[A[M[a,1],M[AI[a],1]],0]

50S>(46a,12a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[A[a,AI[a]]] ~defined[1]
51S>(50b,40a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[1]
52S>(51b,25a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]
53S>(52a,18b) equalish[M[1,A[a,AI[a]]],x] ~equalish[A[M[a,1],M[AI[a],1]],x]
54: 53 {0/x} equalish[M[1,A[a,AI[a]]],0] ~equalish[A[M[a,1],M[AI[a],1]],0]
55S>(54b,49a) equalish[M[1,A[a,AI[a]]],0]

56S>(47a,24b) equalish[A[M[a,1],M[AI[a],1]],0]

57S>(40a,9b) equalish[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]] ~equalish[x,y]
58S>(57b,21a) equalish[M[A[0,x],A[a,AI[a]]],M[x,A[a,AI[a]]] ~defined[x]
59S>(58a,18b) ~defined[x] equalish[M[A[0,x],A[a,AI[a]]],y] ~equalish[M[x,A[a,AI[a]]],y]
60S>(59a,25a) equalish[M[A[0,1],A[a,AI[a]]],x] ~equalish[M[1,A[a,AI[a]]],x]
61: 60 {0/x} equalish[M[A[0,1],A[a,AI[a]]],0] ~equalish[M[1,A[a,AI[a]]],0]
62S>(61b,55a) equalish[M[A[0,1],A[a,AI[a]]],0]

63S>(46a,12a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[A[a,AI[a]]] ~defined[1]
64S>(63b,40a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[1]
65S>(64b,25a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]
66S>(65a,18b) equalish[M[1,A[a,AI[a]]],x] ~equalish[A[M[a,1],M[AI[a],1]],x]
67: 66 {0/x} equalish[M[1,A[a,AI[a]]],0] ~equalish[A[M[a,1],M[AI[a],1]],0]
68S>(67b,56a) equalish[M[1,A[a,AI[a]]],0]

69S>(40a,9b) equalish[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]] ~equalish[x,y]
70S>(69b,16a) equalish[M[A[x,y],A[a,AI[a]]],M[A[y,x],A[a,AI[a]]] ~defined[x] ~defined[y]
71S>(70b,25a) equalish[M[A[1,x],A[a,AI[a]]],M[A[x,1],A[a,AI[a]]] ~defined[x]
72S>(71a,18b) ~defined[x] equalish[M[A[1,x],A[a,AI[a]]],y] ~equalish[M[A[x,1],A[a,AI[a]]],y]
73S>(72a,27a) equalish[M[A[1,0],A[a,AI[a]]],x] ~equalish[M[A[0,1],A[a,AI[a]]],x]
74: 73 {0/x} equalish[M[A[1,0],A[a,AI[a]]],0] ~equalish[M[A[0,1],A[a,AI[a]]],0]
75S>(74b,62a) equalish[M[A[1,0],A[a,AI[a]]],0]

76S>(40a,9b) equalish[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]] ~equalish[x,y]
77S>(76b,21a) equalish[M[A[0,x],A[a,AI[a]]],M[x,A[a,AI[a]]] ~defined[x]
78S>(77a,18b) ~defined[x] equalish[M[A[0,x],A[a,AI[a]]],y] ~equalish[M[x,A[a,AI[a]]],y]
79S>(78a,25a) equalish[M[A[0,1],A[a,AI[a]]],x] ~equalish[M[1,A[a,AI[a]]],x]
80: 79 {0/x} equalish[M[A[0,1],A[a,AI[a]]],0] ~equalish[M[1,A[a,AI[a]]],0]
81S>(80b,68a) equalish[M[A[0,1],A[a,AI[a]]],0]

82S>(33a,18c) equalish[x,a] ~equalish[x,A[0,a]]
83S>(82b,16a) equalish[A[a,0],a] ~defined[a] ~defined[0]
84S>(83b,31a) equalish[A[a,0],a] ~defined[0]
85S>(84b,27a) equalish[A[a,0],a]
86S>(85a,9c) equalish[M[A[a,0],x],M[a,x]] ~defined[x]
87S>(86a,18b) ~defined[x] equalish[M[A[a,0],x],y] ~equalish[M[a,x],y]
88S>(87a,32a) equalish[M[A[a,0],b],x] ~equalish[M[a,b],x]
89S>(88b,28a) equalish[M[A[a,0],b],0]

90S>(40a,2d) equalish[A[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]],M[A[x,y],A[a,AI[a]]] ~defined[x]
~defined[y]
91S>(90b,25a) equalish[A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]],M[A[1,x],A[a,AI[a]]] ~defined[x]
92S>(91a,18b) ~defined[x] equalish[A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]],y]
~equalish[M[A[1,x],A[a,AI[a]]],y]
93S>(92b,24b) ~defined[x] ~equalish[M[A[1,x],A[a,AI[a]]],y]
equalish[y,A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]]]
94S>(93a,27a) ~equalish[M[A[1,0],A[a,AI[a]]],x] equalish[x,A[M[1,A[a,AI[a]]],M[0,A[a,AI[a]]]]
95S>(94a,75a) equalish[0,A[M[1,A[a,AI[a]]],M[0,A[a,AI[a]]]]

96S>(40a,9b) equalish[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]] ~equalish[x,y]
97S>(96b,11a) equalish[M[A[x,y],A[a,AI[a]]],M[A[z,y],A[a,AI[a]]] ~defined[y] ~equalish[x,z]
98S>(97b,25a) equalish[M[A[x,1],A[a,AI[a]]],M[A[y,1],A[a,AI[a]]] ~equalish[x,y]
99S>(98a,18b) ~equalish[x,y] equalish[M[A[x,1],A[a,AI[a]]],z] ~equalish[M[A[y,1],A[a,AI[a]]],z]
100S>(99a,34a) equalish[M[A[A[a,AI[a]],1],A[a,AI[a]]],x] ~equalish[M[A[0,1],A[a,AI[a]]],x]
101: 100 {0/x} equalish[M[A[A[a,AI[a]],1],A[a,AI[a]]],0] ~equalish[M[A[0,1],A[a,AI[a]]],0]
102S>(101b,81a) equalish[M[A[A[a,AI[a]],1],A[a,AI[a]]],0]

103S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]
104S>(103b,27a) defined[M[0,A[a,AI[a]]]]

```

```

105S>(104a,11b) equalish[A[x,M[0,A[a,AI[a]]],A[y,M[0,A[a,AI[a]]]] ~equalish[x,y]
106S>(105b,20a) equalish[A[M[1,x],M[0,A[a,AI[a]]],A[x,M[0,A[a,AI[a]]]] ~defined[x]
107S>(106a,18c) ~defined[x] equalish[y,A[x,M[0,A[a,AI[a]]]]
~equalish[y,A[M[1,x],M[0,A[a,AI[a]]]]
108S>(107a,40a) equalish[x,A[A[a,AI[a]],M[0,A[a,AI[a]]]]
~equalish[x,A[M[1,A[a,AI[a]]],M[0,A[a,AI[a]]]]
109: 108|{0/x} equalish[0,A[A[a,AI[a]],M[0,A[a,AI[a]]]]
~equalish[0,A[M[1,A[a,AI[a]]],M[0,A[a,AI[a]]]]
110S>(109b,95a) equalish[0,A[A[a,AI[a]],M[0,A[a,AI[a]]]]

111S>(32a,2d) equalish[A[M[x,b],M[y,b]],M[A[x,y],b]] ~defined[x] ~defined[y]
112S>(111b,31a) equalish[A[M[a,b],M[x,b]],M[A[a,x],b]] ~defined[x]
113S>(112a,18b) ~defined[x] equalish[A[M[a,b],M[x,b]],y] ~equalish[M[A[a,x],b],y]
114S>(113b,24b) ~defined[x] ~equalish[M[A[a,x],b],y] equalish[y,A[M[a,b],M[x,b]]]
115S>(114a,27a) ~equalish[M[A[a,0],b],x] equalish[x,A[M[a,b],M[0,b]]]
116S>(115a,89a) equalish[0,A[M[a,b],M[0,b]]]

117S>(40a,9b) equalish[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]] ~equalish[x,y]
118S>(117b,16a) equalish[M[A[x,y],A[a,AI[a]]],M[A[y,x],A[a,AI[a]]] ~defined[x] ~defined[y]
119S>(118b,25a) equalish[M[A[1,x],A[a,AI[a]]],M[A[x,1],A[a,AI[a]]] ~defined[x]
120S>(119a,18b) ~defined[x] equalish[M[A[1,x],A[a,AI[a]]],y] ~equalish[M[A[x,1],A[a,AI[a]]],y]
121S>(120a,40a) equalish[M[A[1,A[a,AI[a]]],A[a,AI[a]],x]
~equalish[M[A[A[a,AI[a]],1],A[a,AI[a]],x]
122: 121|{0/x} equalish[M[A[1,A[a,AI[a]]],A[a,AI[a]],0]
~equalish[M[A[A[a,AI[a]],1],A[a,AI[a]],0]
123S>(122b,102a) equalish[M[A[1,A[a,AI[a]]],A[a,AI[a]],0]

124S>(32a,15c) defined[M[x,b]] ~defined[x]
125S>(124b,27a) defined[M[0,b]]
126S>(125a,11b) equalish[A[x,M[0,b]],A[y,M[0,b]] ~equalish[x,y]
127S>(126a,18c) ~equalish[x,y] equalish[z,A[y,M[0,b]]] ~equalish[z,A[x,M[0,b]]]
128S>(127b,24b) ~equalish[x,y] ~equalish[z,A[x,M[0,b]]] equalish[A[y,M[0,b]],z]
129S>(128a,28a) ~equalish[x,A[M[a,b],M[0,b]]] equalish[A[0,M[0,b]],x]
130S>(129a,116a) equalish[A[0,M[0,b]],0]

131S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]
132S>(131b,27a) defined[M[0,A[a,AI[a]]]
133S>(132a,11b) equalish[A[x,M[0,A[a,AI[a]]],A[y,M[0,A[a,AI[a]]]] ~equalish[x,y]
134S>(133a,18c) ~equalish[x,y] equalish[z,A[y,M[0,A[a,AI[a]]]] ~equalish[z,A[x,M[0,A[a,AI[a]]]]
135S>(134b,24b) ~equalish[x,y] ~equalish[z,A[x,M[0,A[a,AI[a]]]] equalish[A[y,M[0,A[a,AI[a]]],z]
136S>(135a,34a) ~equalish[x,A[A[a,AI[a]],M[0,A[a,AI[a]]]] equalish[A[0,M[0,A[a,AI[a]]],x]
137S>(136a,110a) equalish[A[0,M[0,A[a,AI[a]]],0]

138S>(40a,2d) equalish[A[M[x,A[a,AI[a]]],M[y,A[a,AI[a]]],M[A[x,y],A[a,AI[a]]] ~defined[x]
~defined[y]
139S>(138b,25a) equalish[A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]],M[A[1,x],A[a,AI[a]]] ~defined[x]
140S>(139a,18b) ~defined[x] equalish[A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]],y]
~equalish[M[A[1,x],A[a,AI[a]],y]
141S>(140b,24b) ~defined[x] ~equalish[M[A[1,x],A[a,AI[a]],y]
equalish[y,A[M[1,A[a,AI[a]]],M[x,A[a,AI[a]]]]
142S>(141a,40a) ~equalish[M[A[1,A[a,AI[a]]],A[a,AI[a]],x]
equalish[x,A[M[1,A[a,AI[a]]],M[A[a,AI[a]],A[a,AI[a]]]]
143S>(142a,123a) equalish[0,A[M[1,A[a,AI[a]]],M[A[a,AI[a]],A[a,AI[a]]]]

144S>(32a,15c) defined[M[x,b]] ~defined[x]
145S>(144a,21b) ~defined[x] equalish[A[0,M[x,b]],M[x,b]]
146S>(145b,24b) ~defined[x] equalish[M[x,b],A[0,M[x,b]]]
147S>(146b,18b) ~defined[x] equalish[M[x,b],y] ~equalish[A[0,M[x,b]],y]
148S>(147b,24b) ~defined[x] ~equalish[A[0,M[x,b]],y] equalish[y,M[x,b]]
149S>(148a,27a) ~equalish[A[0,M[0,b]],x] equalish[x,M[0,b]]
150S>(149a,130a) equalish[0,M[0,b]]

151S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]
152S>(151a,21b) ~defined[x] equalish[A[0,M[x,A[a,AI[a]]],M[x,A[a,AI[a]]]
153S>(152b,24b) ~defined[x] equalish[M[x,A[a,AI[a]],A[0,M[x,A[a,AI[a]]]]
154S>(153b,18b) ~defined[x] equalish[M[x,A[a,AI[a]],y] ~equalish[A[0,M[x,A[a,AI[a]]],y]
155S>(154b,24b) ~defined[x] ~equalish[A[0,M[x,A[a,AI[a]]],y] equalish[y,M[x,A[a,AI[a]]]]
156S>(155a,27a) ~equalish[A[0,M[0,A[a,AI[a]]],x] equalish[x,M[0,A[a,AI[a]]]]
157S>(156a,137a) equalish[0,M[0,A[a,AI[a]]]

158S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]
159S>(158b,40a) defined[M[A[a,AI[a]],A[a,AI[a]]]
160S>(159a,11b) equalish[A[x,M[A[a,AI[a]],A[a,AI[a]]],A[y,M[A[a,AI[a]],A[a,AI[a]]]]
~equalish[x,y]
161S>(160b,20a) equalish[A[M[1,x],M[A[a,AI[a]],A[a,AI[a]]],A[x,M[A[a,AI[a]],A[a,AI[a]]]]
~defined[x]
162S>(161a,18c) ~defined[x] equalish[y,A[x,M[A[a,AI[a]],A[a,AI[a]]]]
~equalish[y,A[M[1,x],M[A[a,AI[a]],A[a,AI[a]]]]
163S>(162a,40a) equalish[x,A[A[a,AI[a]],M[A[a,AI[a]],A[a,AI[a]]]]
~equalish[x,A[M[1,A[a,AI[a]],M[A[a,AI[a]],A[a,AI[a]]]]

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164: 163 {0/x} equalish[0,A[A[a,AI[a]],M[A[a,AI[a]],A[a,AI[a]]]]  
~equalish[0,A[M[1,A[a,AI[a]]],M[A[a,AI[a]],A[a,AI[a]]]]  
165S>(164b,143a) equalish[0,A[A[a,AI[a]],M[A[a,AI[a]],A[a,AI[a]]]]  
  
166S>(24b,9a) equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[z,x]  
167S>(166b,32a) equalish[M[x,b],M[y,b]] ~equalish[y,x]  
168S>(167a,18c) ~equalish[x,y] equalish[z,M[x,b]] ~equalish[z,M[y,b]]  
169S>(168a,34a) equalish[x,M[A[a,AI[a]],b]] ~equalish[x,M[0,b]]  
170S>(169a,24b) ~equalish[x,M[0,b]] equalish[M[A[a,AI[a]],b],x]  
171S>(170a,150a) equalish[M[A[a,AI[a]],b],0]  
  
172S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]  
173S>(172b,40a) defined[M[A[a,AI[a]],A[a,AI[a]]]  
174S>(173a,11b) equalish[A[x,M[A[a,AI[a]],A[a,AI[a]]],A[y,M[A[a,AI[a]],A[a,AI[a]]]]  
~equalish[x,y]  
175S>(174a,18c) ~equalish[x,y] equalish[z,A[y,M[A[a,AI[a]],A[a,AI[a]]]]  
~equalish[z,A[x,M[A[a,AI[a]],A[a,AI[a]]]]  
176S>(175b,24b) ~equalish[x,y] ~equalish[z,A[x,M[A[a,AI[a]],A[a,AI[a]]]]  
equalish[A[y,M[A[a,AI[a]],A[a,AI[a]]],z]  
177S>(176a,34a) ~equalish[x,A[A[a,AI[a]],M[A[a,AI[a]],A[a,AI[a]]]]  
equalish[A[0,M[A[a,AI[a]],A[a,AI[a]]],x]  
178S>(177a,165a) equalish[A[0,M[A[a,AI[a]],A[a,AI[a]]],0]  
  
179S>(31a,17c) defined[A[x,a]] ~defined[x]  
180S>(179b,27a) defined[A[0,a]]  
  
181S>(24b,18a) equalish[x,y] ~equalish[y,z] ~equalish[z,x]  
182S>(181b,41a) equalish[x,M[A[a,AI[a]],1]] ~equalish[0,x]  
183S>(182b,23a) equalish[0,M[A[a,AI[a]],1]] ~defined[0]  
184S>(183b,27a) equalish[0,M[A[a,AI[a]],1]  
185S>(184a,18c) equalish[x,M[A[a,AI[a]],1]] ~equalish[x,0]  
186S>(185a,45c) ~equalish[M[A[a,AI[a]],x],0] ~defined[x]  
equalish[A[M[a,x],M[AI[a],x]],M[A[a,AI[a]],1]]  
187S>(186a,171a) ~defined[b] equalish[A[M[a,b],M[AI[a],b]],M[A[a,AI[a]],1]]  
188S>(187a,32a) equalish[A[M[a,b],M[AI[a],b]],M[A[a,AI[a]],1]]  
  
189S>(40a,15c) defined[M[x,A[a,AI[a]]] ~defined[x]  
190S>(189a,21b) ~defined[x] equalish[A[0,M[x,A[a,AI[a]]],M[x,A[a,AI[a]]]  
191S>(190b,24b) ~defined[x] equalish[M[x,A[a,AI[a]],A[0,M[x,A[a,AI[a]]]]  
192S>(191b,18b) ~defined[x] equalish[M[x,A[a,AI[a]],y] ~equalish[A[0,M[x,A[a,AI[a]]],y]  
193S>(192b,24b) ~defined[x] ~equalish[A[0,M[x,A[a,AI[a]]],y] equalish[y,M[x,A[a,AI[a]]]]  
194S>(193a,40a) ~equalish[A[0,M[A[a,AI[a]],A[a,AI[a]]],x] equalish[x,M[A[a,AI[a]],A[a,AI[a]]]  
195S>(194a,178a) equalish[0,M[A[a,AI[a]],A[a,AI[a]]]  
  
196S>(24b,20a) equalish[x,M[1,x]] ~defined[x]  
197S>(196a,18b) ~defined[x] equalish[x,y] ~equalish[M[1,x],y]  
198S>(197a,31a) equalish[a,x] ~equalish[M[1,a],x]  
199S>(198a,30a) ~equalish[M[1,a],0]  
  
200S>(24a,18c) ~equalish[x,y] equalish[z,x] ~equalish[z,y]  
201S>(200a,48a) equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
~equalish[x,M[A[a,AI[a]],A[a,AI[a]]]  
202S>(201b,18a) equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]] ~equalish[x,y]  
~equalish[y,M[A[a,AI[a]],A[a,AI[a]]]  
203S>(202b,41a) equalish[M[A[a,AI[a]],1],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
~equalish[0,M[A[a,AI[a]],A[a,AI[a]]]  
204S>(203b,195a) equalish[M[A[a,AI[a]],1],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
205S>(204a,18b) equalish[M[A[a,AI[a]],1],x] ~equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],x]  
206S>(205b,48a) equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],A[a,AI[a]]]  
  
207S>(24b,18a) equalish[x,y] ~equalish[y,z] ~equalish[z,x]  
208S>(207a,24b) ~equalish[x,y] ~equalish[y,z] equalish[x,z]  
209S>(208a,24a) ~equalish[x,y] equalish[z,y] ~equalish[x,z]  
210S>(209a,44a) equalish[x,M[A[a,AI[a]],y]] ~equalish[A[M[a,y],M[AI[a],y]],x] ~defined[y]  
211S>(210a,46a) ~equalish[A[M[a,x],M[AI[a],x]],M[A[a,AI[a]],1]] ~defined[x]  
equalish[M[A[a,AI[a]],x],A[M[a,1],M[AI[a],1]]  
212S>(211a,188a) ~defined[b] equalish[M[A[a,AI[a]],b],A[M[a,1],M[AI[a],1]]  
213S>(212a,32a) equalish[M[A[a,AI[a]],b],A[M[a,1],M[AI[a],1]]  
  
214S>(32a,6b) equalish[M[b,MI[b]],1] equalish[b,0]  
215S>(214b,29a) equalish[M[b,MI[b]],1]  
216S>(215a,24b) equalish[1,M[b,MI[b]]]  
217S>(216a,9c) equalish[M[1,x],M[M[b,MI[b]],x]] ~defined[x]  
218S>(217a,18b) ~defined[x] equalish[M[1,x],y] ~equalish[M[M[b,MI[b]],x],y]  
219S>(218a,31a) equalish[M[1,a],x] ~equalish[M[M[b,MI[b]],a],x]  
220S>(219a,199a) ~equalish[M[M[b,MI[b]],a],0]  
  
221S>(48a,24b) equalish[M[A[a,AI[a]],A[a,AI[a]]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
222S>(221a,18c) equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
~equalish[x,M[A[a,AI[a]],A[a,AI[a]]]

223S>(222a,46a) ~equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],A[a,AI[a]]]  
 equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]  
 224S>(223a,206a) equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]  
 225S>(224a,24b) equalish[A[M[a,1],M[AI[a],1]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 226S>(225a,18c) equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 ~equalish[x,A[M[a,1],M[AI[a],1]]]  
 227S>(226a,45c) ~equalish[M[A[a,AI[a]],x],A[M[a,1],M[AI[a],1]]] ~defined[x]  
 equalish[A[M[a,x],M[AI[a],x]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 228S>(227a,213a) ~defined[b]  
 equalish[A[M[a,b],M[AI[a],b]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 229S>(228a,32a) equalish[A[M[a,b],M[AI[a],b]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
  
 230S>(32a,7b) defined[MI[b]] equalish[b,0]  
 231S>(230b,29a) defined[MI[b]]  
 232S>(231a,15c) defined[M[x,MI[b]]] ~defined[x]  
 233S>(232b,32a) defined[M[b,MI[b]]]  
 234S>(233a,12b) equalish[M[M[b,MI[b]],x],M[x,M[b,MI[b]]] ~defined[x]  
 235S>(234a,18b) ~defined[x] equalish[M[M[b,MI[b]],x],y] ~equalish[M[x,M[b,MI[b]]],y]  
 236S>(235a,31a) equalish[M[M[b,MI[b]],a],x] ~equalish[M[a,M[b,MI[b]]],x]  
 237S>(236a,220a) ~equalish[M[a,M[b,MI[b]]],0]  
  
 238S>(47a,24b) equalish[A[M[a,1],M[AI[a],1]],0]  
  
 239S>(48a,24b) equalish[M[A[a,AI[a]],A[a,AI[a]]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 240S>(239a,18b) equalish[M[A[a,AI[a]],A[a,AI[a]]],x]  
 ~equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],x]  
 241S>(240b,18a) equalish[M[A[a,AI[a]],A[a,AI[a]]],x]  
 ~equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],y] ~equalish[y,x]  
 242S>(241b,24a) equalish[M[A[a,AI[a]],A[a,AI[a]]],x] ~equalish[y,x]  
 ~equalish[y,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 243S>(242b,44a) equalish[M[A[a,AI[a]],A[a,AI[a]]],M[A[a,AI[a]],x]  
 ~equalish[A[M[a,x],M[AI[a],x]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]] ~defined[x]  
 244S>(243a,24b) ~equalish[A[M[a,x],M[AI[a],x]],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]  
 ~defined[x] equalish[M[A[a,AI[a]],x],M[A[a,AI[a]],A[a,AI[a]]]  
 245S>(244a,229a) ~defined[b] equalish[M[A[a,AI[a]],b],M[A[a,AI[a]],A[a,AI[a]]]  
 246S>(245a,32a) equalish[M[A[a,AI[a]],b],M[A[a,AI[a]],A[a,AI[a]]]  
  
 247S>(32a,7b) defined[MI[b]] equalish[b,0]  
 248S>(247b,29a) defined[MI[b]]  
 249S>(248a,1d) equalish[M[x,M[y,MI[b]]],M[M[x,y],MI[b]] ~defined[x] ~defined[y]  
 250S>(249b,31a) equalish[M[a,M[x,MI[b]]],M[M[a,x],MI[b]] ~defined[x]  
 251S>(250b,32a) equalish[M[a,M[b,MI[b]]],M[M[a,b],MI[b]]  
 252S>(251a,18b) equalish[M[a,M[b,MI[b]]],x] ~equalish[M[M[a,b],MI[b]],x]  
 253S>(252a,237a) ~equalish[M[M[a,b],MI[b]],0]  
  
 254S>(46a,12a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[A[a,AI[a]]] ~defined[1]  
 255S>(254b,40a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]] ~defined[1]  
 256S>(255b,25a) equalish[M[1,A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]  
 257S>(256a,18b) equalish[M[1,A[a,AI[a]]],x] ~equalish[A[M[a,1],M[AI[a],1]],x]  
 258: 257{0/x} equalish[M[1,A[a,AI[a]]],0] ~equalish[A[M[a,1],M[AI[a],1]],0]  
 259S>(258b,238a) equalish[M[1,A[a,AI[a]]],0]  
  
 260S>(29a,7c) defined[MI[b]] ~defined[b]  
 261S>(260b,32a) defined[MI[b]]  
 262S>(261a,9b) equalish[M[x,MI[b]],M[y,MI[b]]] ~equalish[x,y]  
 263S>(262a,18b) ~equalish[x,y] equalish[M[x,MI[b]],z] ~equalish[M[y,MI[b]],z]  
 264S>(263a,28a) equalish[M[M[a,b],MI[b]],x] ~equalish[M[0,MI[b]],x]  
 265S>(264b,24a) equalish[M[M[a,b],MI[b]],x] ~equalish[x,M[0,MI[b]]]  
 266S>(265a,253a) ~equalish[0,M[0,MI[b]]]  
  
 267S>(32a,6b) equalish[M[b,MI[b]],1] equalish[b,0]  
 268S>(267b,29a) equalish[M[b,MI[b]],1]  
 269S>(268a,9c) equalish[M[M[b,MI[b]],x],M[1,x]] ~defined[x]  
 270S>(269a,18b) ~defined[x] equalish[M[M[b,MI[b]],x],y] ~equalish[M[1,x],y]  
 271S>(270a,40a) equalish[M[M[b,MI[b]],A[a,AI[a]]],x] ~equalish[M[1,A[a,AI[a]]],x]  
 272: 271{0/x} equalish[M[M[b,MI[b]],A[a,AI[a]]],0] ~equalish[M[1,A[a,AI[a]]],0]  
 273S>(272b,259a) equalish[M[M[b,MI[b]],A[a,AI[a]]],0]  
  
 274S>(40a,15b) defined[M[A[a,AI[a]],x]] ~defined[x]  
 275S>(274b,25a) defined[M[A[a,AI[a]],1]]  
  
 276S>(24a,9c) ~equalish[x,y] equalish[M[y,z],M[x,z]] ~defined[z]  
 277S>(276a,47a) equalish[M[A[M[a,1],M[AI[a],1]],x],M[0,x]] ~defined[x]  
 278S>(277b,7a) equalish[M[A[M[a,1],M[AI[a],1]],MI[x]],M[0,MI[x]] ~defined[x] equalish[x,0]  
 279S>(278b,32a) equalish[M[A[M[a,1],M[AI[a],1]],MI[b]],M[0,MI[b]] equalish[b,0]  
 280S>(279b,29a) equalish[M[A[M[a,1],M[AI[a],1]],MI[b]],M[0,MI[b]]  
 281S>(280a,18c) equalish[x,M[0,MI[b]]] ~equalish[x,M[A[M[a,1],M[AI[a],1]],MI[b]]  
 282S>(281a,266a) ~equalish[0,M[A[M[a,1],M[AI[a],1]],MI[b]]  
  
 283S>(32a,7b) defined[MI[b]] equalish[b,0]  
 284S>(283b,29a) defined[MI[b]]

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285S>(284a,15c) defined[M[x,MI[b]]] ~defined[x]
286S>(285b,32a) defined[M[b,MI[b]]]
287S>(286a,12c) equalish[M[x,M[b,MI[b]]],M[M[b,MI[b]],x]] ~defined[x]
288S>(287a,18b) ~defined[x] equalish[M[x,M[b,MI[b]]],y] ~equalish[M[M[b,MI[b]],x],y]
289S>(288a,40a) equalish[M[A[a,AI[a]],M[b,MI[b]]],x] ~equalish[M[M[b,MI[b]],A[a,AI[a]]],x]
290: 289|{0/x} equalish[M[A[a,AI[a]],M[b,MI[b]]],0] ~equalish[M[M[b,MI[b]],A[a,AI[a]]],0]
291S>(290b,273a) equalish[M[A[a,AI[a]],M[b,MI[b]]],0]

292S>(32a,7b) defined[MI[b]] equalish[b,0]
293S>(292b,29a) defined[MI[b]]
294S>(293a,1d) equalish[M[x,M[y,MI[b]]],M[M[x,y],MI[b]]] ~defined[x] ~defined[y]
295S>(294b,40a) equalish[M[A[a,AI[a]],M[x,MI[b]]],M[M[A[a,AI[a]],x],MI[b]]] ~defined[x]
296S>(295a,24b) ~defined[x] equalish[M[M[A[a,AI[a]],x],MI[b]],M[A[a,AI[a]],M[x,MI[b]]]]
297S>(296b,18b) ~defined[x] equalish[M[M[A[a,AI[a]],x],MI[b]],y]
~equalish[M[A[a,AI[a]],M[x,MI[b]]],y]
298S>(297a,32a) equalish[M[M[A[a,AI[a]],b],MI[b]],x] ~equalish[M[A[a,AI[a]],M[b,MI[b]]],x]
299: 298|{0/x} equalish[M[M[A[a,AI[a]],b],MI[b]],0] ~equalish[M[A[a,AI[a]],M[b,MI[b]]],0]
300S>(299b,291a) equalish[M[M[A[a,AI[a]],b],MI[b]],0]

301S>(36a,7c) defined[MI[A[0,a]]] ~defined[A[0,a]]
302S>(301b,180a) defined[MI[A[0,a]]]
303S>(302a,19b) equalish[A[MI[A[0,a]],AI[MI[A[0,a]]]],0]
304S>(303a,24b) equalish[0,A[MI[A[0,a]],AI[MI[A[0,a]]]]
305S>(304a,9c) equalish[M[0,x],M[A[MI[A[0,a]],AI[MI[A[0,a]]]],x]] ~defined[x]
306S>(305a,18c) ~defined[x] equalish[y,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],x]] ~equalish[y,M[0,x]]
307S>(306a,40a) equalish[x,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],A[a,AI[a]]]
~equalish[x,M[0,A[a,AI[a]]]
308: 307|{0/x} equalish[0,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],A[a,AI[a]]]
~equalish[0,M[0,A[a,AI[a]]]
309S>(308b,157a) equalish[0,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],A[a,AI[a]]]

310S>(36a,7c) defined[MI[A[0,a]]] ~defined[A[0,a]]
311S>(310b,180a) defined[MI[A[0,a]]]
312S>(311a,19b) equalish[A[MI[A[0,a]],AI[MI[A[0,a]]]],0]
313S>(312a,9c) equalish[M[A[MI[A[0,a]],AI[MI[A[0,a]]]],x],M[0,x]] ~defined[x]
314S>(313a,18c) ~defined[x] equalish[y,M[0,x]] ~equalish[y,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],x]]
315S>(314b,24b) ~defined[x] ~equalish[y,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],x]] equalish[M[0,x],y]
316S>(315a,40a) ~equalish[x,M[A[MI[A[0,a]],AI[MI[A[0,a]]]],A[a,AI[a]]]
equalish[M[0,A[a,AI[a]],x]
317S>(316a,309a) equalish[M[0,A[a,AI[a]]],0]

318S>(34a,9c) equalish[M[A[a,AI[a]],x],M[0,x]] ~defined[x]
319S>(318a*45c) ~defined[x] equalish[A[M[a,x],M[AI[a],x]],M[0,x]]
320S>(319b,18b) ~defined[x] equalish[A[M[a,x],M[AI[a],x]],y] ~equalish[M[0,x],y]
321S>(320a,40a) equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],x] ~equalish[M[0,A[a,AI[a]]],x]
322S>(321a,24b) ~equalish[M[0,A[a,AI[a]]],x] equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]
323S>(322a,317a) equalish[0,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]

324S>(34a,18c) equalish[x,0] ~equalish[x,A[a,AI[a]]]
325S>(324a,18b) ~equalish[x,A[a,AI[a]]] equalish[x,y] ~equalish[0,y]
326S>(325a,23a) equalish[A[a,AI[a]],x] ~equalish[0,x] ~defined[A[a,AI[a]]]
327S>(326c,40a) equalish[A[a,AI[a]],x] ~equalish[0,x]
328S>(327b,18a) equalish[A[a,AI[a]],x] ~equalish[0,y] ~equalish[y,x]
329S>(328a,24b) ~equalish[0,x] ~equalish[x,y] equalish[y,A[a,AI[a]]]
330S>(329b,48a) ~equalish[0,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]
equalish[M[A[a,AI[a]],A[a,AI[a]]],A[a,AI[a]]]
331S>(330a,323a) equalish[M[A[a,AI[a]],A[a,AI[a]]],A[a,AI[a]]]

332S>(34a,18c) equalish[x,0] ~equalish[x,A[a,AI[a]]]
333S>(332b,45b) equalish[A[M[a,x],M[AI[a],x]],0] ~defined[x] ~equalish[M[A[a,AI[a]],x],A[a,AI[a]]]
334S>(333b,40a) equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],0]
~equalish[M[A[a,AI[a]],A[a,AI[a]]],A[a,AI[a]]]
335S>(334b,331a) equalish[A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]],0]
336S>(335a,24b) equalish[0,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]
337S>(336a,18c) equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]] ~equalish[x,0]
338S>(337b,41a) equalish[M[A[a,AI[a]],1],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]

339S>(48a,18c) equalish[x,M[A[a,AI[a]],A[a,AI[a]]]
~equalish[x,A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]
340S>(339a,46a) ~equalish[M[A[a,AI[a]],1],A[M[a,A[a,AI[a]]],M[AI[a],A[a,AI[a]]]]
equalish[M[A[a,AI[a]],A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]
341S>(340a,338a) equalish[M[A[a,AI[a]],A[a,AI[a]]],A[M[a,1],M[AI[a],1]]]
342S>(341a,18c) equalish[x,A[M[a,1],M[AI[a],1]]] ~equalish[x,M[A[a,AI[a]],A[a,AI[a]]]
343S>(342a,24b) ~equalish[x,M[A[a,AI[a]],A[a,AI[a]]] equalish[A[M[a,1],M[AI[a],1]],x]
344S>(343a,45b) equalish[A[M[a,1],M[AI[a],1]],A[M[a,x],M[AI[a],x]]] ~defined[x]
~equalish[M[A[a,AI[a]],x],M[A[a,AI[a]],A[a,AI[a]]]
345S>(344b,32a) equalish[A[M[a,1],M[AI[a],1]],A[M[a,b],M[AI[a],b]]]
~equalish[M[A[a,AI[a]],b],M[A[a,AI[a]],A[a,AI[a]]]
346S>(345b,246a) equalish[A[M[a,1],M[AI[a],1]],A[M[a,b],M[AI[a],b]]]

347S>(46a,23a) equalish[M[A[a,AI[a]],1],A[M[a,1],M[AI[a],1]]] ~defined[M[A[a,AI[a]],1]]

```

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348S>(347b,275a) equalish[M[A[a,AI[a]],1],A[M[a,1],M[AI[a],1]]]
349S>(348a,18b) equalish[M[A[a,AI[a]],1],x] ~equalish[A[M[a,1],M[AI[a],1]],x]
350S>(349b,18a) equalish[M[A[a,AI[a]],1],x] ~equalish[A[M[a,1],M[AI[a],1]],y] ~equalish[y,x]
351S>(350a,24b) ~equalish[A[M[a,1],M[AI[a],1]],x] ~equalish[x,y] equalish[y,M[A[a,AI[a]],1]]
352S>(351b,44a) ~equalish[A[M[a,1],M[AI[a],1]],A[M[a,x],M[AI[a],x]]]
equalish[M[A[a,AI[a]],x],M[A[a,AI[a]],1]] ~defined[x]
353S>(352b,24b) ~equalish[A[M[a,1],M[AI[a],1]],A[M[a,x],M[AI[a],x]]] ~defined[x]
equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],x]]
354S>(353a,346a) ~defined[b] equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],b]]
355S>(354a,32a) equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],b]]

356S>(45a,32a) equalish[A[M[a,b],M[AI[a],b]],x] ~equalish[M[A[a,AI[a]],b],x]
357S>(356b,46b) equalish[A[M[a,b],M[AI[a],b]],A[M[a,1],M[AI[a],1]]]
~equalish[M[A[a,AI[a]],1],M[A[a,AI[a]],b]]
358S>(357b,355a) equalish[A[M[a,b],M[AI[a],b]],A[M[a,1],M[AI[a],1]]]
359S>(358a,9c) equalish[M[A[M[a,b],M[AI[a],b]],x],M[A[M[a,1],M[AI[a],1]],x]] ~defined[x]
360S>(359b,7a) equalish[M[A[M[a,b],M[AI[a],b]],MI[x]],M[A[M[a,1],M[AI[a],1]],MI[x]]] ~defined[x]
equalish[x,0]
361S>(360b,32a) equalish[M[A[M[a,b],M[AI[a],b]],MI[b]],M[A[M[a,1],M[AI[a],1]],MI[b]]]
equalish[b,0]
362S>(361b,29a) equalish[M[A[M[a,b],M[AI[a],b]],MI[b]],M[A[M[a,1],M[AI[a],1]],MI[b]]]
363S>(362a,18c) equalish[x,M[A[M[a,1],M[AI[a],1]],MI[b]]]
~equalish[x,M[A[M[a,b],M[AI[a],b]],MI[b]]]
364S>(363a,282a) ~equalish[0,M[A[M[a,b],M[AI[a],b]],MI[b]]]

365S>(44a,9c) ~defined[x] equalish[M[A[M[a,x],M[AI[a],x]],y],M[M[A[a,AI[a]],x],y]] ~defined[y]
366S>(365a,32a) equalish[M[A[M[a,b],M[AI[a],b]],x],M[M[A[a,AI[a]],b],x]] ~defined[x]
367S>(366b,7a) equalish[M[A[M[a,b],M[AI[a],b]],MI[x]],M[M[A[a,AI[a]],b],MI[x]]] ~defined[x]
equalish[x,0]
368S>(367b,32a) equalish[M[A[M[a,b],M[AI[a],b]],MI[b]],M[M[A[a,AI[a]],b],MI[b]]] equalish[b,0]
369S>(368b,29a) equalish[M[A[M[a,b],M[AI[a],b]],MI[b]],M[M[A[a,AI[a]],b],MI[b]]]
370S>(369a,18b) equalish[M[A[M[a,b],M[AI[a],b]],MI[b]],x] ~equalish[M[M[A[a,AI[a]],b],MI[b]],x]
371S>(370a,24b) ~equalish[M[M[A[a,AI[a]],b],MI[b]],x] equalish[x,M[A[M[a,b],M[AI[a],b]],MI[b]]]
372S>(371a,300a) equalish[0,M[A[M[a,b],M[AI[a],b]],MI[b]]]
373S>(372a,364a) []

```

```

PHASE 0: 0 s PHASE 1: 57 s PHASE 2: 463 s Total Time: 543 s
NOD: 15367436 RES: 69401477 FAC: 2456278 T: 26 V: 32 L: 16
CTE: 2338619 CTH: 1119783 CTF: 0 CSZ: 16777216
UTE: 3336862 UTH: 6753884 UTF: 1 SBA: 0
BAS: 32 RED: 50 LEN: 10+325=335

```

## FLD044-3

Predicates: sum product less\_or\_equal defined  
Functions: b a . multiplicative\_identity additive\_identity : add additive\_inverse multiply  
multiplicative\_inverse  
Renamed predicates: S P LE defined  
Renamed functions: b a . 1 0 : A AI M MI

C58: Spawned: 150 xvl 9.14.4 W28 16S defined[z] ~defined[x] ~defined[y] ... ?

Axioms:

```
1 >S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]
2 >P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]
3: LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]
4: LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]
5 >P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]
6 >P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]
7: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
8 >S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]
9 >S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]
10: P[MI[x],x,1] S[0,x,0] ~defined[x]
11: defined[MI[x]] ~defined[x] S[0,x,0]
12: S[0,x,y] ~LE[x,y] ~LE[y,x]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14 >defined[M[x,y]] ~defined[x] ~defined[y]
15 >P[x,y,M[x,y]] ~defined[x] ~defined[y]
16S>defined[z] ~defined[x] ~defined[y] ... ?
17 >S[x,y,A[x,y]] ~defined[x] ~defined[y]
18 >S[AI[x],x,0] ~defined[x]
19 >P[1,x,x] ~defined[x]
20 >defined[AI[x]] ~defined[x]
21 >S[0,x,x] ~defined[x]
22 >P[x,y,z] ~P[y,x,z]
23 >S[x,y,z] ~S[y,x,z]
24: ~S[0,0,1]
25 >defined[1]
26 >defined[0]
```

Negated conclusion:

```
27S>~P[AI[a],b,AI[M[a,b]]]
28S>defined[a]
29S>defined[b]
```

-----  
Phase 0 clauses used in proof:

```
30S>(29a*21b) S[0,b,b]
32S>(29a*19b) P[1,b,b]
33S>(29a*18b) S[AI[b],b,0]
35S>(28a*20b) defined[AI[a]]
37S>(28a*18b) S[AI[a],a,0]
38S>(27a*22a) ~P[b,AI[a],AI[M[a,b]]]
39 >(26a*21b) S[0,0,0]
40 >(26a*20b) defined[AI[0]]
41 >(26a*19b) P[1,0,0]
43 >(25a*21b) S[0,1,1]
```

Phases 1 and 2 clauses used in proof:

```
44S>(40a,16b) defined[x] ~defined[y] ... ?
45S>(44b,40a) defined[x] ... ?

46S>(40a,16b) defined[x] ~defined[y] ... ?
47S>(46b,40a) defined[x] ... ?

48: (22b,15a) P[x,y,M[y,x]] ~defined[y] ~defined[x]
49S>(48b,45a) P[x,y,M[y,x]] ~defined[x] ... ?
50S>(49b,47a) P[x,y,M[y,x]] ... ?
51S>(50a,1d) S[M[x,y],z,u] ~S[y,v,w] ~P[w,x,u] ~P[v,x,z] ... ?
52S>(51a,23b) ~S[x,y,z] ~P[z,u,v] ~P[y,u,w] S[w,M[u,x],v] ... ?
53S>(52b,32a) ~S[x,y,1] ~P[y,b,z] S[z,M[b,x],b] ... ?
54S>(53a,43a) ~P[1,b,x] S[x,M[b,0],b] ... ?
55S>(54a,32a) S[b,M[b,0],b] ... ?

56S>(29a,14b) defined[M[b,x]] ~defined[x]
57S>(56b,47a) defined[M[b,x]] ... ?
58S>(57a,17b) S[M[b,x],y,A[M[b,x],y]] ~defined[y] ... ?
59S>(58b,45a) S[M[b,x],y,A[M[b,x],y]] ... ?
60S>(59a,23b) S[x,M[b,y],A[M[b,y],x]] ... ?
61S>(60a,9b) S[A[M[b,x],y],z,u] ~S[M[b,x],z,v] ~S[y,v,u] ... ?
62S>(61b,23a) S[A[M[b,x],y],z,u] ~S[y,v,u] ~S[z,M[b,x],v] ... ?
63S>(62b,33a) S[A[M[b,x],AI[b]],y,0] ~S[y,M[b,x],b] ... ?
```

64: 63{|0/x,b/y} S[A[M[b,0],AI[b]],b,0] ~S[b,M[b,0],b] ... ?  
65S>(64b,55a) S[A[M[b,0],AI[b]],b,0] ... ?

66S>(29a,14b) defined[M[b,x]] ~defined[x]  
67S>(66b,47a) defined[M[b,x]] ... ?  
68S>(67a,17b) S[M[b,x],y,A[M[b,x],y]] ~defined[y] ... ?  
69S>(68b,45a) S[M[b,x],y,A[M[b,x],y]] ... ?  
70S>(69a,8b) S[M[b,x],y,z] ~S[u,v,y] ~S[A[M[b,x],u],v,z] ... ?  
71S>(70b,33a) S[M[b,x],0,y] ~S[A[M[b,x],AI[b]],b,y] ... ?  
72S>(71a,23b) ~S[A[M[b,x],AI[b]],b,y] S[0,M[b,x],y] ... ?  
73: 72{|0/x,0/y} ~S[A[M[b,0],AI[b]],b,0] S[0,M[b,0],0] ... ?  
74S>(73a,65a) S[0,M[b,0],0] ... ?

75: (26a,14c) defined[M[x,0]] ~defined[x]  
76S>(75b,47a) defined[M[x,0]] ... ?  
77S>(76a,17c) S[x,M[y,0],A[x,M[y,0]]] ~defined[x] ... ?  
78S>(77b,47a) S[x,M[y,0],A[x,M[y,0]]] ... ?  
79S>(78a,8c) S[x,A[y,M[z,0]],u] ~S[x,y,v] ~S[v,M[z,0],u] ... ?  
80S>(79b,18a) S[AI[x],A[x,M[y,0]],z] ~S[0,M[y,0],z] ~defined[x] ... ?  
81S>(80c,47a) S[AI[x],A[x,M[y,0]],z] ~S[0,M[y,0],z] ... ?  
82: 81{|b/y,0/z} S[AI[x],A[x,M[b,0]],0] ~S[0,M[b,0],0] ... ?  
83S>(82b,74a) S[AI[x],A[x,M[b,0]],0] ... ?

84: (26a,14c) defined[M[x,0]] ~defined[x]  
85S>(84b,47a) defined[M[x,0]] ... ?  
86S>(85a,17c) S[x,M[y,0],A[x,M[y,0]]] ~defined[x] ... ?  
87S>(86b,47a) S[x,M[y,0],A[x,M[y,0]]] ... ?  
88S>(87a,9c) S[x,M[y,0],z] ~S[u,v,x] ~S[u,A[v,M[y,0]],z] ... ?  
89S>(88b,33a) S[0,M[x,0],y] ~S[AI[b],A[b,M[x,0]],y] ... ?  
90S>(89a,23b) ~S[AI[b],A[b,M[x,0]],y] S[M[x,0],0,y] ... ?  
91: 90{|b/x,0/y} ~S[AI[b],A[b,M[b,0]],0] S[M[b,0],0,0] ... ?  
92S>(91a,83a) S[M[b,0],0,0] ... ?

93S>(40a,16b) defined[x] ~defined[y] ... ?  
94S>(93a,19b) ~defined[x] P[1,y,y] ... ?  
95S>(94a,47a) P[1,y,y] ... ?

96: (43a,1b) S[x,y,z] ~P[1,u,z] ~P[0,u,x] ~P[1,u,y]  
97: (96b,19a) S[x,y,z] ~P[0,z,x] ~P[1,z,y] ~defined[z]  
98: (97b\*15a) S[M[0,x],y,x] ~P[1,x,y] ~defined[x] ~defined[0]  
99S>(98c,47a) S[M[0,x],y,x] ~P[1,x,y] ~defined[0] ... ?  
100S>(99c,26a) S[M[0,x],y,x] ~P[1,x,y] ... ?  
101S>(100a,8b) ~P[1,x,y] S[M[0,x],z,u] ~S[y,v,z] ~S[x,v,u] ... ?  
102S>(101b,23b) ~P[1,x,y] ~S[y,z,u] ~S[x,z,v] S[u,M[0,x],v] ... ?  
103S>(102bc) ~P[1,x,x] ~S[x,y,z] S[z,M[0,x],z] ... ?  
104S>(103a,95a) ~S[x,y,z] S[z,M[0,x],z] ... ?  
105S>(104a,37a) S[0,M[0,AI[a]],0] ... ?

106: (21b,14a) S[0,M[x,y],M[x,y]] ~defined[x] ~defined[y]  
107S>(106b,47a) S[0,M[x,y],M[x,y]] ~defined[y] ... ?  
108S>(107b,45a) S[0,M[x,y],M[x,y]] ... ?  
109S>(108a,23b) S[M[x,y],0,M[x,y]] ... ?  
110S>(109a,9b) S[M[x,y],z,u] ~S[0,z,v] ~S[M[x,y],v,u] ... ?  
111S>(110b,23a) S[M[x,y],z,u] ~S[M[x,y],v,u] ~S[z,0,v] ... ?  
112S>(111bc) S[M[x,y],M[x,y],0] ~S[M[x,y],0,0] ... ?  
113: 112{|b/x,0/y} S[M[b,0],M[b,0],0] ~S[M[b,0],0,0] ... ?  
114S>(113b,92a) S[M[b,0],M[b,0],0] ... ?

115S>(35a,14c) defined[M[x,AI[a]]] ~defined[x]  
116S>(115b,47a) defined[M[x,AI[a]]] ... ?  
117S>(116a,21b) S[0,M[x,AI[a]],M[x,AI[a]]] ... ?  
118S>(117a,23b) S[M[x,AI[a]],0,M[x,AI[a]]] ... ?  
119S>(118a,9c) S[x,0,y] ~S[z,M[u,AI[a]],x] ~S[z,M[u,AI[a]],y] ... ?  
120S>(119b,21a) S[M[x,AI[a]],0,y] ~S[0,M[x,AI[a]],y] ~defined[M[x,AI[a]]] ... ?  
121S>(120c,116a) S[M[x,AI[a]],0,y] ~S[0,M[x,AI[a]],y] ... ?  
122S>(121a,8b) ~S[0,M[x,AI[a]],y] S[M[x,AI[a]],z,u] ~S[0,v,z] ~S[y,v,u] ... ?  
123S>(122ad) S[M[x,AI[a]],y,0] ~S[0,M[x,AI[a]],y] ~S[0,M[x,AI[a]],0] ... ?  
124S>(123b,117a) S[M[x,AI[a]],M[x,AI[a]],0] ~S[0,M[x,AI[a]],0] ... ?  
125: 124{|0/x} S[M[0,AI[a]],M[0,AI[a]],0] ~S[0,M[0,AI[a]],0] ... ?  
126S>(125b,105a) S[M[0,AI[a]],M[0,AI[a]],0] ... ?

127: (40a,14c) defined[M[x,AI[0]]] ~defined[x]  
128S>(127b,47a) defined[M[x,AI[0]]] ... ?  
129S>(128a,16c) defined[x] ~defined[y] ... ?  
130S>(129b,45a) defined[x] ... ?  
131S>(130a,15c) P[x,y,M[x,y]] ~defined[x] ... ?  
132S>(131b,130a) P[x,y,M[x,y]] ... ?  
133S>(132a,22b) P[x,y,M[y,x]] ... ?  
134S>(133a,2d) P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~S[w,M[y,v],z] ... ?  
135S>(134a,22b) ~S[x,y,z] ~P[x,u,v] ~S[v,M[u,y],w] P[u,z,w] ... ?  
136S>(135b,133a) ~S[x,y,z] ~S[M[u,x],M[u,y],v] P[u,z,v] ... ?

137S>(136a,39a)  $\sim S[M[x,0],M[x,0],y] P[x,0,y] \dots ?$   
138: 137|{b/x,0/y}  $\sim S[M[b,0],M[b,0],0] P[b,0,0] \dots ?$   
139S>(138a,114a)  $P[b,0,0] \dots ?$

140S>(40a,16b) defined[x]  $\sim$ defined[y] ... ?  
141S>(140a,14b)  $\sim$ defined[x] defined[M[y,z]]  $\sim$ defined[z] ... ?  
142S>(141a,47a) defined[M[y,z]]  $\sim$ defined[z] ... ?  
143S>(142b,35a) defined[M[x,AI[a]]] ... ?

144S>(35a,15c)  $P[x,AI[a],M[x,AI[a]]] \sim$ defined[x]  
145S>(144b,47a)  $P[x,AI[a],M[x,AI[a]]] \dots ?$   
146S>(145a,6c)  $P[x,AI[a],y] \sim P[z,u,x] \sim P[z,M[u,AI[a]],y] \dots ?$   
147S>(146b,41a)  $P[0,AI[a],x] \sim P[1,M[0,AI[a]],x] \dots ?$   
148S>(147b,19a)  $P[0,AI[a],M[0,AI[a]]] \sim$ defined[M[0,AI[a]]] ... ?  
149S>(148b,143a)  $P[0,AI[a],M[0,AI[a]]] \dots ?$   
150S>(149a,2d)  $P[x,AI[a],y] \sim S[z,0,x] \sim P[z,AI[a],u] \sim S[u,M[0,AI[a]],y] \dots ?$   
151S>(150a,22b)  $\sim S[x,0,y] \sim P[x,AI[a],z] \sim S[z,M[0,AI[a]],u] P[AI[a],y,u] \dots ?$   
152S>(151b,145a)  $\sim S[x,0,y] \sim S[M[x,AI[a]],M[0,AI[a]],z] P[AI[a],y,z] \dots ?$   
153S>(152a,39a)  $\sim S[M[0,AI[a]],M[0,AI[a]],x] P[AI[a],0,x] \dots ?$   
154S>(153a,126a)  $P[AI[a],0,0] \dots ?$

155S>(40a,16c) defined[x]  $\sim$ defined[y] ... ?  
156S>(155b,45a) defined[x] ... ?  
157S>(156a,19b)  $P[1,x,x] \dots ?$   
158S>(157a,5d)  $P[x,y,z] \sim P[x,u,1] \sim P[u,z,y] \dots ?$   
159S>(158b,19a)  $P[1,x,y] \sim P[1,y,x] \sim$ defined[1] ... ?  
160S>(159c,25a)  $P[1,x,y] \sim P[1,y,x] \dots ?$   
161S>(160a,6d)  $\sim P[1,x,y] P[z,u,x] \sim P[1,v,z] \sim P[v,u,y] \dots ?$   
162S>(161b,22b)  $\sim P[1,x,y] \sim P[1,z,u] \sim P[z,v,y] P[v,u,x] \dots ?$   
163S>(162b,32a)  $\sim P[1,x,y] \sim P[b,z,y] P[z,b,x] \dots ?$   
164S>(163a,41a)  $\sim P[b,x,0] P[x,b,0] \dots ?$   
165S>(164a,139a)  $P[0,b,0] \dots ?$

166S>(35a,15c)  $P[x,AI[a],M[x,AI[a]]] \sim$ defined[x]  
167S>(166b,47a)  $P[x,AI[a],M[x,AI[a]]] \dots ?$   
168S>(167a,6b)  $P[M[x,AI[a]],y,z] \sim P[AI[a],y,u] \sim P[x,u,z] \dots ?$   
169S>(168bc)  $P[M[AI[a],AI[a]],x,x] \sim P[AI[a],x,x] \dots ?$   
170S>(169a,5d)  $\sim P[AI[a],x,x] P[y,z,x] \sim P[y,u,M[AI[a],AI[a]]] \sim P[u,x,z] \dots ?$   
171S>(170ad)  $P[x,y,y] \sim P[x,AI[a],M[AI[a],AI[a]]] \sim P[AI[a],y,y] \dots ?$   
172S>(171a,22b)  $\sim P[x,AI[a],M[AI[a],AI[a]]] \sim P[AI[a],y,y] P[y,x,y] \dots ?$   
173S>(172a,167a)  $\sim P[AI[a],x,x] P[x,AI[a],x] \dots ?$   
174S>(173a,154a)  $P[0,AI[a],0] \dots ?$

175S>(35a,15c)  $P[x,AI[a],M[x,AI[a]]] \sim$ defined[x]  
176S>(175b,47a)  $P[x,AI[a],M[x,AI[a]]] \dots ?$   
177S>(176a,22b)  $P[AI[a],x,M[x,AI[a]]] \dots ?$   
178S>(177a,1d)  $S[M[x,AI[a]],y,z] \sim S[AI[a],u,v] \sim P[v,x,z] \sim P[u,x,y] \dots ?$   
179S>(178b,18a)  $S[M[x,AI[a]],y,z] \sim P[0,x,z] \sim P[a,x,y] \sim$ defined[a] ... ?  
180S>(179d,28a)  $S[M[x,AI[a]],y,z] \sim P[0,x,z] \sim P[a,x,y] \dots ?$   
181S>(180a,23b)  $\sim P[0,x,y] \sim P[a,x,z] S[z,M[x,AI[a]],y] \dots ?$   
182S>(181b,15a)  $\sim P[0,x,y] S[M[a,x],M[x,AI[a]],y] \sim$ defined[a]  $\sim$ defined[x] ... ?  
183S>(182c,28a)  $\sim P[0,x,y] S[M[a,x],M[x,AI[a]],y] \sim$ defined[x] ... ?  
184S>(183c,47a)  $\sim P[0,x,y] S[M[a,x],M[x,AI[a]],y] \dots ?$   
185: 184|{b/x,0/y}  $\sim P[0,b,0] S[M[a,b],M[b,AI[a]],0] \dots ?$   
186S>(185a,165a)  $S[M[a,b],M[b,AI[a]],0] \dots ?$

187: (20b,14a) defined[AI[M[x,y]]]  $\sim$ defined[x]  $\sim$ defined[y]  
188S>(187b,47a) defined[AI[M[x,y]]]  $\sim$ defined[y] ... ?  
189S>(188b,45a) defined[AI[M[x,y]]] ... ?  
190S>(189a,21b)  $S[0,AI[M[x,y]],AI[M[x,y]]] \dots ?$   
191S>(190a,23b)  $S[AI[M[x,y]],0,AI[M[x,y]]] \dots ?$

192S>(30a,2b)  $P[b,x,y] \sim P[0,x,z] \sim P[b,x,u] \sim S[z,u,y]$   
193S>(192a,38a)  $\sim P[0,AI[a],x] \sim P[b,AI[a],y] \sim S[x,y,AI[M[a,b]]]$   
194S>(193b,15a)  $\sim P[0,AI[a],x] \sim S[x,M[b,AI[a]],AI[M[a,b]]] \sim$ defined[b]  $\sim$ defined[AI[a]]  
195S>(194c,29a)  $\sim P[0,AI[a],x] \sim S[x,M[b,AI[a]],AI[M[a,b]]] \sim$ defined[AI[a]]  
196S>(195c,35a)  $\sim P[0,AI[a],x] \sim S[x,M[b,AI[a]],AI[M[a,b]]]$   
197S>(196b,9a)  $\sim P[0,AI[a],x] \sim S[y,z,x] \sim S[z,M[b,AI[a]],u] \sim S[y,u,AI[M[a,b]]]$   
198S>(197b,18a)  $\sim P[0,AI[a],0] \sim S[x,M[b,AI[a]],y] \sim S[AI[x],y,AI[M[a,b]]] \sim$ defined[x]  
199S>(198a,174a)  $\sim S[x,M[b,AI[a]],y] \sim S[AI[x],y,AI[M[a,b]]] \sim$ defined[x] ... ?  
200S>(199c,47a)  $\sim S[x,M[b,AI[a]],y] \sim S[AI[x],y,AI[M[a,b]]] \dots ?$   
201S>(200b,8a)  $\sim S[x,M[b,AI[a]],y] \sim S[AI[x],z,u] \sim S[z,v,y] \sim S[u,v,AI[M[a,b]]] \dots ?$   
202S>(201bd)  $\sim S[M[a,b],M[b,AI[a]],x] \sim S[y,y,x] \sim S[AI[M[a,b]],y,AI[M[a,b]]] \dots ?$   
203S>(202b,39a)  $\sim S[M[a,b],M[b,AI[a]],0] \sim S[AI[M[a,b]],0,AI[M[a,b]]] \dots ?$   
204S>(203a,186a)  $\sim S[AI[M[a,b]],0,AI[M[a,b]]] \dots ?$   
205S>(204a,191a) [] ... ?

Inferences derived from proof before variable replaced by constant  
30 32 33 35 37 38 39 41 43 48 98 106 127 144 166 175 198

Inferences derived when variable is replaced by original constant

45 47 57 67 76 85 116 140 141 156 188

PHASE 0: 0 s PHASE 1: 21 s PHASE 2: 75 s Total Time: 96 s  
NOD: 3166539 RES: 8104011 FAC: 1520970 T: 9 V: 32 L: 16  
CTE: 1281132 CTH: 992103 CTF: 0 CSZ: 16777216  
UTE: 134908 UTH: 2242027 UTF: 0 SBA: 0  
BAS: 29 RED: 43 LEN: 10+162=172

Proof found to theorem by slave 58!

Axioms:

1S>S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]  
2S>P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]  
3: LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]  
4: LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]  
5S>P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]  
6S>P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]  
7: LE[x,y] LE[y,x] ~defined[x] ~defined[y]  
8S>S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]  
9S>S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]  
10: P[MI[x],x,1] S[0,x,0] ~defined[x]  
11: defined[MI[x]] ~defined[x] S[0,x,0]  
12: S[0,x,y] ~LE[x,y] ~LE[y,x]  
13: LE[x,y] ~LE[x,z] ~LE[z,y]  
14S>defined[M[x,y]] ~defined[x] ~defined[y]  
15S>P[x,y,M[x,y]] ~defined[x] ~defined[y]  
16S defined[A[x,y]] ~defined[x] ~defined[y]  
17S S[x,y,A[x,y]] ~defined[x] ~defined[y]  
18S S[AI[x],x,0] ~defined[x]  
19S>P[1,x,x] ~defined[x]  
20S>defined[AI[x]] ~defined[x]  
21S>S[0,x,x] ~defined[x]  
22S>P[x,y,z] ~P[y,x,z]  
23S>S[x,y,z] ~S[y,x,z]  
24: ~S[0,0,1]  
25S>defined[1]  
26S>defined[0]

Negated conclusion:

27S ~P[AI[a],b,AI[M[a,b]]]  
28S>defined[a]  
29S>defined[b]

-----

Phase 0 clauses used in proof:

31S>P[1,b,b]  
32S>S[AI[b],b,0]  
33S>defined[AI[a]]  
34S>S[AI[a],a,0]  
36S>S[0,0,0]  
37S>P[1,0,0]  
38S>S[0,1,1]  
39S>P[x,y,M[y,x]] ~defined[y] ~defined[x]  
40S>S[M[0,x],y,x] ~P[1,x,y] ~defined[x] ~defined[0]  
41S>S[0,M[x,y],M[x,y]] ~defined[x] ~defined[y]  
43S>P[x,AI[a],M[x,AI[a]]] ~defined[x]  
44S>~P[0,AI[a],0] ~S[x,M[b,AI[a]],y] ~S[AI[x],y,AI[M[a,b]]] ~defined[x]  
53S>(40d\*26a) S[M[0,x],y,x] ~P[1,x,y] ~defined[x]

Phases 1 and 2 clauses used in proof:

54S>(39b,29a) P[x,b,M[b,x]] ~defined[x]  
55S>(54a,1d) ~defined[x] S[M[b,x],y,z] ~S[x,u,v] ~P[v,b,z] ~P[u,b,y]  
56S>(55b,23b) ~defined[x] ~S[x,y,z] ~P[z,b,u] ~P[y,b,v] S[v,M[b,x],u]  
57S>(56b,38a) ~defined[0] ~P[1,b,x] ~P[1,b,y] S[y,M[b,0],x]  
58S>(57a,26a) ~P[1,b,x] ~P[1,b,y] S[y,M[b,0],x]  
59S>(58ab) ~P[1,b,x] S[x,M[b,0],x]  
60S>(59a,31a) S[b,M[b,0],b]  
  
61S>(41b,29a) S[0,M[b,x],M[b,x]] ~defined[x]  
62S>(61b,26a) S[0,M[b,0],M[b,0]]  
63S>(62a,8d) S[x,y,M[b,0]] ~S[x,z,0] ~S[z,M[b,0],y]  
64S>(63a,9b) ~S[x,y,0] ~S[y,M[b,0],z] S[M[b,0],u,v] ~S[z,u,w] ~S[x,w,v]  
65S>(64a,32a) ~S[b,M[b,0],x] S[M[b,0],y,z] ~S[x,y,u] ~S[AI[b],u,z]  
66S>(65ac) S[M[b,0],M[b,0],x] ~S[b,M[b,0],b] ~S[AI[b],b,x]  
67S>(66b,60a) S[M[b,0],M[b,0],x] ~S[AI[b],b,x]  
68S>(67b,32a) S[M[b,0],M[b,0],0]  
  
69S>(39b,29a) P[x,b,M[b,x]] ~defined[x]  
70S>(69b,26a) P[0,b,M[b,0]]  
71S>(70a,2d) P[x,b,y] ~S[z,0,x] ~P[z,b,u] ~S[u,M[b,0],y]  
72S>(71a,22b) ~S[x,0,y] ~P[x,b,z] ~S[z,M[b,0],u] P[b,y,u]

```

73S>(72b,70a) ~S[0,0,x] ~S[M[b,0],M[b,0],y] P[b,x,y]
74S>(73a,36a) ~S[M[b,0],M[b,0],x] P[b,0,x]
75S>(74a,68a) P[b,0,0]

76S>(53b*19a) S[M[0,x],x,x] ~defined[x]
77S>(76a,8b) ~defined[x] S[M[0,x],y,z] ~S[x,u,y] ~S[x,u,z]
78S>(77b,23b) ~defined[x] ~S[x,y,z] ~S[x,y,u] S[z,M[0,x],u]
79S>(78b,34a) ~defined[AI[a]] ~S[AI[a],a,x] S[0,M[0,AI[a]],x]
80S>(79a,33a) ~S[AI[a],a,x] S[0,M[0,AI[a]],x]
81S>(80a,34a) S[0,M[0,AI[a]],0]

82S>(39a,6d) ~defined[x] ~defined[y] P[z,u,M[x,y]] ~P[y,v,z] ~P[v,u,x]
83S>(82b,25a) ~defined[x] P[y,z,M[x,1]] ~P[1,u,y] ~P[u,z,x]
84S>(83b,22b) ~defined[x] ~P[1,y,z] ~P[y,u,x] P[u,z,M[x,1]]
85S>(84b,31a) ~defined[x] ~P[b,y,x] P[y,b,M[x,1]]
86S>(85a,26a) ~P[b,x,0] P[x,b,M[0,1]]
87S>(86a,75a) P[0,b,M[0,1]]

88S>(41b,26a) S[0,M[0,x],M[0,x]] ~defined[x]
89S>(88b,33a) S[0,M[0,AI[a]],M[0,AI[a]]]
90S>(89a,8d) S[x,y,M[0,AI[a]]] ~S[x,z,0] ~S[z,M[0,AI[a]],y]
91S>(90a,9b) ~S[x,y,0] ~S[y,M[0,AI[a]],z] S[M[0,AI[a]],u,v] ~S[z,u,w] ~S[x,w,v]
92S>(91a,36a) ~S[0,M[0,AI[a]],x] S[M[0,AI[a]],y,z] ~S[x,y,u] ~S[0,u,z]
93S>(92ac) S[M[0,AI[a]],M[0,AI[a]],x] ~S[0,M[0,AI[a]],0] ~S[0,0,x]
94S>(93b,81a) S[M[0,AI[a]],M[0,AI[a]],x] ~S[0,0,x]
95S>(94b,36a) S[M[0,AI[a]],M[0,AI[a]],0]

96S>(36a,2b) P[0,x,y] ~P[0,x,z] ~P[0,x,u] ~S[z,u,y]
97S>(96b,43a) P[0,AI[a],x] ~P[0,AI[a],y] ~S[M[0,AI[a]],y,x] ~defined[0]
98S>(97d,26a) P[0,AI[a],x] ~P[0,AI[a],y] ~S[M[0,AI[a]],y,x]
99S>(98b,15a) P[0,AI[a],x] ~S[M[0,AI[a]],M[0,AI[a]],x] ~defined[0] ~defined[AI[a]]
100S>(99c,26a) P[0,AI[a],x] ~S[M[0,AI[a]],M[0,AI[a]],x] ~defined[AI[a]]
101S>(100c,33a) P[0,AI[a],x] ~S[M[0,AI[a]],M[0,AI[a]],x]
102S>(101a,22b) ~S[M[0,AI[a]],M[0,AI[a]],x] P[AI[a],0,x]
103S>(102a,95a) P[AI[a],0,0]

104S>(37a,5d) P[x,y,0] ~P[x,z,1] ~P[z,0,y]
105S>(104b,19a) P[1,x,0] ~P[1,0,x] ~defined[1]
106S>(105c,25a) P[1,x,0] ~P[1,0,x]
107S>(106b,39a) P[1,M[0,1],0] ~defined[0] ~defined[1]
108S>(107b,26a) P[1,M[0,1],0] ~defined[1]
109S>(108b,25a) P[1,M[0,1],0]
110S>(109a,6d) P[x,y,0] ~P[1,z,x] ~P[z,y,M[0,1]]
111S>(110b,37a) P[0,x,0] ~P[0,x,M[0,1]]
112: 111|{b/x} P[0,b,0] ~P[0,b,M[0,1]]
113S>(112b,87a) P[0,b,0]

114S>(29a,14c) defined[M[x,b]] ~defined[x]
115S>(114a,20b) ~defined[x] defined[AI[M[x,b]]]
116S>(115a,28a) defined[AI[M[a,b]]]

117S>(29a,14c) defined[M[x,b]] ~defined[x]
118S>(117b,28a) defined[M[a,b]]

119S>(39b,28a) P[x,a,M[a,x]] ~defined[x]
120S>(119a,22b) ~defined[x] P[a,x,M[a,x]]
121S>(120a,29a) P[a,b,M[a,b]]

122S>(43a,22b) ~defined[x] P[AI[a],x,M[x,AI[a]]]
123S>(122a,29a) P[AI[a],b,M[b,AI[a]]]

124S>(44a,22a) ~S[x,M[b,AI[a]],y] ~S[AI[x],y,AI[M[a,b]]] ~defined[x] ~P[AI[a],0,0]
125S>(124d,103a) ~S[x,M[b,AI[a]],y] ~S[AI[x],y,AI[M[a,b]]] ~defined[x]
126S>(125b,23a) ~S[x,M[b,AI[a]],y] ~defined[x] ~S[y,AI[x],AI[M[a,b]]]
127S>(126a,23a) ~defined[x] ~S[y,AI[x],AI[M[a,b]]] ~S[M[b,AI[a]],x,y]
128S>(127b,21a) ~defined[M[a,b]] ~S[M[b,AI[a]],M[a,b],0] ~defined[AI[M[a,b]]]
129S>(128a,118a) ~S[M[b,AI[a]],M[a,b],0] ~defined[AI[M[a,b]]]
130S>(129b,116a) ~S[M[b,AI[a]],M[a,b],0]
131S>(130a,1a) ~S[x,y,z] ~P[z,u,0] ~P[x,u,M[b,AI[a]]] ~P[y,u,M[a,b]]
132S>(131a,34a) ~P[0,x,0] ~P[AI[a],x,M[b,AI[a]]] ~P[a,x,M[a,b]]
133S>(132a,113a) ~P[AI[a],b,M[b,AI[a]]] ~P[a,b,M[a,b]]
134S>(133a,123a) ~P[a,b,M[a,b]]
135S>(134a,121a) []

```

```

PHASE 0: 0 s PHASE 1: 25 s PHASE 2: 28 s Total Time: 149 s
NOD: 1735704 RES: 4525268 FAC: 1048348 T: 9 V: 32 L: 16
CTE: 1629511 CTH: 704338 CTF: 0 CSZ: 16777216
UTE: 424297 UTH: 1797195 UTF: 0 SBA: 0
BAS: 29 RED: 57 LEN: 13+82=95

```

## FLD047-3

Predicates: sum product less\_or\_equal defined  
Functions: additive\_identity c(S) b(S) a(S) . multiplicative\_identity : additive\_inverse add multiply  
multiplicative\_inverse  
Renamed predicates: S P LE defined  
Renamed functions: 0 c b a . 1 : AI A M MI

C39: Spawned: 150 xvl 13.14.6 W19 27S ~P[a,x,M[a,c],MI[M[b,c]]] ... ?

Axioms:

```
1 >S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]
2 >P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]
3: LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]
4: LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]
5 >P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]
6: P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]
7: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
8 >S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]
9 >S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]
10 >P[MI[x],x,1] S[0,x,0] ~defined[x]
11 >defined[MI[x]] ~defined[x] S[0,x,0]
12: S[0,x,y] ~LE[x,y] ~LE[y,x]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14 >defined[M[x,y]] ~defined[x] ~defined[y]
15 >P[x,y,M[x,y]] ~defined[x] ~defined[y]
16: defined[A[x,y]] ~defined[x] ~defined[y]
17: S[x,y,A[x,y]] ~defined[x] ~defined[y]
18 >S[AI[x],x,0] ~defined[x]
19 >P[1,x,x] ~defined[x]
20: defined[AI[x]] ~defined[x]
21 >S[0,x,x] ~defined[x]
22 >P[x,y,z] ~P[y,x,z]
23 >S[x,y,z] ~S[y,x,z]
24: ~S[0,0,1]
25 >defined[1]
26 >defined[0]
```

Negated conclusion:

```
27S>~P[a,x,M[a,c],MI[M[b,c]]] ... ?
28S>~S[0,b,0]
29S>~S[0,c,0]
30S>defined[a]
31S>defined[b]
32S>defined[c]
```

Phase 0 clauses used in proof:

```
33S>(32a*21b) S[0,c,c]
35S>(32a*19b) P[1,c,c]
36S>(32a*18b) S[AI[c],c,0]
37S>(31a*21b) S[0,b,b]
45S>(29a*23a) ~S[c,0,0]
```

Phases 1 and 2 clauses used in proof:

```
49: (23b,21a) S[x,0,x] ~defined[x]
50: (49b,25a) S[1,0,1]

51: (18a,9d) ~defined[x] S[y,z,0] ~S[AI[x],u,y] ~S[u,z,x]
52: (51a,25a) S[x,y,0] ~S[AI[1],z,x] ~S[z,y,1]
53S>(52a,45a) ~S[AI[1],x,c] ~S[x,0,1]
54: 53|{1/x} ~S[AI[1],1,c] ~S[1,0,1]
55: (54b,50a) ~S[AI[1],1,c]

56S>(35a,1e) S[x,c,y] ~S[z,1,u] ~P[u,c,y] ~P[z,c,x]
57S>(56b,21a) S[x,c,y] ~P[1,c,y] ~P[0,c,x] ~defined[1]
58S>(57d,25a) S[x,c,y] ~P[1,c,y] ~P[0,c,x]
59S>(58a,23b) ~P[1,c,x] ~P[0,c,y] S[c,y,x]
60S>(59b,15a) ~P[1,c,x] S[c,M[0,c],x] ~defined[0] ~defined[c]
61S>(60a,35a) S[c,M[0,c],c] ~defined[0] ~defined[c]
62S>(61b,26a) S[c,M[0,c],c] ~defined[c]
63S>(62b,32a) S[c,M[0,c],c]

64S>(33a,8d) S[x,y,c] ~S[x,z,0] ~S[z,c,y]
65S>(64b,18a) S[AI[x],y,c] ~S[x,c,y] ~defined[x]
66S>(65b,23a) S[AI[x],y,c] ~defined[x] ~S[c,x,y]
67S>(66b,25a) S[AI[1],x,c] ~S[c,1,x]
68S>(67a,55a) ~S[c,1,1]

69: (23b,21a) S[x,0,x] ~defined[x]
```

```

70: (69b,25a) S[1,0,1]
71S>(32a,15c) P[x,c,M[x,c]] ~defined[x]
72S>(71b,26a) P[0,c,M[0,c]]
73S>(72a,1e) S[x,M[0,c],y] ~S[z,0,u] ~P[u,c,y] ~P[z,c,x]
74S>(73a,9c) ~S[x,0,y] ~P[y,c,z] ~P[x,c,u] S[v,M[0,c],w] ~S[t,u,v] ~S[t,z,w]
75S>(74b,35a) ~S[x,0,1] ~P[x,c,y] S[z,M[0,c],u] ~S[v,y,z] ~S[v,c,u]
76S>(75b,35a) ~S[1,0,1] S[x,M[0,c],y] ~S[z,c,x] ~S[z,c,y]
77S>(76a,70a) S[x,M[0,c],y] ~S[z,c,x] ~S[z,c,y]
78S>(77b,36a) S[0,M[0,c],x] ~S[AI[c],c,x]
79S>(78b,36a) S[0,M[0,c],0]
80S>(32a,14c) defined[M[x,c]] ~defined[x]
81S>(80a,21b) ~defined[x] S[0,M[x,c],M[x,c]]
82S>(81b,8d) ~defined[x] S[y,z,M[x,c]] ~S[y,u,0] ~S[u,M[x,c],z]
83S>(82b,9b) ~defined[x] ~S[y,z,0] ~S[z,M[x,c],u] S[M[x,c],v,w] ~S[u,v,t] ~S[y,t,w]
84S>(83a,26a) ~S[x,y,0] ~S[y,M[0,c],z] S[M[0,c],u,v] ~S[z,u,w] ~S[x,w,v]
85S>(84a,36a) ~S[c,M[0,c],x] S[M[0,c],y,z] ~S[x,y,u] ~S[AI[c],u,z]
86S>(85ac) S[M[0,c],M[0,c],x] ~S[c,M[0,c],c] ~S[AI[c],c,x]
87S>(86b,63a) S[M[0,c],M[0,c],x] ~S[AI[c],c,x]
88S>(87b,36a) S[M[0,c],M[0,c],0]
89S>(31a,10c) P[MI[b],b,1] S[0,b,0]
90S>(89b,28a) P[MI[b],b,1]
91S>(90a,22b) P[b,MI[b],1]
92S>(91a,1e) S[x,1,y] ~S[z,b,u] ~P[u,MI[b],y] ~P[z,MI[b],x]
93S>(92b,37a) S[x,1,y] ~P[b,MI[b],y] ~P[0,MI[b],x]
94S>(93b,91a) S[x,1,1] ~P[0,MI[b],x]
95S>(94a,68a) ~P[0,MI[b],c]
96S>(32a,15c) P[x,c,M[x,c]] ~defined[x]
97S>(96b,30a) P[a,c,M[a,c]]
98S>(32a,14c) defined[M[x,c]] ~defined[x]
99S>(98b,30a) defined[M[a,c]]
100S>(32a,15c) P[x,c,M[x,c]] ~defined[x]
101S>(100b,26a) P[0,c,M[0,c]]
102S>(101a,2d) P[x,c,y] ~S[z,0,x] ~P[z,c,u] ~S[u,M[0,c],y]
103S>(102b,8a) ~P[x,c,y] ~P[z,c,u] ~S[u,M[0,c],y] ~S[z,v,w] ~S[v,t,0] ~S[w,t,x]
104S>(103b,101a) P[x,c,y] ~S[M[0,c],M[0,c],y] ~S[0,z,u] ~S[z,v,0] ~S[u,v,x]
105S>(104bd) P[x,c,0] ~S[0,M[0,c],y] ~S[M[0,c],M[0,c],0] ~S[y,M[0,c],x]
106S>(105c,88a) P[x,c,0] ~S[0,M[0,c],y] ~S[y,M[0,c],x]
107S>(106bc) P[0,c,0] ~S[0,M[0,c],0]
108S>(107b,79a) P[0,c,0]
109S>(27a,5a) ~P[a,x,y] ~P[x,z,u] ~P[y,z,M[a,c],MI[M[b,c]]] ... ?
110S>(109b,15a) ~P[a,x,y] ~P[y,z,M[a,c],MI[M[b,c]]] ~defined[z] ... ?
111S>(110b*15a) ~P[a,x,M[a,c]] ~defined[x] ~defined[MI[M[b,c]]] ~defined[M[a,c]] ... ?
112S>(111d,99a) ~P[a,x,M[a,c]] ~defined[x] ~defined[MI[M[b,c]]] ... ?
113S>(112b,32a) ~P[a,c,M[a,c]] ~defined[MI[M[b,c]]] ... ?
114S>(113a,97a) ~defined[MI[M[b,c]]] ... ?
115S>(31a,10c) P[MI[b],b,1] S[0,b,0]
116S>(115b,28a) P[MI[b],b,1]
117S>(116a,5b) P[MI[b],x,y] ~P[b,z,x] ~P[1,z,y]
118S>(117a,22b) ~P[b,x,y] ~P[1,x,z] P[y,MI[b],z]
119S>(118b,35a) ~P[b,c,x] P[x,MI[b],c]
120: 119|{0/x} ~P[b,c,0] P[0,MI[b],c]
121S>(120b,95a) ~P[b,c,0]
122S>(32a,15c) P[x,c,M[x,c]] ~defined[x]
123S>(122b,31a) P[b,c,M[b,c]]
124: (14a,11b) ~defined[x] ~defined[y] defined[MI[M[x,y]]] S[0,M[x,y],0]
125S>(124a,31a) ~defined[x] defined[MI[M[b,x]]] S[0,M[b,x],0]
126S>(125a,32a) defined[MI[M[b,c]]] S[0,M[b,c],0]
127S>(126a,114a) S[0,M[b,c],0] ... ?
128S>(127a,2e) P[x,y,0] ~S[z,u,x] ~P[z,y,0] ~P[u,y,M[b,c]] ... ?
129S>(128b,37a) P[b,x,0] ~P[0,x,0] ~P[b,x,M[b,c]] ... ?
130S>(129a,121a) ~P[0,c,0] ~P[b,c,M[b,c]] ... ?
131S>(130a,108a) ~P[b,c,M[b,c]] ... ?
132S>(131a,123a) [] ... ?

```

Inferences derived from proof before variable replaced by constant  
33 35 36 37 45 50 55 63 68 79 88 95 97 99 108 121 123 126

Inferences derived when variable is replaced by original constant  
109 110 112

PHASE 0: 0 s, PHASE 1: 243 s, PHASE 2: 147 s, Total Time: 390 s  
 NOD: 9240325 RES: 30564395 FAC: 6170207 T: 13 V: 32 L: 16  
 CTE: 5090745 CTH: 1949473 CTF: 5877505 CSZ: 8388608  
 UTE: 3297659 UTH: 5957315 UTF: 1406150 SBA: 0  
 BAS: 32 RED: 48 LEN: 5+84=89

Axioms:

1S>S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]  
 2S>P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]  
 3: LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]  
 4: LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]  
 5S>P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]  
 6: P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]  
 7: LE[x,y] LE[y,x] ~defined[x] ~defined[y]  
 8S S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]  
 9S S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]  
 10S>P[MI[x],x,1] S[0,x,0] ~defined[x]  
 11S>defined[MI[x]] ~defined[x] S[0,x,0]  
 12: S[0,x,y] ~LE[x,y] ~LE[y,x]  
 13: LE[x,y] ~LE[x,z] ~LE[z,y]  
 14S>defined[M[x,y]] ~defined[x] ~defined[y]  
 15S P[x,y,M[x,y]] ~defined[x] ~defined[y]  
 16: defined[A[x,y]] ~defined[x] ~defined[y]  
 17: S[x,y,A[x,y]] ~defined[x] ~defined[y]  
 18S S[AI[x],x,0] ~defined[x]  
 19S>P[1,x,x] ~defined[x]  
 20: defined[AI[x]] ~defined[x]  
 21S>S[0,x,x] ~defined[x]  
 22S>P[x,y,z] ~P[y,x,z]  
 23S S[x,y,z] ~S[y,x,z]  
 24: ~S[0,0,1]  
 25S defined[1]  
 26S defined[0]

Negated conclusion:

27S ~P[a,MI[b],M[M[a,c],MI[M[b,c]]]]  
 28S>~S[0,b,0]  
 29S ~S[0,c,0]  
 30S>defined[a]  
 31S>defined[b]  
 32S>defined[c]

-----

Phase 0 clauses used in proof:

34S>P[1,c,c]  
 36S>S[0,b,b]  
 47S>P[0,c,0]  
 48S>~P[b,c,0]  
 49S>P[b,c,M[b,c]]  
 50S>defined[MI[M[b,c]]] S[0,M[b,c],0]  
 53S>~P[c,MI[M[b,c]],MI[b]] ~defined[a] ~defined[c] ~defined[MI[M[b,c]]]  
 54S>(53c\*32a) ~P[c,MI[M[b,c]],MI[b]] ~defined[a] ~defined[MI[M[b,c]]]  
 56S>(54b\*30a) ~P[c,MI[M[b,c]],MI[b]] ~defined[MI[M[b,c]]]

Phases 1 and 2 clauses used in proof:

57S>(22b,5a) P[x,y,z] ~P[y,u,v] ~P[u,w,x] ~P[v,w,z]  
 58S>(57b,10a) P[x,MI[y],z] ~P[y,u,x] ~P[1,u,z] S[0,y,0] ~defined[y]  
 59S>(58b,49a) P[M[b,c],MI[b],x] ~P[1,c,x] S[0,b,0] ~defined[b]  
 60S>(59c,28a) P[M[b,c],MI[b],x] ~P[1,c,x] ~defined[b]  
 61S>(60c,31a) P[M[b,c],MI[b],x] ~P[1,c,x]  
 62S>(61b,34a) P[M[b,c],MI[b],c]  
 63S>(32a,14c) defined[M[x,c]] ~defined[x]  
 64S>(63b,31a) defined[M[b,c]]  
 65S>(49a,2d) P[x,c,y] ~S[z,b,x] ~P[z,c,u] ~S[u,M[b,c],y]  
 66S>(65b,21a) P[b,c,x] ~P[0,c,y] ~S[y,M[b,c],x] ~defined[b]  
 67S>(66d,31a) P[b,c,x] ~P[0,c,y] ~S[y,M[b,c],x]  
 68S>(67a,48a) ~P[0,c,x] ~S[x,M[b,c],0]  
 69S>(68a,47a) ~S[0,M[b,c],0]  
 70S>(50b,2e) defined[MI[M[b,c]]] P[x,y,0] ~S[z,u,x] ~P[z,y,0] ~P[u,y,M[b,c]]  
 71S>(70b,48a) defined[MI[M[b,c]]] ~S[x,y,b] ~P[x,c,0] ~P[y,c,M[b,c]]  
 72S>(71b,36a) defined[MI[M[b,c]]] ~P[0,c,0] ~P[b,c,M[b,c]]  
 73S>(72b,47a) defined[MI[M[b,c]]] ~P[b,c,M[b,c]]  
 74S>(73b,49a) defined[MI[M[b,c]]]  
 75S>(47a,1d) S[0,x,y] ~S[0,z,u] ~P[u,c,y] ~P[z,c,x]  
 76S>(75b,11c) S[0,x,y] ~P[0,c,y] ~P[z,c,x] defined[MI[z]] ~defined[z]  
 77S>(76b,47a) S[0,x,0] ~P[y,c,x] defined[MI[y]] ~defined[y]

```

78S>(77b,49a) S[0,M[b,c],0] defined[MI[b]] ~defined[b]
79S>(78a,69a) defined[MI[b]] ~defined[b]
80S>(79b,31a) defined[MI[b]]

81S>(56a,22a) ~defined[MI[M[b,c]]] ~P[MI[M[b,c]],c,MI[b]]
82S>(81a,74a) ~P[MI[M[b,c]],c,MI[b]]
83S>(82a,5a) ~P[MI[M[b,c]],x,y] ~P[x,z,c] ~P[y,z,MI[b]]
84S>(83a,10a) ~P[M[b,c],x,c] ~P[1,x,MI[b]] S[0,M[b,c],0] ~defined[M[b,c]]
85S>(84c,69a) ~P[M[b,c],x,c] ~P[1,x,MI[b]] ~defined[M[b,c]]
86S>(85c,64a) ~P[M[b,c],x,c] ~P[1,x,MI[b]]
87S>(86b,19a) ~P[M[b,c],MI[b],c] ~defined[MI[b]]
88S>(87a,62a) ~defined[MI[b]]
89S>(88a,80a) []

PHASE 0: 0 s, PHASE 1: 15 s, PHASE 2: 1 s, Total Time: 406 s
NOD: 576606 RES: 1579853 FAC: 391933 T: 13 V: 32 L: 16
CTE: 1147739 CTH: 324809 CTF: 0 CSZ: 8388608
UTE: 533556 UTH: 901441 UTF: 0 SBA: 0
BAS: 32 RED: 55 LEN: 9+33=42

```

# FLD057-1

Predicates: equalish defined less\_or\_equal  
Functions: multiplicative\_identity additive\_identity : . add multiplicative\_inverse multiply  
additive\_inverse  
Renamed predicates: equalish defined LE  
Renamed functions: 1 0 : . A MI M AI

C47: xvl 16.10.4 t39 W23 9S equalish[M[x,y],u] ~defined[y] ~equalish[x,z] ... ?

Axioms:

```
1: equalish[M[x,M[y,z]],M[M[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
2: equalish[A[M[x,y],M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y]
3: equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
4: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
5: LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
6: equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]
7: defined[MI[x]] ~defined[x] equalish[x,0]
8: LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]
9S>equalish[M[x,y],u] ~defined[y] ~equalish[x,z] ... ?
10: LE[x,y] ~LE[z,y] ~equalish[z,x]
11: equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]
12: equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: equalish[x,y] ~LE[x,y] ~LE[y,x]
15: defined[M[x,y]] ~defined[x] ~defined[y]
16: equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]
17: defined[A[x,y]] ~defined[x] ~defined[y]
18 >equalish[x,y] ~equalish[x,z] ~equalish[z,y]
19 >equalish[A[x,AI[x]],0] ~defined[x]
20: equalish[M[1,x],x] ~defined[x]
21: equalish[A[0,x],x] ~defined[x]
22 >defined[AI[x]] ~defined[x]
23: equalish[x,x] ~defined[x]
24 >equalish[x,y] ~equalish[y,x]
25 >defined[1]
26 >~equalish[0,1]
27: defined[0]
```

Negated conclusion:

28S ~LE[0,1]

Phase 0 clauses used in proof:

```
34 >(26a*24a) ~equalish[1,0]
36 >(25a*22b) defined[AI[1]]
39 >(25a*19b) equalish[A[1,AI[1]],0]
```

Phases 1 and 2 clauses used in proof:

```
40S>(39a,9c) equalish[M[A[1,AI[1]],x],y] ~defined[x] ... ?
41S>(40a,24b) ~defined[x] equalish[y,M[A[1,AI[1]],x]] ... ?
42S>(41a,36a) equalish[x,M[A[1,AI[1]],AI[1]]] ... ?

43S>(39a,9c) equalish[M[A[1,AI[1]],x],y] ~defined[x] ... ?
44S>(43a,18c) ~defined[x] equalish[y,z] ~equalish[y,M[A[1,AI[1]],x]] ... ?
45S>(44a,36a) equalish[x,y] ~equalish[x,M[A[1,AI[1]],AI[1]]] ... ?
46S>(45b,42a) equalish[x,y] ... ?
47S>(46a,34a) [] ... ?
```

Inferences derived from proof before weakened clause repaired

34 36

Inferences derived when weakened clause replaced with given clause

40 42 43 46

```
PHASE 0: 0 s PHASE 1: 0 s PHASE 2: 0 s Total Time: 0 s
NOD: 33 RES: 124 FAC: 4 T: 16 V: 32 L: 16
CTE: 65 CTH: 0 CTF: 0 CSZ: 16777216
UTE: 328 UTH: 112 UTF: 0 SBA: 0
BAS: 28 RED: 39 LEN: 3+8=11
```

Proof found to theorem by slave 47!

Axioms:

```
1 >equalish[M[x,M[y,z]],M[M[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
2 >equalish[A[M[x,y],M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y]
3 >equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
4 >LE[x,y] LE[y,x] ~defined[x] ~defined[y]
5 >LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
6 >equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]
```

```

7 >defined[MI[x]] ~defined[x] equalish[x,0]
8 >LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]
9S>equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[x,z]
10 >LE[x,y] ~LE[z,y] ~equalish[z,x]
11 >equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]
12 >equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]
13 >LE[x,y] ~LE[x,z] ~LE[z,y]
14 >equalish[x,y] ~LE[x,y] ~LE[y,x]
15 >defined[M[x,y]] ~defined[x] ~defined[y]
16 >equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]
17 >defined[A[x,y]] ~defined[x] ~defined[y]
18S>equalish[x,y] ~equalish[x,z] ~equalish[z,y]
19S>equalish[A[x,AI[x]],0] ~defined[x]
20 >equalish[M[1,x],x] ~defined[x]
21 >equalish[A[0,x],x] ~defined[x]
22S>defined[AI[x]] ~defined[x]
23 >equalish[x,x] ~defined[x]
24S>equalish[x,y] ~equalish[y,x]
25S>defined[1]
26S>~equalish[0,1]
27 >defined[0]

```

Negated conclusion:

```
28S>~LE[0,1]
```

-----

Phase 0 clauses used in proof:

```

29S>~equalish[1,0]
30S>defined[AI[1]]
31S>equalish[M[A[1,AI[1]],x],M[0,x]] ~defined[x]
33S>equalish[M[0,AI[1]],M[0,AI[1]]]
35S>(31b*27a) equalish[M[A[1,AI[1]],0],M[0,0]]
37S>(30a*23b) equalish[AI[1],AI[1]]
38S>(30a*22b) defined[AI[AI[1]]]
40S>(30a*20b) equalish[M[1,AI[1]],AI[1]]
41S>(30a*19b) equalish[A[AI[1],AI[AI[1]]],0]
42 >(27a*23b) equalish[0,0]

```

Phases 1 and 2 clauses used in proof:

```

43S>(25a,4c) LE[1,x] LE[x,1] ~defined[x]
44S>(43ab) LE[1,1] ~defined[1]
45S>(44b,25a) LE[1,1]

```

```

46S>(24b,21a) equalish[x,A[0,x]] ~defined[x]
47S>(46a,10c) ~defined[x] LE[A[0,x],y] ~LE[x,y]
48S>(47a,25a) LE[A[0,1],x] ~LE[1,x]
49: 48|{1/x} LE[A[0,1],1] ~LE[1,1]
50S>(49b,45a) LE[A[0,1],1]

```

```

51S>(24b,11a) equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[z,x]
52S>(51a,10c) ~defined[x] ~equalish[y,z] LE[A[y,x],u] ~LE[A[z,x],u]
53S>(52a,25a) ~equalish[x,y] LE[A[x,1],z] ~LE[A[y,1],z]
54S>(53a,41a) LE[A[A[AI[1],AI[AI[1]]],1],x] ~LE[A[0,1],x]
55: 54|{1/x} LE[A[A[AI[1],AI[AI[1]]],1],1] ~LE[A[0,1],1]
56S>(55b,50a) LE[A[A[AI[1],AI[AI[1]]],1],1]

```

```
57: (27a,20b) equalish[M[1,0],0]
```

```

58S>(38a,17c) defined[A[x,AI[AI[1]]]] ~defined[x]
59S>(58a,16b) ~defined[x] equalish[A[A[x,AI[AI[1]]],y],A[y,A[x,AI[AI[1]]]] ~defined[y]
60S>(59b,10c) ~defined[x] ~defined[y] LE[A[y,A[x,AI[AI[1]]],z] ~LE[A[A[x,AI[AI[1]]],y],z]
61S>(60b,25a) ~defined[x] LE[A[1,A[x,AI[AI[1]]],y] ~LE[A[A[x,AI[AI[1]]],1],y]
62S>(61a,30a) LE[A[1,A[AI[1],AI[AI[1]]],x] ~LE[A[A[AI[1],AI[AI[1]]],1],x]
63: 62|{1/x} LE[A[1,A[AI[1],AI[AI[1]]],1] ~LE[A[A[AI[1],AI[AI[1]]],1],1]
64S>(63b,56a) LE[A[1,A[AI[1],AI[AI[1]]],1]

```

```

65S>(21a,9c) ~defined[x] equalish[M[A[0,x],y],M[x,y]] ~defined[y]
66S>(65b,18b) ~defined[x] ~defined[y] equalish[M[A[0,x],y],z] ~equalish[M[x,y],z]
67S>(66a,25a) ~defined[x] equalish[M[A[0,1],x],y] ~equalish[M[1,x],y]
68S>(67a,27a) equalish[M[A[0,1],0],x] ~equalish[M[1,0],x]
69: 68|{0/x} equalish[M[A[0,1],0],0] ~equalish[M[1,0],0]
70: (69b,57a) equalish[M[A[0,1],0],0]

```

```
71: (27a,20b) equalish[M[1,0],0]
```

```

72S>(30a,15b) defined[M[AI[1],x]] ~defined[x]
73S>(72b,27a) defined[M[AI[1],0]]

```

```

74S>(35a,18c) equalish[x,M[0,0]] ~equalish[x,M[A[1,AI[1]],0]]
75S>(74b,2a) equalish[A[M[1,0],M[AI[1],0]],M[0,0]] ~defined[AI[1]] ~defined[0]
76S>(75b,25a) equalish[A[M[1,0],M[AI[1],0]],M[0,0]] ~defined[AI[1]] ~defined[0]
77S>(76b,30a) equalish[A[M[1,0],M[AI[1],0]],M[0,0]] ~defined[0]

```

```

78S>(77b,27a) equalish[A[M[1,0],M[AI[1],0]],M[0,0]]
79S>(78a,24b) equalish[M[0,0],A[M[1,0],M[AI[1],0]]]
80S>(79a,18b) equalish[M[0,0],x] ~equalish[A[M[1,0],M[AI[1],0]],x]
81S>(80b,11a) equalish[M[0,0],A[x,M[AI[1],0]]] ~defined[M[AI[1],0]] ~equalish[M[1,0],x]
82S>(81b,73a) equalish[M[0,0],A[x,M[AI[1],0]]] ~equalish[M[1,0],x]
83: 82|{0/x} equalish[M[0,0],A[0,M[AI[1],0]]] ~equalish[M[1,0],0]
84: (83b,71a) equalish[M[0,0],A[0,M[AI[1],0]]]

85S>(27a,9b) equalish[M[x,0],M[y,0]] ~equalish[x,y]
86S>(85b,16a) equalish[M[A[x,y],0],M[A[y,x],0]] ~defined[x] ~defined[y]
87S>(86a,18b) ~defined[x] ~defined[y] equalish[M[A[x,y],0],z] ~equalish[M[A[y,x],0],z]
88S>(87a,25a) ~defined[x] equalish[M[A[1,x],0],y] ~equalish[M[A[x,1],0],y]
89S>(88a,27a) equalish[M[A[1,0],0],x] ~equalish[M[A[0,1],0],x]
90: 89|{0/x} equalish[M[A[1,0],0],0] ~equalish[M[A[0,1],0],0]
91: (90b,70a) equalish[M[A[1,0],0],0]

92S>(38a,3d) equalish[A[x,A[y,AI[AI[1]]],A[A[x,y],AI[AI[1]]]] ~defined[x] ~defined[y]
93S>(92b,25a) equalish[A[1,A[x,AI[AI[1]]],A[A[1,x],AI[AI[1]]]] ~defined[x]
94S>(93a,10c) ~defined[x] LE[A[A[1,x],AI[AI[1]]],y] ~LE[A[1,A[x,AI[AI[1]]],y]
95S>(94a,30a) LE[A[A[1,AI[1]],AI[AI[1]]],x] ~LE[A[1,A[AI[1]],AI[AI[1]]],x]
96: 95|{1/x} LE[A[A[1,AI[1]],AI[AI[1]]],1] ~LE[A[1,A[AI[1]],AI[AI[1]]],1]
97S>(96b,64a) LE[A[A[1,AI[1]],AI[AI[1]]],1]

98S>(28a,10a) ~LE[x,1] ~equalish[x,0]
99S>(98b,20a) ~LE[M[1,0],1] ~defined[0]
100S>(99b,27a) ~LE[M[1,0],1]

101S>(30a,15b) defined[M[AI[1],x]] ~defined[x]
102S>(101a,21b) ~defined[x] equalish[A[0,M[AI[1],x]],M[AI[1],x]]
103S>(102b,18c) ~defined[x] equalish[y,M[AI[1],x]] ~equalish[y,A[0,M[AI[1],x]]]
104S>(103b,24b) ~defined[x] ~equalish[y,A[0,M[AI[1],x]]] equalish[M[AI[1],x],y]
105S>(104b,18a) ~defined[x] equalish[M[AI[1],x],y] ~equalish[y,z] ~equalish[z,A[0,M[AI[1],x]]]
106S>(105a,27a) equalish[M[AI[1],0],x] ~equalish[x,y] ~equalish[y,A[0,M[AI[1],0]]]
107S>(106b,35a) equalish[M[AI[1],0],M[A[1,AI[1]],0]] ~equalish[M[0,0],A[0,M[AI[1],0]]]
108S>(107b,84a) equalish[M[AI[1],0],M[A[1,AI[1]],0]]

109: (27a,2d) equalish[A[M[x,0],M[y,0]],M[A[x,y],0]] ~defined[x] ~defined[y]
110S>(109b,25a) equalish[A[M[1,0],M[x,0]],M[A[1,x],0]] ~defined[x]
111S>(110a,18b) ~defined[x] equalish[A[M[1,0],M[x,0]],y] ~equalish[M[A[1,x],0],y]
112S>(111b,24b) ~defined[x] ~equalish[M[A[1,x],0],y] equalish[y,A[M[1,0],M[x,0]]]
113S>(112a,27a) ~equalish[M[A[1,0],0],x] equalish[x,A[M[1,0],M[0,0]]]
114S>(113a,91a) equalish[0,A[M[1,0],M[0,0]]]

115S>(38a,11b) equalish[A[x,AI[AI[1]]],A[y,AI[AI[1]]]] ~equalish[x,y]
116S>(115b,19a) equalish[A[A[x,AI[x]],AI[AI[1]]],A[0,AI[AI[1]]]] ~defined[x]
117S>(116b,25a) equalish[A[A[1,AI[1]],AI[AI[1]]],A[0,AI[AI[1]]]]
118S>(117a,10c) LE[A[0,AI[AI[1]]],x] ~LE[A[A[1,AI[1]],AI[AI[1]]],x]
119: 118|{1/x} LE[A[0,AI[AI[1]]],1] ~LE[A[A[1,AI[1]],AI[AI[1]]],1]
120S>(119b,97a) LE[A[0,AI[AI[1]]],1]

121: (27a,15c) defined[M[x,0]] ~defined[x]
122: (121a,11b) ~defined[x] equalish[A[y,M[x,0]],A[z,M[x,0]]] ~equalish[y,z]
123S>(122b,18c) ~defined[x] ~equalish[y,z] equalish[u,A[z,M[x,0]]] ~equalish[u,A[y,M[x,0]]]
124S>(123b,20a) ~defined[x] equalish[y,A[z,M[x,0]]] ~equalish[y,A[M[1,z],M[x,0]]] ~defined[z]
125S>(124b,24b) ~defined[x] ~equalish[y,A[M[1,z],M[x,0]]] ~defined[z] equalish[A[z,M[x,0]],y]
126S>(125a,27a) ~equalish[x,A[M[1,y],M[0,0]]] ~defined[y] equalish[A[y,M[0,0]],x]
127S>(126b,27a) ~equalish[x,A[M[1,0],M[0,0]]] equalish[A[0,M[0,0]],x]
128S>(127a,114a) equalish[A[0,M[0,0]],0]

129S>(27a,9b) equalish[M[x,0],M[y,0]] ~equalish[x,y]
130S>(129a,18b) ~equalish[x,y] equalish[M[x,0],z] ~equalish[M[y,0],z]
131S>(130a,24a) equalish[M[x,0],y] ~equalish[M[z,0],y] ~equalish[z,x]
132S>(131b,18a) equalish[M[x,0],y] ~equalish[z,x] ~equalish[M[z,0],u] ~equalish[u,y]
133S>(132a,24b) ~equalish[x,y] ~equalish[M[x,0],z] ~equalish[z,u] equalish[u,M[y,0]]
134S>(133a,37a) ~equalish[M[AI[1],0],x] ~equalish[x,y] equalish[y,M[AI[1],0]]
135S>(134b,35a) ~equalish[M[AI[1],0],M[A[1,AI[1]],0]] equalish[M[0,0],M[AI[1],0]]
136S>(135a,108a) equalish[M[0,0],M[AI[1],0]]

137S>(28a,10a) ~LE[x,1] ~equalish[x,0]
138S>(137b,19a) ~LE[A[x,AI[x]],1] ~defined[x]
139S>(138a,13a) ~defined[x] ~LE[A[x,AI[x]],y] ~LE[y,1]
140S>(139b,8a) ~defined[x] ~LE[A[y,AI[x]],1] ~defined[AI[x]] ~LE[x,y]
141S>(140a,30a) ~LE[A[x,AI[AI[1]]],1] ~defined[AI[AI[1]]] ~LE[AI[1],x]
142S>(141a,120a) ~defined[AI[AI[1]]] ~LE[AI[1],0]
143S>(142a,38a) ~LE[AI[1],0]

144: (27a,15b) defined[M[0,x]] ~defined[x]
145: (144a,21b) ~defined[x] equalish[A[0,M[0,x]],M[0,x]]
146S>(145b,24b) ~defined[x] equalish[M[0,x],A[0,M[0,x]]]
147S>(146b,18b) ~defined[x] equalish[M[0,x],y] ~equalish[A[0,M[0,x]],y]
148S>(147b,24b) ~defined[x] ~equalish[A[0,M[0,x]],y] equalish[y,M[0,x]]

```

```

149S>(148a,27a) ~equalish[A[0,M[0,0]],x] equalish[x,M[0,0]]
150S>(149a,128a) equalish[0,M[0,0]]

151: (27a,12c) equalish[M[x,0],M[0,x]] ~defined[x]
152S>(151a,18c) ~defined[x] equalish[y,M[0,x]] ~equalish[y,M[x,0]]
153S>(152b,18b) ~defined[x] ~equalish[y,M[x,0]] equalish[y,z] ~equalish[M[0,x],z]
154S>(153b*31a) ~defined[0] equalish[M[A[1,AI[1]],0],x] ~equalish[M[0,0],x]
155S>(154a,27a) equalish[M[A[1,AI[1]],0],x] ~equalish[M[0,0],x]
156S>(155a,152c) ~equalish[M[0,0],M[x,0]] ~defined[x] equalish[M[A[1,AI[1]],0],M[0,x]]
157S>(156b,30a) ~equalish[M[0,0],M[AI[1],0]] equalish[M[A[1,AI[1]],0],M[0,AI[1]]]
158S>(157a,136a) equalish[M[A[1,AI[1]],0],M[0,AI[1]]]

159: (27a,4d) LE[x,0] LE[0,x] ~defined[x]
160: (159b,10b) LE[x,0] ~defined[x] LE[y,x] ~equalish[0,y]
161S>(160b,30a) LE[AI[1],0] LE[x,AI[1]] ~equalish[0,x]
162S>(161a,143a) LE[x,AI[1]] ~equalish[0,x]
163S>(162b,42a) LE[0,AI[1]]

164S>(24b,21a) equalish[x,A[0,x]] ~defined[x]
165S>(164a,18b) ~defined[x] equalish[x,y] ~equalish[A[0,x],y]
166S>(165b,29a) ~defined[1] ~equalish[A[0,1],0]
167S>(166a,25a) ~equalish[A[0,1],0]

168S>(25a,4c) LE[1,x] LE[x,1] ~defined[x]
169S>(168ab) LE[1,1] ~defined[1]
170S>(169b,25a) LE[1,1]

171S>(24b,18a) equalish[x,y] ~equalish[y,z] ~equalish[z,x]
172S>(171b,35a) equalish[x,M[A[1,AI[1]],0]] ~equalish[M[0,0],x]
173S>(172b*12a) equalish[M[0,0],M[A[1,AI[1]],0]] ~defined[0]
174S>(173b,27a) equalish[M[0,0],M[A[1,AI[1]],0]]
175S>(174a,171c) equalish[M[A[1,AI[1]],0],x] ~equalish[x,M[0,0]]
176S>(175a,24b) ~equalish[x,M[0,0]] equalish[x,M[A[1,AI[1]],0]]
177S>(176a,150a) equalish[0,M[A[1,AI[1]],0]]

178S>(24b,18a) equalish[x,y] ~equalish[y,z] ~equalish[z,x]
179S>(178b,33a) equalish[x,M[0,AI[1]]] ~equalish[M[0,AI[1]],x]
180S>(179b,18a) equalish[x,M[0,AI[1]]] ~equalish[M[0,AI[1]],y] ~equalish[y,x]
181S>(180b,24a) equalish[x,M[0,AI[1]]] ~equalish[y,x] ~equalish[y,M[0,AI[1]]]
182S>(181b,35a) equalish[M[0,0],M[0,AI[1]]] ~equalish[M[A[1,AI[1]],0],M[0,AI[1]]]
183S>(182b,158a) equalish[M[0,0],M[0,AI[1]]]

184S>(24b,11a) equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[z,x]
185S>(184a,18b) ~defined[x] ~equalish[y,z] equalish[A[z,x],u] ~equalish[A[y,x],u]
186S>(185a,25a) ~equalish[x,y] equalish[A[y,1],z] ~equalish[A[x,1],z]
187S>(186a,41a) equalish[A[0,1],x] ~equalish[A[A[AI[1],AI[AI[1]]],1],x]
188S>(187a,167a) ~equalish[A[A[AI[1],AI[AI[1]]],1],0]

189S>(24b,21a) equalish[x,A[0,x]] ~defined[x]
190S>(189a,10c) ~defined[x] LE[A[0,x],y] ~LE[x,y]
191S>(190a,25a) LE[A[0,1],x] ~LE[1,x]
192: 191|{1/x} LE[A[0,1],1] ~LE[1,1]
193S>(192b,170a) LE[A[0,1],1]

194S>(24a,18c) ~equalish[x,y] equalish[z,x] ~equalish[z,y]
195S>(194b,18c) ~equalish[x,y] ~equalish[z,y] equalish[u,x] ~equalish[u,z]
196S>(195b,35a) ~equalish[x,M[0,0]] equalish[y,x] ~equalish[y,M[A[1,AI[1]],0]]
197S>(196a*12a) equalish[x,M[0,0]] ~equalish[x,M[A[1,AI[1]],0]] ~defined[0]
198S>(197c,27a) equalish[x,M[0,0]] ~equalish[x,M[A[1,AI[1]],0]]
199S>(198a,195d) ~equalish[x,M[A[1,AI[1]],0]] ~equalish[y,z] ~equalish[M[0,0],z] equalish[x,y]
200S>(199b,33a) ~equalish[x,M[A[1,AI[1]],0]] ~equalish[M[0,0],M[0,AI[1]]]
equalish[x,M[0,AI[1]]]
201S>(200a,177a) ~equalish[M[0,0],M[0,AI[1]]] equalish[0,M[0,AI[1]]]
202S>(201a,183a) equalish[0,M[0,AI[1]]]

203S>(38a,17c) defined[A[x,AI[AI[1]]]] ~defined[x]
204S>(203a,16b) ~defined[x] equalish[A[A[x,AI[AI[1]]],y],A[y,A[x,AI[AI[1]]]] ~defined[y]
205S>(204b,18b) ~defined[x] ~defined[y] equalish[A[A[x,AI[AI[1]]],y],z]
~equalish[A[y,A[x,AI[AI[1]]],z]
206S>(205b,25a) ~defined[x] equalish[A[A[x,AI[AI[1]]],1],y] ~equalish[A[1,A[x,AI[AI[1]]],y]
207S>(206a,30a) equalish[A[A[AI[1],AI[AI[1]]],1],x] ~equalish[A[1,A[AI[1],AI[AI[1]]],x]
208S>(207a,188a) ~equalish[A[1,A[AI[1],AI[AI[1]]],0]

209S>(38a,3d) equalish[A[x,A[y,AI[AI[1]]],A[A[x,y],AI[AI[1]]]] ~defined[x] ~defined[y]
210S>(209b,25a) equalish[A[1,A[x,AI[AI[1]]],A[A[1,x],AI[AI[1]]]] ~defined[x]
211S>(210a,18b) ~defined[x] equalish[A[1,A[x,AI[AI[1]]],y] ~equalish[A[A[1,x],AI[AI[1]]],y]
212S>(211a,30a) equalish[A[1,A[AI[1],AI[AI[1]]],x] ~equalish[A[A[1,AI[1],AI[AI[1]]],x]
213S>(212a,208a) ~equalish[A[A[1,AI[1],AI[AI[1]]],0]

214S>(19a,11c) ~defined[x] equalish[A[A[x,AI[x]],y],A[0,y]] ~defined[y]
215S>(214a,25a) equalish[A[A[1,AI[1]],x],A[0,x]] ~defined[x]

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216S>(215a,18b) ~defined[x] equalish[A[A[1,AI[1]],x],y] ~equalish[A[0,x],y]  
217S>(216a,38a) equalish[A[A[1,AI[1]],AI[AI[1]],x] ~equalish[A[0,AI[AI[1]]],x]  
218S>(217a,213a) ~equalish[A[0,AI[AI[1]]],0]

219S>(18b,11a) equalish[A[x,y],z] ~equalish[A[u,y],z] ~defined[y] ~equalish[x,u]  
220S>(219b,41a) equalish[A[x,AI[AI[1]]],0] ~defined[AI[AI[1]]] ~equalish[x,AI[1]]  
221S>(220b,38a) equalish[A[x,AI[AI[1]]],0] ~equalish[x,AI[1]]  
222S>(221b,24a) equalish[A[x,AI[AI[1]]],0] ~equalish[AI[1],x]  
223S>(222a,218a) ~equalish[AI[1],0]

224S>(30a,6b) equalish[M[AI[1],MI[AI[1]]],1] equalish[AI[1],0]  
225S>(224b,223a) equalish[M[AI[1],MI[AI[1]]],1]  
226S>(225a,24b) equalish[1,M[AI[1],MI[AI[1]]]  
227S>(226a,10c) LE[M[AI[1],MI[AI[1]]],x] ~LE[1,x]  
228S>(227b,4b) LE[M[AI[1],MI[AI[1]]],x] LE[x,1] ~defined[x] ~defined[1]  
229S>(228d,25a) LE[M[AI[1],MI[AI[1]]],x] LE[x,1] ~defined[x]  
230S>(229b,28a) LE[M[AI[1],MI[AI[1]]],0] ~defined[0]  
231S>(230b,27a) LE[M[AI[1],MI[AI[1]]],0]

232S>(19a,18c) ~defined[x] equalish[y,0] ~equalish[y,A[x,AI[x]]]  
233S>(232a,30a) equalish[x,0] ~equalish[x,A[AI[1],AI[AI[1]]]  
234S>(233b,11a) equalish[A[x,AI[AI[1]]],0] ~defined[AI[AI[1]]] ~equalish[x,AI[1]]  
235S>(234a,218a) ~defined[AI[AI[1]]] ~equalish[0,AI[1]]  
236S>(235a,38a) ~equalish[0,AI[1]]

237S>(41a,18c) equalish[x,0] ~equalish[x,A[AI[1],AI[AI[1]]]  
238S>(237b,11a) equalish[A[x,AI[AI[1]]],0] ~defined[AI[AI[1]]] ~equalish[x,AI[1]]  
239S>(238b,38a) equalish[A[x,AI[AI[1]]],0] ~equalish[x,AI[1]]  
240S>(239b,18a) equalish[A[x,AI[AI[1]]],0] ~equalish[x,y] ~equalish[y,AI[1]]  
241S>(240b,24a) equalish[A[x,AI[AI[1]]],0] ~equalish[y,AI[1]] ~equalish[y,x]  
242S>(241b,40a) equalish[A[x,AI[AI[1]]],0] ~equalish[M[1,AI[1]],x]  
243S>(242a,218a) ~equalish[M[1,AI[1]],0]

244S>(40a,18b) equalish[M[1,AI[1]],x] ~equalish[AI[1],x]  
245S>(244b,6c) equalish[M[1,AI[1]],0] equalish[M[AI[1],MI[AI[1]]],1] ~defined[AI[1]]  
246S>(245a,243a) equalish[M[AI[1],MI[AI[1]]],1] ~defined[AI[1]]  
247S>(246b,30a) equalish[M[AI[1],MI[AI[1]]],1]  
248S>(247a,9c) equalish[M[M[AI[1],MI[AI[1]]],x],M[1,x] ~defined[x]  
249S>(248a,10c) ~defined[x] LE[M[1,x],y] ~LE[M[M[AI[1],MI[AI[1]]],x],y]  
250S>(249a,27a) LE[M[1,0],x] ~LE[M[M[AI[1],MI[AI[1]]],0],x]  
251S>(250a,100a) ~LE[M[M[AI[1],MI[AI[1]]],0],1]

252S>(40a,18b) equalish[M[1,AI[1]],x] ~equalish[AI[1],x]  
253S>(252b,7c) equalish[M[1,AI[1]],0] defined[MI[AI[1]]] ~defined[AI[1]]  
254S>(253a,243a) defined[MI[AI[1]]] ~defined[AI[1]]  
255S>(254b,30a) defined[MI[AI[1]]]  
256S>(255a,1c) equalish[M[x,M[MI[AI[1]],y]],M[M[x,MI[AI[1]]],y] ~defined[x] ~defined[y]  
257S>(256a,10c) ~defined[x] ~defined[y] LE[M[M[x,MI[AI[1]]],y],z] ~LE[M[x,M[MI[AI[1]],y]],z]  
258S>(257b,27a) ~defined[x] LE[M[M[x,MI[AI[1]]],0],y] ~LE[M[x,M[MI[AI[1]],0]],y]  
259S>(258a,30a) LE[M[M[AI[1],MI[AI[1]]],0],x] ~LE[M[AI[1],M[MI[AI[1]],0]],x]  
260S>(259a,251a) ~LE[M[AI[1],M[MI[AI[1]],0]],1]

261S>(30a,7b) defined[MI[AI[1]]] equalish[AI[1],0]  
262S>(261b,223a) defined[MI[AI[1]]]  
263S>(262a,15b) defined[M[MI[AI[1]],x] ~defined[x]  
264S>(263a,12b) ~defined[x] equalish[M[M[MI[AI[1]],x],y],M[y,M[MI[AI[1]],x]] ~defined[y]  
265S>(264b,10c) ~defined[x] ~defined[y] LE[M[y,M[MI[AI[1]],x]],z] ~LE[M[M[MI[AI[1]],x],y],z]  
266S>(265a,27a) ~defined[x] LE[M[x,M[MI[AI[1]],0]],y] ~LE[M[M[MI[AI[1]],0],x],y]  
267S>(266a,30a) LE[M[AI[1],M[MI[AI[1]],0]],x] ~LE[M[M[MI[AI[1]],0],AI[1]],x]  
268S>(267a,260a) ~LE[M[M[MI[AI[1]],0],AI[1]],1]

269S>(40a,18b) equalish[M[1,AI[1]],x] ~equalish[AI[1],x]  
270S>(269b,6c) equalish[M[1,AI[1]],0] equalish[M[AI[1],MI[AI[1]]],1] ~defined[AI[1]]  
271S>(270a,243a) equalish[M[AI[1],MI[AI[1]]],1] ~defined[AI[1]]  
272S>(271b,30a) equalish[M[AI[1],MI[AI[1]]],1]  
273S>(272a,18c) equalish[x,1] ~equalish[x,M[AI[1],MI[AI[1]]]  
274S>(273b,14a) equalish[x,1] ~LE[x,M[AI[1],MI[AI[1]]] ~LE[M[AI[1],MI[AI[1]]],x]  
275S>(274b,5a) equalish[0,1] ~LE[M[AI[1],MI[AI[1]]],0] ~LE[0,AI[1]] ~LE[0,MI[AI[1]]]  
276S>(275a,26a) ~LE[M[AI[1],MI[AI[1]]],0] ~LE[0,AI[1]] ~LE[0,MI[AI[1]]]  
277S>(276a,231a) ~LE[0,AI[1]] ~LE[0,MI[AI[1]]]  
278S>(277a,163a) ~LE[0,MI[AI[1]]]

279S>(30a,7b) defined[MI[AI[1]]] equalish[AI[1],0]  
280S>(279b,223a) defined[MI[AI[1]]]  
281S>(280a,1b) equalish[M[MI[AI[1]],M[x,y]],M[M[MI[AI[1]],x],y] ~defined[x] ~defined[y]  
282S>(281b,27a) equalish[M[MI[AI[1]],M[0,x]],M[M[MI[AI[1]],0],x] ~defined[x]  
283S>(282a,10c) ~defined[x] LE[M[M[MI[AI[1]],0],x],y] ~LE[M[MI[AI[1]],M[0,x]],y]  
284S>(283a,30a) LE[M[M[MI[AI[1]],0],AI[1]],x] ~LE[M[MI[AI[1]],M[0,AI[1]]],x]  
285S>(284a,268a) ~LE[M[MI[AI[1]],M[0,AI[1]]],1]

286S>(30a,7b) defined[MI[AI[1]]] equalish[AI[1],0]

```

287S>(286b,223a) defined[MI[AI[1]]]
288S>(287a,4d) LE[x,MI[AI[1]]] LE[MI[AI[1]],x] ~defined[x]
289S>(288b,8c) LE[x,MI[AI[1]]] ~defined[x] LE[A[MI[AI[1]],y],A[x,y]] ~defined[y]
290S>(289b,27a) LE[0,MI[AI[1]]] LE[A[MI[AI[1]],x],A[0,x]] ~defined[x]
291S>(290a,278a) LE[A[MI[AI[1]],x],A[0,x]] ~defined[x]
292S>(291a,13b) ~defined[x] LE[A[MI[AI[1]],x],y] ~LE[A[0,x],y]
293S>(292a,25a) LE[A[MI[AI[1]],1],x] ~LE[A[0,1],x]
294: 293{|/x} LE[A[MI[AI[1]],1],1] ~LE[A[0,1],1]
295S>(294b,193a) LE[A[MI[AI[1]],1],1]

296S>(30a,7b) defined[MI[AI[1]]] equalish[AI[1],0]
297S>(296b,223a) defined[MI[AI[1]]]
298S>(297a,12c) equalish[M[x,MI[AI[1]]],M[MI[AI[1]],x]] ~defined[x]
299S>(298b,15a) equalish[M[x,y],MI[AI[1]]],M[MI[AI[1]],M[x,y]] ~defined[x] ~defined[y]
300S>(299b,27a) equalish[M[M[0,x],MI[AI[1]]],M[MI[AI[1]],M[0,x]]] ~defined[x]
301S>(300a,10c) ~defined[x] LE[M[MI[AI[1]],M[0,x]],y] ~LE[M[M[0,x],MI[AI[1]]],y]
302S>(301a,30a) LE[M[MI[AI[1]],M[0,MI[1]]],x] ~LE[M[M[0,MI[1]],MI[AI[1]]],x]
303S>(302a,285a) ~LE[M[M[0,MI[1]],MI[AI[1]]],1]

304S>(41a,18b) equalish[A[AI[1],AI[AI[1]]],x] ~equalish[0,x]
305S>(304b,18a) equalish[A[AI[1],AI[AI[1]]],x] ~equalish[0,y] ~equalish[y,x]
306S>(305a,9c) ~equalish[0,x] ~equalish[x,y] equalish[M[A[AI[1],AI[AI[1]]],z],M[y,z]] ~defined[z]
307S>(306b,33a) ~equalish[0,M[0,AI[1]]] equalish[M[A[AI[1],AI[AI[1]]],x],M[M[0,AI[1]],x]]
~defined[x]
308S>(307a,202a) equalish[M[A[AI[1],AI[AI[1]]],x],M[M[0,AI[1]],x]] ~defined[x]
309S>(308b,7a) equalish[M[A[AI[1],AI[AI[1]]],MI[x]],M[M[0,AI[1]],MI[x]]] ~defined[x]
equalish[x,0]
310S>(309a,10c) ~defined[x] equalish[x,0] LE[M[M[0,AI[1]],MI[x]],y]
~LE[M[A[AI[1],AI[AI[1]]],MI[x]],y]
311S>(310a,30a) equalish[AI[1],0] LE[M[M[0,AI[1]],MI[AI[1]]],x]
~LE[M[A[AI[1],AI[AI[1]]],MI[AI[1]]],x]
312S>(311a,223a) LE[M[M[0,AI[1]],MI[AI[1]]],x] ~LE[M[A[AI[1],AI[AI[1]]],MI[AI[1]]],x]
313S>(312a,303a) ~LE[M[A[AI[1],AI[AI[1]]],MI[AI[1]]],1]

314S>(25a,16c) equalish[A[x,1],A[1,x]] ~defined[x]
315S>(314b,7a) equalish[A[MI[x],1],A[1,MI[x]]] ~defined[x] equalish[x,0]
316S>(315b,22a) equalish[A[MI[AI[x]],1],A[1,MI[AI[x]]] equalish[AI[x],0] ~defined[x]
317S>(316b,24b) equalish[A[MI[AI[x]],1],A[1,MI[AI[x]]] ~defined[x] equalish[0,AI[x]]
318S>(317a,10c) ~defined[x] equalish[0,AI[x]] LE[A[1,MI[AI[x]],y] ~LE[A[MI[AI[x]],1],y]
319S>(318a,25a) equalish[0,AI[1]] LE[A[1,MI[AI[1]],x] ~LE[A[MI[AI[1]],1],x]
320S>(319a,236a) LE[A[1,MI[AI[1]],x] ~LE[A[MI[AI[1]],1],x]
321: 320{|/x} LE[A[1,MI[AI[1]],1] ~LE[A[MI[AI[1]],1],1]
322S>(321b,295a) LE[A[1,MI[AI[1]],1]

323S>(40a,18b) equalish[M[1,AI[1]],x] ~equalish[AI[1],x]
324S>(323b,7c) equalish[M[1,AI[1]],0] defined[MI[AI[1]]] ~defined[AI[1]]
325S>(324a,243a) defined[MI[AI[1]]] ~defined[AI[1]]
326S>(325b,30a) defined[MI[AI[1]]]
327S>(326a,9b) equalish[M[x,MI[AI[1]]],M[y,MI[AI[1]]] ~equalish[x,y]
328S>(327b,19a) equalish[M[Ax,AI[x]],MI[AI[1]],M[0,MI[AI[1]]] ~defined[x]
329S>(328a,24b) ~defined[x] equalish[M[0,MI[AI[1]]],M[Ax,AI[x]],MI[AI[1]]]
330S>(329b,10c) ~defined[x] LE[M[Ax,AI[x]],MI[AI[1]],y] ~LE[M[0,MI[AI[1]],y]
331S>(330a,30a) LE[M[Ax,AI[x]],MI[AI[1]],x] ~LE[M[0,MI[AI[1]],x]
332S>(331a,313a) ~LE[M[0,MI[AI[1]],1]

333S>(40a,18b) equalish[M[1,AI[1]],x] ~equalish[AI[1],x]
334S>(333b,6c) equalish[M[1,AI[1]],0] equalish[M[AI[1],MI[AI[1]]],1] ~defined[AI[1]]
335S>(334a,243a) equalish[M[AI[1],MI[AI[1]]],1] ~defined[AI[1]]
336S>(335b,30a) equalish[M[AI[1],MI[AI[1]]],1]
337S>(336a,24b) equalish[1,M[AI[1],MI[AI[1]]]
338S>(337a,11c) equalish[A[1,x],A[M[AI[1],MI[AI[1]]],x]] ~defined[x]
339S>(338b,7a) equalish[A[1,MI[x]],A[M[AI[1],MI[AI[1]]],MI[x]] ~defined[x] equalish[x,0]
340S>(339a,10c) ~defined[x] equalish[x,0] LE[A[M[AI[1],MI[AI[1]]],MI[x]],y] ~LE[A[1,MI[x]],y]
341S>(340a,30a) equalish[AI[1],0] LE[A[M[AI[1],MI[AI[1]]],MI[AI[1]],x] ~LE[A[1,MI[AI[1]],x]
342S>(341a,223a) LE[A[M[AI[1],MI[AI[1]]],MI[AI[1]],x] ~LE[A[1,MI[AI[1]],x]
343: 342{|/x} LE[A[M[AI[1],MI[AI[1]]],MI[AI[1]],1] ~LE[A[1,MI[AI[1]],1]
344S>(343b,322a) LE[A[M[AI[1],MI[AI[1]]],MI[AI[1]],1]

345S>(31a,18c) ~defined[x] equalish[y,M[0,x]] ~equalish[y,M[A[1,AI[1]],x]]
346S>(345b,24b) ~defined[x] ~equalish[y,M[A[1,AI[1]],x]] equalish[M[0,x],y]
347S>(346b*2a) ~defined[x] equalish[M[0,x],A[M[1,x],M[AI[1],x]]] ~defined[1] ~defined[AI[1]]
348S>(347c,25a) ~defined[x] equalish[M[0,x],A[M[1,x],M[AI[1],x]]] ~defined[AI[1]]
349S>(348c,30a) ~defined[x] equalish[M[0,x],A[M[1,x],M[AI[1],x]]]
350S>(349b,24b) ~defined[x] equalish[A[M[1,x],M[AI[1],x]],M[0,x]]
351S>(350a,7a) equalish[A[M[1,MI[x]],M[AI[1],MI[x]],M[0,MI[x]]] ~defined[x] equalish[x,0]
352S>(351a,10c) ~defined[x] equalish[x,0] LE[M[0,MI[x]],y] ~LE[A[M[1,MI[x]],M[AI[1],MI[x]],y]
353S>(352a,30a) equalish[AI[1],0] LE[M[0,MI[AI[1]],x]
~LE[A[M[1,MI[AI[1]]],M[AI[1],MI[AI[1]]],x]
354S>(353a,223a) LE[M[0,MI[AI[1]],x] ~LE[A[M[1,MI[AI[1]]],M[AI[1],MI[AI[1]]],x]
355S>(354a,332a) ~LE[A[M[1,MI[AI[1]]],M[AI[1],MI[AI[1]]],1]

```

```

356S>(40a,18b) equalish[M[1,AI[1]],x] ~equalish[AI[1],x]
357S>(356b,7c) equalish[M[1,AI[1]],0] defined[MI[AI[1]]] ~defined[AI[1]]
358S>(357a,243a) defined[MI[AI[1]]] ~defined[AI[1]]
359S>(358b,30a) defined[MI[AI[1]]]
360S>(359a,15c) defined[M[x,MI[AI[1]]] ~defined[x]
361S>(360b,30a) defined[M[AI[1],MI[AI[1]]]
362S>(361a,16b) equalish[A[M[AI[1],MI[AI[1]]],x],A[x,M[AI[1],MI[AI[1]]]] ~defined[x]
363S>(362a,10c) ~defined[x] LE[A[x,M[AI[1],MI[AI[1]]],y] ~LE[A[M[AI[1],MI[AI[1]]],x],y]
364S>(363a,359a) LE[A[M[AI[1],M[AI[1],MI[AI[1]]],x] ~LE[A[M[AI[1],MI[AI[1]]],MI[AI[1]]],x]
365: 364 {1/x} LE[A[M[AI[1]],M[AI[1],MI[AI[1]]],1] ~LE[A[M[AI[1],MI[AI[1]]],MI[AI[1]],1]
366S>(365b,344a) LE[A[M[AI[1]],M[AI[1],MI[AI[1]]],1]

367S>(30a,7b) defined[MI[AI[1]]] equalish[AI[1],0]
368S>(367b,223a) defined[MI[AI[1]]]
369S>(368a,15c) defined[M[x,MI[AI[1]]] ~defined[x]
370S>(369a,11b) ~defined[x] equalish[A[y,M[x,MI[AI[1]]],A[z,M[x,MI[AI[1]]]] ~equalish[y,z]
371S>(370b,24b) ~defined[x] ~equalish[y,z] equalish[A[z,M[x,MI[AI[1]]],A[y,M[x,MI[AI[1]]]]
372S>(371b,20a) ~defined[x] equalish[A[y,M[x,MI[AI[1]]],A[M[1,y],M[x,MI[AI[1]]]] ~defined[y]
373S>(372b,10c) ~defined[x] ~defined[y] LE[A[M[1,y],M[x,MI[AI[1]]],z]
~LE[A[y,M[x,MI[AI[1]]],z]
374S>(373b,368a) ~defined[x] LE[A[M[1,MI[AI[1]]],M[x,MI[AI[1]]],y]
~LE[A[M[AI[1]],M[x,MI[AI[1]]],y]
375S>(374a,30a) LE[A[M[1,MI[AI[1]],M[AI[1],MI[AI[1]]],x]
~LE[A[M[AI[1]],M[AI[1],MI[AI[1]]],x]
376S>(375a,355a) ~LE[A[M[AI[1]],M[AI[1],MI[AI[1]]],1]
377S>(376a,366a) []

```

```

PHASE 0: 0 s PHASE 1: 116 s PHASE 2: 388 s Total Time: 504 s
NOD: 17594303 RES: 55265584 FAC: 6456236 T: 16 V: 32 L: 16
CTE: 6626557 CTH: 3916814 CTF: 168 CSZ: 16777216
UTE: 2789819 UTH: 11535321 UTF: 92 SBA: 0
BAS: 28 RED: 43 LEN: 10+335=345

```

# FLD062-1

Predicates: equalish defined less\_or\_equal  
Functions: a b . additive\_identity multiplicative\_identity : multiplicative\_inverse additive\_inverse  
add multiply  
Renamed predicates: equalish defined LE  
Renamed functions: a b . 0 1 : MI AI A M

C18: Spawned: 150 xvl 10.10.4 W8 28S ~LE[AI[b],x] ... ?

Axioms:

```
1: equalish[M[x,M[y,z]],M[M[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
2: equalish[A[M[x,y],M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y]
3: equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
4: >LE[x,y] LE[y,x] ~defined[x] ~defined[y]
5: LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
6: equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]
7: defined[MI[x]] ~defined[x] equalish[x,0]
8: LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]
9: equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[x,z]
10: LE[x,y] ~LE[z,y] ~equalish[z,x]
11: equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]
12: equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: equalish[x,y] ~LE[x,y] ~LE[y,x]
15: defined[M[x,y]] ~defined[x] ~defined[y]
16: equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]
17: defined[A[x,y]] ~defined[x] ~defined[y]
18: equalish[x,y] ~equalish[x,z] ~equalish[z,y]
19: equalish[A[x,AI[x]],0] ~defined[x]
20: equalish[M[1,x],x] ~defined[x]
21: equalish[A[0,x],x] ~defined[x]
22: >defined[AI[x]] ~defined[x]
23: equalish[x,x] ~defined[x]
24: equalish[x,y] ~equalish[y,x]
25: defined[1]
26: ~equalish[0,1]
27: defined[0]
```

Negated conclusion:

```
28S>~LE[AI[b],x] ... ?
29S LE[a,b]
30S defined[a]
31S>defined[b]
```

-----

Phase 0 clauses used in proof:

```
33S>(31a*22b) defined[AI[b]]
```

Phases 1 and 2 clauses used in proof:

```
47S>(33a,4d) LE[x,AI[b]] LE[AI[b],x] ~defined[x]
48S>(47b,28a) LE[x,AI[b]] ~defined[x] ... ?
49S>(48a,28a) ~defined[AI[b]] ... ?
50S>(49a,33a) [] ... ?
```

Inferences derived from proof before variable replaced by constant

33

Inferences derived when variable is replaced by original constant

48

```
PHASE 0: 0 s PHASE 1: 0 s PHASE 2: -NA- Total Time: 0 s
NOD: 639 RES: 3301 FAC: 93 T: 10 V: 32 L: 16
CTE: 1484 CTH: 25 CTF: 0 CSZ: 16777216
UTE: 1957 UTH: 889 UTF: 0 SBA: 0
BAS: 31 RED: 46 LEN: 1+4=5 OPT: kl ml z0 h1 n1 c1 bl t3600/2700
```

Proof found to theorem by slave 18!

Axioms:

```
1: equalish[M[x,M[y,z]],M[M[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
2: equalish[A[M[x,y],M[z,y]],M[A[x,z],y]] ~defined[x] ~defined[z] ~defined[y]
3: >equalish[A[x,A[y,z]],A[A[x,y],z]] ~defined[x] ~defined[y] ~defined[z]
4S>LE[x,y] LE[y,x] ~defined[x] ~defined[y]
5: LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
6: equalish[M[x,MI[x]],1] ~defined[x] equalish[x,0]
7: defined[MI[x]] ~defined[x] equalish[x,0]
8: >LE[A[x,y],A[z,y]] ~defined[y] ~LE[x,z]
9: equalish[M[x,y],M[z,y]] ~defined[y] ~equalish[x,z]
10 >LE[x,y] ~LE[z,y] ~equalish[z,x]
```

```

11 >equalish[A[x,y],A[z,y]] ~defined[y] ~equalish[x,z]
12: equalish[M[x,y],M[y,x]] ~defined[x] ~defined[y]
13 >LE[x,y] ~LE[x,z] ~LE[z,y]
14: equalish[x,y] ~LE[x,y] ~LE[y,x]
15: defined[M[x,y]] ~defined[x] ~defined[y]
16 >equalish[A[x,y],A[y,x]] ~defined[x] ~defined[y]
17 >defined[A[x,y]] ~defined[x] ~defined[y]
18 >equalish[x,y] ~equalish[x,z] ~equalish[z,y]
19 >equalish[A[x,AI[x]],0] ~defined[x]
20: equalish[M[1,x],x] ~defined[x]
21 >equalish[A[0,x],x] ~defined[x]
22S>defined[AI[x]] ~defined[x]
23 >equalish[x,x] ~defined[x]
24 >equalish[x,y] ~equalish[y,x]
25: defined[1]
26: ~equalish[0,1]
27: defined[0]

Negated conclusion:
28S>~LE[AI[b],AI[a]]
29S>LE[a,b]
30S>defined[a]
31S>defined[b]

-----
Phase 0 clauses used in proof:
32S>defined[AI[b]]
36S>(32a*23b) equalish[AI[b],AI[b]]
38S>(32a*21b) equalish[A[0,AI[b]],AI[b]]
44S>(31a*19b) equalish[A[b,AI[b]],0]
45S>(30a*23b) equalish[a,a]
46S>(30a*22b) defined[AI[a]]

Phases 1 and 2 clauses used in proof:
47S>(46a,4c) LE[AI[a],x] LE[x,AI[a]] ~defined[x]
48S>(47ab) LE[AI[a],AI[a]] ~defined[AI[a]]
49S>(48b,46a) LE[AI[a],AI[a]]

50S>(46a,21b) equalish[A[0,AI[a]],AI[a]]
51S>(50a,24b) equalish[AI[a],A[0,AI[a]]]
52S>(51a,10c) LE[A[0,AI[a]],x] ~LE[AI[a],x]
53S>(52b,13a) LE[A[0,AI[a]],x] ~LE[AI[a],y] ~LE[y,x]
54S>(53bc) LE[A[0,AI[a]],AI[a]] ~LE[AI[a],AI[a]]
55S>(54b,49a) LE[A[0,AI[a]],AI[a]]

56S>(46a,11b) equalish[A[x,AI[a]],A[y,AI[a]]] ~equalish[x,y]
57S>(56b,24a) equalish[A[x,AI[a]],A[y,AI[a]]] ~equalish[y,x]
58S>(57a,10c) ~equalish[x,y] LE[A[x,AI[a]],z] ~LE[A[y,AI[a]],z]
59S>(58b,13c) ~equalish[x,y] ~LE[A[y,AI[a]],z] LE[u,z] ~LE[u,A[x,AI[a]]]
60S>(59a,44a) ~LE[A[0,AI[a]],x] LE[y,x] ~LE[y,A[A[b,AI[b]],AI[a]]]
61S>(60b,28a) ~LE[A[0,AI[a]],AI[a]] ~LE[AI[b],A[A[b,AI[b]],AI[a]]]
62S>(61a,55a) ~LE[AI[b],A[A[b,AI[b]],AI[a]]]

63S>(46a,8b) LE[A[x,AI[a]],A[y,AI[a]]] ~LE[x,y]
64S>(63a,10b) ~LE[x,y] LE[z,A[y,AI[a]]] ~equalish[A[x,AI[a]],z]
65S>(64a,8a) LE[x,A[A[y,z],AI[a]]] ~equalish[A[A[u,z],AI[a]],x] ~defined[z] ~LE[u,y]
66S>(65a,10b) ~equalish[A[A[x,y],AI[a]],z] ~defined[y] ~LE[x,u] LE[v,A[A[u,y],AI[a]]]
~equalish[z,v]
67S>(66a,24a) ~defined[x] ~LE[y,z] LE[u,A[A[z,x],AI[a]]] ~equalish[v,u]
~equalish[v,A[A[y,x],AI[a]]]
68S>(67b,29a) ~defined[x] LE[y,A[A[b,x],AI[a]]] ~equalish[z,y] ~equalish[z,A[A[a,x],AI[a]]]
69S>(68a,32a) LE[x,A[A[b,AI[b]],AI[a]]] ~equalish[y,x] ~equalish[y,A[A[a,AI[b]],AI[a]]]
70S>(69b,36a) LE[AI[b],A[A[b,AI[b]],AI[a]]] ~equalish[AI[b],A[A[a,AI[b]],AI[a]]]
71S>(70a,62a) ~equalish[AI[b],A[A[a,AI[b]],AI[a]]]

72S>(46a,11b) equalish[A[x,AI[a]],A[y,AI[a]]] ~equalish[x,y]
73S>(72a,18c) ~equalish[x,y] equalish[z,A[y,AI[a]]] ~equalish[z,A[x,AI[a]]]
74S>(73a,11a) equalish[x,A[A[y,z],AI[a]]] ~equalish[x,A[A[u,z],AI[a]]] ~defined[z] ~equalish[u,y]
75S>(74a,18c) ~equalish[x,A[A[y,z],AI[a]]] ~defined[z] ~equalish[y,u] equalish[v,A[A[u,z],AI[a]]]
~equalish[v,x]
76S>(75a,24a) ~defined[x] ~equalish[y,z] equalish[u,A[A[z,x],AI[a]]] ~equalish[u,v]
~equalish[A[A[y,x],AI[a]],v]
77S>(76a,32a) ~equalish[x,y] equalish[z,A[A[y,AI[b]],AI[a]]] ~equalish[z,u]
~equalish[A[A[x,AI[b]],AI[a]],u]
78S>(77a,45a) equalish[x,A[A[a,AI[b]],AI[a]]] ~equalish[x,y] ~equalish[A[A[a,AI[b]],AI[a]],y]
79S>(78b,36a) equalish[AI[b],A[A[a,AI[b]],AI[a]]] ~equalish[A[A[a,AI[b]],AI[a]],AI[b]]
80S>(79a,71a) ~equalish[A[A[a,AI[b]],AI[a]],AI[b]]

81S>(30a,19b) equalish[A[a,AI[a]],0]
82S>(81a,24b) equalish[0,A[a,AI[a]]]

```

```

83S>(46a,16c) equalish[A[x,AI[a]],A[AI[a],x]] ~defined[x]
84S>(83a,18b) ~defined[x] equalish[A[x,AI[a]],y] ~equalish[A[AI[a],x],y]
85S>(84a,17a) equalish[A[A[x,y],AI[a]],z] ~equalish[A[AI[a],A[x,y]],z] ~defined[x] ~defined[y]
86S>(85a,18b) ~equalish[A[AI[a],A[x,y]],z] ~defined[x] ~defined[y] equalish[A[A[x,y],AI[a]],u]
~equalish[z,u]
87S>(86a,24a) ~defined[x] ~defined[y] equalish[A[A[x,y],AI[a]],z] ~equalish[u,z]
~equalish[u,A[AI[a],A[x,y]]]
88S>(87a,30a) ~defined[x] equalish[A[A[a,x],AI[a]],y] ~equalish[z,y]
~equalish[z,A[AI[a],A[a,x]]]
89S>(88a,32a) equalish[A[A[a,AI[b]],AI[a]],x] ~equalish[y,x] ~equalish[y,A[AI[a],A[a,AI[b]]]]
90S>(89b,36a) equalish[A[A[a,AI[b]],AI[a]],AI[b]] ~equalish[AI[b],A[AI[a],A[a,AI[b]]]]
91S>(90a,80a) ~equalish[AI[b],A[AI[a],A[a,AI[b]]]]

92S>(46a,16c) equalish[A[x,AI[a]],A[AI[a],x]] ~defined[x]
93S>(92a,18c) ~defined[x] equalish[y,A[AI[a],x]] ~equalish[y,A[x,AI[a]]]
94S>(93b,24b) ~defined[x] ~equalish[y,A[x,AI[a]]] equalish[A[AI[a],x],y]
95S>(94a,30a) ~equalish[x,A[a,AI[a]]] equalish[A[AI[a],a],x]
96S>(95a,82a) equalish[A[AI[a],a],0]

97S>(38a,18c) equalish[x,AI[b]] ~equalish[x,A[0,AI[b]]]
98S>(97a,24b) ~equalish[x,A[0,AI[b]]] equalish[AI[b],x]
99S>(98a,18a) equalish[AI[b],x] ~equalish[x,y] ~equalish[y,A[0,AI[b]]]
100S>(99b,3a) equalish[AI[b],A[x,A[y,z]]] ~equalish[A[A[x,y],z],A[0,AI[b]]] ~defined[x]
~defined[y] ~defined[z]
101S>(100b*11a) equalish[AI[b],A[x,A[y,AI[b]]]] ~defined[x] ~defined[y] ~defined[AI[b]]
~equalish[A[x,y],0]
102S>(101d,32a) equalish[AI[b],A[x,A[y,AI[b]]]] ~defined[x] ~defined[y] ~equalish[A[x,y],0]
103S>(102b,46a) equalish[AI[b],A[AI[a],A[x,AI[b]]]] ~defined[x] ~equalish[A[AI[a],x],0]
104S>(103a,91a) ~defined[a] ~equalish[A[AI[a],a],0]
105S>(104a,30a) ~equalish[A[AI[a],a],0]
106S>(105a,96a) []

```

```

PHASE 0: 0 s PHASE 1: 36 s PHASE 2: 77 s Total Time: 113 s
NOD: 3864021 RES: 13685887 FAC: 1639251 T: 10 V: 32 L: 16
CTE: 2957687 CTH: 977430 CTF: 0 CSZ: 16777216
UTE: 1035008 UTH: 3388062 UTF: 0 SBA: 0
BAS: 31 RED: 44 LEN: 6+60=66

```

# FLD073-3

Predicates: sum product less\_or\_equal defined  
Functions: additive\_identity b a . multiplicative\_identity : add additive\_inverse multiply  
multiplicative\_inverse  
Renamed predicates: S P LE defined  
Renamed functions: 0 b a . 1 : A AI M MI

C33: Spawned: 150 xvl 9.14.4 W16 19S P[y,x,x] ~defined[x] ... ?

## Axioms:

```
1 >S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]
2: P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]
3: LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]
4: LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]
5 >P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]
6 >P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]
7: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
8: S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]
9: S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]
10: P[MI[x],x,1] S[0,x,0] ~defined[x]
11: defined[MI[x]] ~defined[x] S[0,x,0]
12: S[0,x,y] ~LE[x,y] ~LE[y,x]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: defined[M[x,y]] ~defined[x] ~defined[y]
15: P[x,y,M[x,y]] ~defined[x] ~defined[y]
16: defined[A[x,y]] ~defined[x] ~defined[y]
17: S[x,y,A[x,y]] ~defined[x] ~defined[y]
18: S[AI[x],x,0] ~defined[x]
19S>P[y,x,x] ~defined[x] ... ?
20: defined[AI[x]] ~defined[x]
21 >S[0,x,x] ~defined[x]
22 >P[x,y,z] ~P[y,x,z]
23: S[x,y,z] ~S[y,x,z]
24 >~S[0,0,1]
25 >defined[1]
26 >defined[0]
```

## Negated conclusion:

```
27S ~LE[0,M[a,b]]
28S LE[a,0]
29S LE[b,0]
30S defined[a]
31S defined[b]
```

## Phase 0 clauses used in proof:

```
42S>(26a*19b) P[x,0,0] ... ?
44 >(25a*21b) S[0,1,1]
46S>(25a*19b) P[x,1,1] ... ?
```

## Phases 1 and 2 clauses used in proof:

```
47S>(46a,22b) P[1,x,1] ... ?
48S>(47a,5c) P[x,1,y] ~P[x,1,z] ~P[z,u,y] ... ?
49S>(48b,47a) P[1,1,x] ~P[1,y,x] ... ?
50S>(49b,42a) P[1,1,0] ... ?

51S>(46a,22b) P[1,x,1] ... ?
52S>(51a,6c) P[x,y,z] ~P[u,1,x] ~P[u,1,z] ... ?
53S>(52b,51a) P[1,x,y] ~P[1,1,y] ... ?
54: 53|{0/y} P[1,x,0] ~P[1,1,0] ... ?
55S>(54b,50a) P[1,x,0] ... ?

56S>(46a,22b) P[1,x,1] ... ?
57S>(56a,1c) S[x,y,1] ~S[z,u,1] ~P[z,v,x] ~P[u,v,y] ... ?
58S>(57a,24a) ~S[x,y,1] ~P[x,z,0] ~P[y,z,0] ... ?
59S>(58a,44a) ~P[0,x,0] ~P[1,x,0] ... ?
60S>(59b,55a) ~P[0,x,0] ... ?
61S>(60a,42a) [] ... ?
```

Inferences derived from proof before variable replaced by constant  
44

Inferences derived when variable is replaced by original constant  
42 46 49 59

```
PHASE 0: 0 s PHASE 1: 0 s PHASE 2: 0 s Total Time: 0 s
NOD: 35 RES: 73 FAC: 3 T: 9 V: 32 L: 16
CTE: 59 CTH: 0 CTF: 0 CSZ: 16777216
UTE: 84 UTH: 43 UTF: 0 SBA: 0
```

Proof found to theorem by slave 33!

Axioms:

```

1S>S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]
2 >P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]
3 >LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]
4 >LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]
5S>P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]
6S>P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]
7 >LE[x,y] LE[y,x] ~defined[x] ~defined[y]
8 >S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]
9 >S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]
10 >P[MI[x],x,1] S[0,x,0] ~defined[x]
11 >defined[MI[x]] ~defined[x] S[0,x,0]
12 >S[0,x,y] ~LE[x,y] ~LE[y,x]
13 >LE[x,y] ~LE[x,z] ~LE[z,y]
14 >defined[M[x,y]] ~defined[x] ~defined[y]
15 >P[x,y,M[x,y]] ~defined[x] ~defined[y]
16: defined[A[x,y]] ~defined[x] ~defined[y]
17 >S[x,y,A[x,y]] ~defined[x] ~defined[y]
18 >S[AI[x],x,0] ~defined[x]
19S>P[1,x,x] ~defined[x]
20 >defined[AI[x]] ~defined[x]
21S>S[0,x,x] ~defined[x]
22S>P[x,y,z] ~P[y,x,z]
23 >S[x,y,z] ~S[y,x,z]
24S>~S[0,0,1]
25S>defined[1]
26S>defined[0]

```

Negated conclusion:

```

27S>~LE[0,M[a,b]]
28S>LE[a,0]
29S>LE[b,0]
30S>defined[a]
31S>defined[b]

```

Phase 0 clauses used in proof:

```

32S>S[0,1,1]
33S>P[1,0,0]
34S>P[1,1,1]
36S>~P[0,1,0] ~P[1,1,0]
37S>(33a*22b) P[0,1,0]
38S>(32a*23b) S[1,0,1]
40S>(31a*20b) defined[AI[b]]
41S>(31a*19b) P[1,b,b]
42S>(31a*18b) S[AI[b],b,0]
43S>(30a*21b) S[0,a,a]
44S>(30a*20b) defined[AI[a]]
45S>(30a*19b) P[1,a,a]
46S>(30a*18b) S[AI[a],a,0]
47S>(37a*36a) ~P[1,1,0]

```

Phases 1 and 2 clauses used in proof:

```

48S>(45a,1c) S[x,y,a] ~S[z,u,1] ~P[z,a,x] ~P[u,a,y]
49S>(48a,23b) ~S[x,y,1] ~P[x,a,z] ~P[y,a,u] S[u,z,a]
50S>(49b,15a) ~S[x,y,1] ~P[y,a,z] S[z,M[x,a],a] ~defined[x] ~defined[a]
51S>(50e,30a) ~S[x,y,1] ~P[y,a,z] S[z,M[x,a],a] ~defined[x]
52S>(51a,32a) ~P[1,a,x] S[x,M[0,a],a] ~defined[0]
53S>(52a,45a) S[a,M[0,a],a] ~defined[0]
54S>(53b,26a) S[a,M[0,a],a]

55S>(30a,14c) defined[M[x,a]] ~defined[x]
56S>(55a,21b) ~defined[x] S[0,M[x,a],M[x,a]]
57S>(56b,8d) ~defined[x] S[y,z,M[x,a]] ~S[y,u,0] ~S[u,M[x,a],z]
58S>(57a,26a) S[x,y,M[0,a]] ~S[x,z,0] ~S[z,M[0,a],y]
59S>(58a,9b) ~S[x,y,0] ~S[y,M[0,a],z] S[M[0,a],u,v] ~S[z,u,w] ~S[x,w,v]
60S>(59a,46a) ~S[a,M[0,a],x] S[M[0,a],y,z] ~S[x,y,u] ~S[AI[a],u,z]
61S>(60ac) S[M[0,a],M[0,a],x] ~S[a,M[0,a],a] ~S[AI[a],a,x]
62S>(61b,54a) S[M[0,a],M[0,a],x] ~S[AI[a],a,x]
63S>(62b,46a) S[M[0,a],M[0,a],0]

64S>(30a,15c) P[x,a,M[x,a]] ~defined[x]
65S>(64b,26a) P[0,a,M[0,a]]
66S>(65a,2d) P[x,a,y] ~S[z,0,x] ~P[z,a,u] ~S[u,M[0,a],y]
67S>(66b,21a) P[0,a,x] ~P[0,a,y] ~S[y,M[0,a],x] ~defined[0]
68S>(67d,26a) P[0,a,x] ~P[0,a,y] ~S[y,M[0,a],x]
69S>(68a,22b) ~P[0,a,x] ~S[x,M[0,a],y] P[a,0,y]

```

70S>(69a,65a) ~S[M[0,a],M[0,a],x] P[a,0,x]  
71S>(70a,63a) P[a,0,0]

72S>(40a,7c) LE[AI[b],x] LE[x,AI[b]] ~defined[x]  
73S>(72ab) LE[AI[b],AI[b]] ~defined[AI[b]]  
74S>(73b,40a) LE[AI[b],AI[b]]

75S>(44a,15b) P[AI[a],x,M[AI[a],x]] ~defined[x]  
76S>(75b,26a) P[AI[a],0,M[AI[a],0]]

77S>(26a,14c) defined[M[x,0]] ~defined[x]  
78S>(77a,21b) ~defined[x] S[0,M[x,0],M[x,0]]  
79S>(78b,23b) ~defined[x] S[M[x,0],0,M[x,0]]  
80S>(79b,2e) ~defined[x] P[y,z,M[x,0]] ~S[u,v,y] ~P[u,z,M[x,0]] ~P[v,z,0]  
81S>(80b,22b) ~defined[x] ~S[y,z,u] ~P[y,v,M[x,0]] ~P[z,v,0] P[v,u,M[x,0]]  
82S>(81a,44a) ~S[x,y,z] ~P[x,u,M[AI[a],0]] ~P[y,u,0] P[u,z,M[AI[a],0]]  
83S>(82a,46a) ~P[AI[a],x,M[AI[a],0]] ~P[a,x,0] P[x,0,M[AI[a],0]]  
84S>(83a,76a) ~P[a,0,0] P[0,0,M[AI[a],0]]  
85S>(84a,71a) P[0,0,M[AI[a],0]]

86S>(41a,1c) S[x,y,b] ~S[z,u,1] ~P[z,b,x] ~P[u,b,y]  
87S>(86a,23b) ~S[x,y,1] ~P[x,b,z] ~P[y,b,u] S[u,z,b]  
88S>(87b,15a) ~S[x,y,1] ~P[y,b,z] S[z,M[x,b],b] ~defined[x] ~defined[b]  
89S>(88e,31a) ~S[x,y,1] ~P[y,b,z] S[z,M[x,b],b] ~defined[x]  
90S>(89a,32a) ~P[1,b,x] S[x,M[0,b],b] ~defined[0]  
91S>(90a,41a) S[b,M[0,b],b] ~defined[0]  
92S>(91b,26a) S[b,M[0,b],b]

93S>(31a,14c) defined[M[x,b]] ~defined[x]  
94S>(93a,21b) ~defined[x] S[0,M[x,b],M[x,b]]  
95S>(94a,30a) S[0,M[a,b],M[a,b]]

96S>(31a,14c) defined[M[x,b]] ~defined[x]  
97S>(96b,30a) defined[M[a,b]]

98S>(44a,7c) LE[AI[a],x] LE[x,AI[a]] ~defined[x]  
99S>(98ab) LE[AI[a],AI[a]] ~defined[AI[a]]  
100S>(99b,44a) LE[AI[a],AI[a]]

101S>(26a,14c) defined[M[x,0]] ~defined[x]  
102S>(101a,21b) ~defined[x] S[0,M[x,0],M[x,0]]  
103S>(102b,23b) ~defined[x] S[M[x,0],0,M[x,0]]  
104S>(103b,2e) ~defined[x] P[y,z,M[x,0]] ~S[u,v,y] ~P[u,z,M[x,0]] ~P[v,z,0]  
105S>(104b,22b) ~defined[x] ~S[y,z,u] ~P[y,v,M[x,0]] ~P[z,v,0] P[v,u,M[x,0]]  
106S>(105a,44a) ~S[x,y,z] ~P[x,u,M[AI[a],0]] ~P[y,u,0] P[u,z,M[AI[a],0]]  
107S>(106a,32a) ~P[0,x,M[AI[a],0]] ~P[1,x,0] P[x,1,M[AI[a],0]]  
108S>(107a,85a) ~P[1,0,0] P[0,1,M[AI[a],0]]  
109S>(108a,33a) P[0,1,M[AI[a],0]]

110S>(31a,14c) defined[M[x,b]] ~defined[x]  
111S>(110a,21b) ~defined[x] S[0,M[x,b],M[x,b]]  
112S>(111b,8d) ~defined[x] S[y,z,M[x,b]] ~S[y,u,0] ~S[u,M[x,b],z]  
113S>(112a,26a) S[x,y,M[0,b]] ~S[x,z,0] ~S[z,M[0,b],y]  
114S>(113a,9b) ~S[x,y,0] ~S[y,M[0,b],z] S[M[0,b],u,v] ~S[z,u,w] ~S[x,w,v]  
115S>(114a,42a) ~S[b,M[0,b],x] S[M[0,b],y,z] ~S[x,y,u] ~S[AI[b],u,z]  
116S>(115ac) S[M[0,b],M[0,b],x] ~S[b,M[0,b],b] ~S[AI[b],b,x]  
117S>(116b,92a) S[M[0,b],M[0,b],x] ~S[AI[b],b,x]  
118S>(117b,42a) S[M[0,b],M[0,b],0]

119S>(46a,8b) S[AI[a],x,y] ~S[a,z,x] ~S[0,z,y]  
120S>(119a,4d) ~S[a,x,y] ~S[0,x,z] LE[u,z] ~LE[v,AI[a]] ~S[v,y,u]  
121S>(120b,21a) ~S[a,x,y] LE[z,x] ~LE[u,AI[a]] ~S[u,y,z] ~defined[x]  
122S>(121a,23a) LE[x,y] ~LE[z,AI[a]] ~S[z,u,x] ~defined[y] ~S[y,a,u]  
123S>(122a,27a) ~LE[x,AI[a]] ~S[x,y,0] ~defined[M[a,b]] ~S[M[a,b],a,y]  
124S>(123c,97a) ~LE[x,AI[a]] ~S[x,y,0] ~S[M[a,b],a,y]  
125S>(124b,46a) ~LE[AI[a],AI[a]] ~S[M[a,b],a,a]  
126S>(125a,100a) ~S[M[a,b],a,a]

127S>(44a,14b) defined[M[AI[a],x]] ~defined[x]  
128S>(127b,31a) defined[M[AI[a],b]]

129S>(46a,8c) S[x,0,y] ~S[x,AI[a],z] ~S[z,a,y]  
130S>(129b,23a) S[x,0,y] ~S[z,a,y] ~S[AI[a],x,z]  
131S>(130b,43a) S[x,0,a] ~S[AI[a],x,0]  
132S>(131b,46a) S[a,0,a]

133S>(46a,23b) S[a,AI[a],0]  
134S>(133a,8c) S[x,0,y] ~S[x,a,z] ~S[z,AI[a],y]  
135S>(134b,43a) S[0,0,x] ~S[a,AI[a],x]  
136S>(135b,133a) S[0,0,0]

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137S>(22a,6c) ~P[x,y,z] P[u,x,v] ~P[w,y,u] ~P[w,z,v]
138S>(137a,15a) P[x,y,z] ~P[u,v,x] ~P[u,M[y,v],z] ~defined[y] ~defined[v]
139S>(138b,33a) P[0,x,y] ~P[1,M[x,0],y] ~defined[x] ~defined[0]
140S>(139d,26a) P[0,x,y] ~P[1,M[x,0],y] ~defined[x]
141S>(140b,5a) P[0,x,y] ~defined[x] ~P[1,z,u] ~P[z,v,M[x,0]] ~P[u,v,y]
142S>(141a,22b) ~defined[x] ~P[1,y,z] ~P[y,u,M[x,0]] ~P[z,u,v] P[x,0,v]
143S>(142b,33a) ~defined[x] ~P[0,y,M[x,0]] ~P[0,y,z] P[x,0,z]
144S>(143a,44a) ~P[0,x,M[AI[a],0]] ~P[0,x,y] P[AI[a],0,y]
145S>(144b,37a) ~P[0,1,M[AI[a],0]] P[AI[a],0,0]
146S>(145a,109a) P[AI[a],0,0]

147S>(31a,15c) P[x,b,M[x,b]] ~defined[x]
148S>(147b,26a) P[0,b,M[0,b]]
149S>(148a,2d) P[x,b,y] ~S[z,0,x] ~P[z,b,u] ~S[u,M[0,b],y]
150S>(149b,21a) P[0,b,x] ~P[0,b,y] ~S[y,M[0,b],x] ~defined[0]
151S>(150d,26a) P[0,b,x] ~P[0,b,y] ~S[y,M[0,b],x]
152S>(151a,22b) ~P[0,b,x] ~S[x,M[0,b],y] P[b,0,y]
153S>(152a,148a) ~S[M[0,b],M[0,b],x] P[b,0,x]
154S>(153a,118a) P[b,0,0]

155S>(31a,15c) P[x,b,M[x,b]] ~defined[x]
156S>(155a,22b) ~defined[x] P[b,x,M[x,b]]
157S>(156b,1e) ~defined[x] S[y,M[x,b],z] ~S[u,b,v] ~P[v,x,z] ~P[u,x,y]
158S>(157b,23b) ~defined[x] ~S[y,b,z] ~P[z,x,u] ~P[y,x,v] S[M[x,b],v,u]
159S>(158a,30a) ~S[x,b,y] ~P[y,a,z] ~P[x,a,u] S[M[a,b],u,z]
160S>(159b,45a) ~S[x,b,1] ~P[x,a,y] S[M[a,b],y,a]
161S>(160b,45a) ~S[1,b,1] S[M[a,b],a,a]
162S>(161b,126a) ~S[1,b,1]

163S>(40a,15b) P[AI[b],x,M[AI[b],x]] ~defined[x]
164S>(163b,26a) P[AI[b],0,M[AI[b],0]]

165S>(46a,23b) S[a,AI[a],0]

166S>(19a,6b) ~defined[x] P[x,y,z] ~P[x,y,u] ~P[1,u,z]
167S>(166a,44a) P[AI[a],x,y] ~P[AI[a],x,z] ~P[1,z,y]
168S>(167a,22b) ~P[AI[a],x,y] ~P[1,y,z] P[x,AI[a],z]
169S>(168b,33a) ~P[AI[a],x,0] P[x,AI[a],0]
170S>(169a,146a) P[0,AI[a],0]

171S>(26a,14c) defined[M[x,0]] ~defined[x]
172S>(171a,21b) ~defined[x] S[0,M[x,0],M[x,0]]
173S>(172b,23b) ~defined[x] S[M[x,0],0,M[x,0]]
174S>(173b,2e) ~defined[x] P[y,z,M[x,0]] ~S[u,v,y] ~P[u,z,M[x,0]] ~P[v,z,0]
175S>(174b,22b) ~defined[x] ~S[y,z,u] ~P[y,v,M[x,0]] ~P[z,v,0] P[v,u,M[x,0]]
176S>(175a,40a) ~S[x,y,z] ~P[x,u,M[AI[b],0]] ~P[y,u,0] P[u,z,M[AI[b],0]]
177S>(176a,42a) ~P[AI[b],x,M[AI[b],0]] ~P[b,x,0] P[x,0,M[AI[b],0]]
178S>(177a,164a) ~P[b,0,0] P[0,0,M[AI[b],0]]
179S>(178a,154a) P[0,0,M[AI[b],0]]

180S>(26a,17c) S[x,0,A[x,0]] ~defined[x]
181S>(180b,26a) S[0,0,A[0,0]]
182S>(181a,8c) S[x,A[0,0],y] ~S[x,0,z] ~S[z,0,y]
183S>(182a,9d) ~S[x,0,y] ~S[y,0,z] S[u,v,z] ~S[x,w,u] ~S[w,v,A[0,0]]
184S>(183b,38a) ~S[x,0,1] S[y,z,1] ~S[x,u,y] ~S[u,z,A[0,0]]
185S>(184ac) S[1,x,1] ~S[y,0,1] ~S[0,x,A[0,0]]
186S>(185b,38a) S[1,x,1] ~S[0,x,A[0,0]]
187S>(186a,162a) ~S[0,b,A[0,0]]

188S>(26a,14c) defined[M[x,0]] ~defined[x]
189S>(188b,26a) defined[M[0,0]]

190S>(44a,21b) S[0,AI[a],AI[a]]
191S>(190a,4d) LE[x,AI[a]] ~LE[y,0] ~S[y,AI[a],x]
192S>(191b,28a) LE[x,AI[a]] ~S[a,AI[a],x]
193: 192 {0/x} LE[0,AI[a]] ~S[a,AI[a],0]
194S>(193b,165a) LE[0,AI[a]]

195S>(47a,5a) ~P[1,x,y] ~P[x,z,1] ~P[y,z,0]
196S>(195a,34a) ~P[1,x,1] ~P[1,x,0]
197S>(196b,33a) ~P[1,0,1]

198S>(26a,14c) defined[M[x,0]] ~defined[x]
199S>(198a,21b) ~defined[x] S[0,M[x,0],M[x,0]]
200S>(199b,23b) ~defined[x] S[M[x,0],0,M[x,0]]
201S>(200b,2e) ~defined[x] P[y,z,M[x,0]] ~S[u,v,y] ~P[u,z,M[x,0]] ~P[v,z,0]
202S>(201b,22b) ~defined[x] ~S[y,z,u] ~P[y,v,M[x,0]] ~P[z,v,0] P[v,u,M[x,0]]
203S>(202a,40a) ~S[x,y,z] ~P[x,u,M[AI[b],0]] ~P[y,u,0] P[u,z,M[AI[b],0]]
204S>(203a,32a) ~P[0,x,M[AI[b],0]] ~P[1,x,0] P[x,1,M[AI[b],0]]
205S>(204a,179a) ~P[1,0,0] P[0,1,M[AI[b],0]]
206S>(205a,33a) P[0,1,M[AI[b],0]]

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207S>(26a,17c) S[x,0,A[x,0]] ~defined[x]  
208S>(207b,26a) S[0,0,A[0,0]]  
209S>(208a,9d) S[x,y,A[0,0]] ~S[0,z,x] ~S[z,y,0]  
210S>(209b,23a) S[x,y,A[0,0]] ~S[z,y,0] ~S[z,0,x]  
211S>(210b,42a) S[x,b,A[0,0]] ~S[AI[b],0,x]  
212S>(211a,187a) ~S[AI[b],0,0]  
  
213S>(38a,2b) P[1,x,y] ~P[1,x,z] ~P[0,x,u] ~S[z,u,y]  
214S>(213a,22b) ~P[1,x,y] ~P[0,x,z] ~S[y,z,u] P[x,1,u]  
215S>(214b,15a) ~P[1,x,y] ~S[y,M[0,x],z] P[x,1,z] ~defined[0] ~defined[x]  
216S>(215d,26a) ~P[1,x,y] ~S[y,M[0,x],z] P[x,1,z] ~defined[x]  
217S>(216b,21a) ~P[1,x,0] P[x,1,M[0,x]] ~defined[x] ~defined[M[0,x]]  
218S>(217a,33a) P[0,1,M[0,0]] ~defined[0] ~defined[M[0,0]]  
219S>(218b,26a) P[0,1,M[0,0]] ~defined[M[0,0]]  
220S>(219b,189a) P[0,1,M[0,0]]  
  
221S>(47a,2a) ~S[x,y,1] ~P[x,1,z] ~P[y,1,u] ~S[z,u,0]  
222S>(221a,21a) ~P[0,1,x] ~P[1,1,y] ~S[x,y,0] ~defined[1]  
223S>(222d,25a) ~P[0,1,x] ~P[1,1,y] ~S[x,y,0]  
224S>(223a,37a) ~P[1,1,x] ~S[0,x,0]  
225S>(224a,34a) ~S[0,1,0]  
  
226S>(22a,6c) ~P[x,y,z] P[u,x,v] ~P[w,y,u] ~P[w,z,v]  
227S>(226a,15a) P[x,y,z] ~P[u,v,x] ~P[u,M[y,v],z] ~defined[y] ~defined[v]  
228S>(227b,33a) P[0,x,y] ~P[1,M[x,0],y] ~defined[x] ~defined[0]  
229S>(228d,26a) P[0,x,y] ~P[1,M[x,0],y] ~defined[x]  
230S>(229b,5a) P[0,x,y] ~defined[x] ~P[1,z,u] ~P[z,v,M[x,0]] ~P[u,v,y]  
231S>(230a,22b) ~defined[x] ~P[1,y,z] ~P[y,u,M[x,0]] ~P[z,u,v] P[x,0,v]  
232S>(231b,33a) ~defined[x] ~P[0,y,M[x,0]] ~P[0,y,z] P[x,0,z]  
233S>(232a,40a) ~P[0,x,M[AI[b],0]] ~P[0,x,y] P[AI[b],0,y]  
234S>(233b,37a) ~P[0,1,M[AI[b],0]] P[AI[b],0,0]  
235S>(234a,206a) P[AI[b],0,0]  
  
236S>(25a,20b) defined[AI[1]]  
237S>(236a,17b) S[AI[1],x,A[AI[1],x]] ~defined[x]  
238S>(237b,25a) S[AI[1],1,A[AI[1],1]]  
239S>(238a,9c) S[x,1,y] ~S[z,AI[1],x] ~S[z,A[AI[1],1],y]  
240S>(239b,18a) S[0,1,x] ~S[AI[AI[1]],A[AI[1],1],x] ~defined[AI[1]]  
241S>(240a,225a) ~S[AI[AI[1]],A[AI[1],1],0] ~defined[AI[1]]  
242S>(241b,236a) ~S[AI[AI[1]],A[AI[1],1],0]  
  
243S>(33a,6b) P[0,x,y] ~P[0,x,z] ~P[1,z,y]  
244S>(243b,15a) P[0,x,y] ~P[1,M[0,x],y] ~defined[0] ~defined[x]  
245S>(244c,26a) P[0,x,y] ~P[1,M[0,x],y] ~defined[x]  
246S>(245b,5a) P[0,x,y] ~defined[x] ~P[1,z,u] ~P[z,v,M[0,x]] ~P[u,v,y]  
247S>(246a,22b) ~defined[x] ~P[1,y,z] ~P[y,u,M[0,x]] ~P[z,u,v] P[x,0,v]  
248S>(247a,26a) ~P[1,x,y] ~P[x,z,M[0,0]] ~P[y,z,u] P[0,0,u]  
249S>(248a,33a) ~P[0,x,M[0,0]] ~P[0,x,y] P[0,0,y]  
250S>(249b,37a) ~P[0,1,M[0,0]] P[0,0,0]  
251S>(250a,220a) P[0,0,0]  
  
252S>(19a,6b) ~defined[x] P[x,y,z] ~P[x,y,u] ~P[1,u,z]  
253S>(252a,40a) P[AI[b],x,y] ~P[AI[b],x,z] ~P[1,z,y]  
254S>(253a,22b) ~P[AI[b],x,y] ~P[1,y,z] P[x,AI[b],z]  
255S>(254b,33a) ~P[AI[b],x,0] P[x,AI[b],0]  
256S>(255a,235a) P[0,AI[b],0]  
  
257S>(40a,21b) S[0,AI[b],AI[b]]  
258S>(257a,23b) S[AI[b],0,AI[b]]  
  
259S>(25a,20b) defined[AI[1]]  
  
260S>(42a,23b) S[b,AI[b],0]  
  
261S>(40a,21b) S[0,AI[b],AI[b]]  
262S>(261a,4d) LE[x,AI[b]] ~LE[y,0] ~S[y,AI[b],x]  
263S>(262b,29a) LE[x,AI[b]] ~S[b,AI[b],x]  
264: 263 {0/x} LE[0,AI[b]] ~S[b,AI[b],0]  
265S>(264b,260a) LE[0,AI[b]]  
  
266S>(46a,23b) S[a,AI[a],0]  
267S>(266a,8c) S[x,0,y] ~S[x,a,z] ~S[z,AI[a],y]  
268S>(267b,43a) S[0,0,x] ~S[a,AI[a],x]  
269S>(268b,266a) S[0,0,0]  
  
270S>(46a,4c) LE[0,x] ~LE[AI[a],y] ~S[y,a,x]  
271S>(270a,27a) ~LE[AI[a],x] ~S[x,a,M[a,b]]  
272S>(271a,13a) ~S[x,a,M[a,b]] ~LE[AI[a],y] ~LE[y,x]  
273S>(272bc) ~S[AI[a],a,M[a,b]] ~LE[AI[a],AI[a]]  
274S>(273b,100a) ~S[AI[a],a,M[a,b]]

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275S>(31a,14c) defined[M[x,b]] ~defined[x]
276S>(275a,21b) ~defined[x] S[0,M[x,b],M[x,b]]
277S>(276b,8d) ~defined[x] S[y,z,M[x,b]] ~S[y,u,0] ~S[u,M[x,b],z]
278S>(277a,30a) S[x,y,M[a,b]] ~S[x,z,0] ~S[z,M[a,b],y]
279S>(278b,46a) S[AI[a],x,M[a,b]] ~S[a,M[a,b],x]
280S>(279a,274a) ~S[a,M[a,b],a]

281S>(31a,15c) P[x,b,M[x,b]] ~defined[x]
282S>(281a,22b) ~defined[x] P[b,x,M[x,b]]
283S>(282b,5c) ~defined[x] P[y,M[x,b],z] ~P[y,b,u] ~P[u,x,z]
284S>(283a,26a) P[x,M[0,b],y] ~P[x,b,z] ~P[z,0,y]
285S>(284a,22b) ~P[x,b,y] ~P[y,0,z] P[M[0,b],x,z]
286S>(285a,41a) ~P[b,0,x] P[M[0,b],1,x]
287S>(286a,154a) P[M[0,b],1,0]

288S>(31a,15c) P[x,b,M[x,b]] ~defined[x]
289S>(288a,22b) ~defined[x] P[b,x,M[x,b]]
290S>(289b,1d) ~defined[x] S[M[x,b],y,z] ~S[b,u,v] ~P[v,x,z] ~P[u,x,y]
291S>(290b,23b) ~defined[x] ~S[b,y,z] ~P[z,x,u] ~P[y,x,v] S[v,M[x,b],u]
292S>(291a,30a) ~S[b,x,y] ~P[y,a,z] ~P[x,a,u] S[u,M[a,b],z]
293S>(292b,45a) ~S[b,x,1] ~P[x,a,y] S[y,M[a,b],a]
294S>(293b,45a) ~S[b,1,1] S[a,M[a,b],a]
295S>(294b,280a) ~S[b,1,1]

296S>(31a,15c) P[x,b,M[x,b]] ~defined[x]
297S>(296a,22b) ~defined[x] P[b,x,M[x,b]]
298S>(297b,5b) ~defined[x] P[b,y,z] ~P[x,u,y] ~P[M[x,b],u,z]
299S>(298a,26a) P[b,x,y] ~P[0,z,x] ~P[M[0,b],z,y]
300S>(299a,22b) ~P[0,x,y] ~P[M[0,b],x,z] P[y,b,z]
301S>(300a,37a) ~P[M[0,b],1,x] P[0,b,x]
302S>(301a,287a) P[0,b,0]

303S>(31a,17b) S[b,x,A[b,x]] ~defined[x]
304S>(303b,25a) S[b,1,A[b,1]]
305S>(304a,8b) S[b,x,y] ~S[l,z,x] ~S[A[b,1],z,y]
306S>(305b,9a) S[b,x,y] ~S[A[b,1],z,y] ~S[u,v,1] ~S[v,z,w] ~S[u,w,x]
307S>(306be) S[b,x,x] ~S[A[b,1],y,1] ~S[y,z,z] ~S[A[b,1],z,x]
308S>(307bd) S[b,1,1] ~S[x,x,x] ~S[A[b,1],x,1]
309S>(308a,295a) ~S[x,x,x] ~S[A[b,1],x,1]
310S>(309a,136a) ~S[A[b,1],0,1]

311S>(42a,8b) S[AI[b],x,y] ~S[b,z,x] ~S[0,z,y]
312S>(311a,4d) ~S[b,x,y] ~S[0,x,z] LE[u,z] ~LE[v,AI[b]] ~S[v,y,u]
313S>(312b,21a) ~S[b,x,y] LE[z,x] ~LE[u,AI[b]] ~S[u,y,z] ~defined[x]
314S>(313a,23a) LE[x,y] ~LE[z,AI[b]] ~S[z,u,x] ~defined[y] ~S[y,b,u]
315S>(314a,27a) ~LE[x,AI[b]] ~S[x,y,0] ~defined[M[a,b]] ~S[M[a,b],b,y]
316S>(315c,97a) ~LE[x,AI[b]] ~S[x,y,0] ~S[M[a,b],b,y]
317S>(316b,42a) ~LE[AI[b],AI[b]] ~S[M[a,b],b,b]
318S>(317a,74a) ~S[M[a,b],b,b]

319S>(46a,23b) S[a,AI[a],0]
320S>(319a,1b) S[x,y,z] ~P[0,u,z] ~P[a,u,x] ~P[AI[a],u,y]
321S>(320a,23b) ~P[0,x,y] ~P[a,x,z] ~P[AI[a],x,u] S[u,z,y]
322S>(321b,15a) ~P[0,x,y] ~P[AI[a],x,z] S[z,M[a,x],y] ~defined[a] ~defined[x]
323S>(322d,30a) ~P[0,x,y] ~P[AI[a],x,z] S[z,M[a,x],y] ~defined[x]
324S>(323b*15a) ~P[0,x,y] S[M[AI[a],x],M[a,x],y] ~defined[x] ~defined[AI[a]]
325S>(324d,44a) ~P[0,x,y] S[M[AI[a],x],M[a,x],y] ~defined[x]
326S>(325b,4c) ~P[0,x,y] ~defined[x] LE[y,z] ~LE[M[AI[a],x],u] ~S[u,M[a,x],z]
327S>(326b,31a) ~P[0,b,x] LE[x,y] ~LE[M[AI[a],b],z] ~S[z,M[a,b],y]
328S>(327b,27a) ~P[0,b,0] ~LE[M[AI[a],b],x] ~S[x,M[a,b],M[a,b]]
329S>(328a,302a) ~LE[M[AI[a],b],x] ~S[x,M[a,b],M[a,b]]
330: 329|{0/x} ~LE[M[AI[a],b],0] ~S[0,M[a,b],M[a,b]]
331S>(330b,95a) ~LE[M[AI[a],b],0]

332S>(31a,17b) S[b,x,A[b,x]] ~defined[x]
333S>(332a,8c) ~defined[x] S[y,A[b,x],z] ~S[y,b,u] ~S[u,x,z]
334S>(333b,23b) ~defined[x] ~S[y,b,z] ~S[z,x,u] S[A[b,x],y,u]
335S>(334b,12a) ~defined[x] ~S[y,x,z] S[A[b,x],0,z] ~LE[b,y] ~LE[y,b]
336S>(335b,32a) ~defined[1] S[A[b,1],0,1] ~LE[b,0] ~LE[0,b]
337S>(336a,25a) S[A[b,1],0,1] ~LE[b,0] ~LE[0,b]
338S>(337a,310a) ~LE[b,0] ~LE[0,b]
339S>(338a,29a) ~LE[0,b]

340S>(31a,15c) P[x,b,M[x,b]] ~defined[x]
341S>(340b,30a) P[a,b,M[a,b]]
342S>(341a,1e) S[x,M[a,b],y] ~S[z,a,u] ~P[u,b,y] ~P[z,b,x]
343S>(342a,23b) ~S[x,a,y] ~P[y,b,z] ~P[x,b,u] S[M[a,b],u,z]
344S>(343b,41a) ~S[x,a,1] ~P[x,b,y] S[M[a,b],y,b]
345S>(344b,41a) ~S[l,a,1] S[M[a,b],b,b]
346S>(345b,318a) ~S[l,a,1]

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347S>(28a,12c) S[0,0,a] ~LE[0,a]
348S>(347a,9b) ~LE[0,a] S[a,x,y] ~S[0,x,z] ~S[0,z,y]
349S>(348b,23b) ~LE[0,a] ~S[0,x,y] ~S[0,y,z] S[x,a,z]
350S>(349b,32a) ~LE[0,a] ~S[0,1,x] S[1,a,x]
351S>(350b,32a) ~LE[0,a] S[1,a,1]
352S>(351b,346a) ~LE[0,a]

353S>(26a,17c) S[x,0,A[x,0]] ~defined[x]
354S>(353b,26a) S[0,0,A[0,0]]
355S>(354a,9d) S[x,y,A[0,0]] ~S[0,z,x] ~S[z,y,0]
356S>(355b,11c) S[0,x,A[0,0]] ~S[y,x,0] defined[MI[y]] ~defined[y]
357S>(356b,42a) S[0,b,A[0,0]] defined[MI[AI[b]]] ~defined[AI[b]]
358S>(357a,187a) defined[MI[AI[b]]] ~defined[AI[b]]
359S>(358b,40a) defined[MI[AI[b]]]

360S>(28a,4b) LE[x,y] ~S[a,z,x] ~S[0,z,y]
361S>(360b,23a) LE[x,y] ~S[0,z,y] ~S[z,a,x]
362S>(361b,23a) LE[x,y] ~S[z,a,x] ~S[z,0,y]
363S>(362b,8a) LE[x,y] ~S[z,0,y] ~S[z,u,v] ~S[u,w,a] ~S[v,w,x]
364S>(363bd) LE[x,a] ~S[y,y,z] ~S[y,0,a] ~S[z,0,x]
365S>(364bd) LE[0,a] ~S[0,0,a] ~S[0,0,0]
366S>(365a,352a) ~S[0,0,a] ~S[0,0,0]
367S>(366b,269a) ~S[0,0,a]

368: (23b,10b) S[x,0,0] P[MI[x],x,1] ~defined[x]
369S>(368b,5d) S[x,0,0] ~defined[x] P[y,z,1] ~P[y,u,MI[x]] ~P[u,x,z]
370S>(369b,40a) S[AI[b],0,0] P[x,y,1] ~P[x,z,MI[AI[b]]] ~P[z,AI[b],y]
371S>(370a,212a) P[x,y,1] ~P[x,z,MI[AI[b]]] ~P[z,AI[b],y]
372S>(371b,19a) P[1,x,1] ~P[MI[AI[b]],AI[b],x] ~defined[MI[AI[b]]]
373S>(372a,197a) ~P[MI[AI[b]],AI[b],0] ~defined[MI[AI[b]]]
374S>(373b,359a) ~P[MI[AI[b]],AI[b],0]

375S>(23b,21a) S[x,0,x] ~defined[x]
376S>(375b,44a) S[AI[a],0,AI[a]]
377S>(376a,8b) S[AI[a],x,y] ~S[0,z,x] ~S[AI[a],z,y]
378S>(377a,8c) ~S[0,x,y] ~S[AI[a],x,z] S[u,z,v] ~S[u,AI[a],w] ~S[w,y,v]
379S>(378ae) ~S[AI[a],x,y] S[z,y,x] ~S[z,AI[a],0] ~S[0,x,x]
380S>(379a,46a) S[x,0,a] ~S[x,AI[a],0] ~S[0,a,a]
381S>(380a,367a) ~S[0,AI[a],0] ~S[0,a,a]
382S>(381b,43a) ~S[0,AI[a],0]

383S>(33a,6b) P[0,x,y] ~P[0,x,z] ~P[1,z,y]
384S>(383b,15a) P[0,x,y] ~P[1,M[0,x],y] ~defined[0] ~defined[x]
385S>(384c,26a) P[0,x,y] ~P[1,M[0,x],y] ~defined[x]
386S>(385b,5a) P[0,x,y] ~defined[x] ~P[1,z,u] ~P[z,v,M[0,x]] ~P[u,v,y]
387S>(386a,22b) ~defined[x] ~P[1,y,z] ~P[y,u,M[0,x]] ~P[z,u,v] P[x,0,v]
388S>(387a,26a) ~P[1,x,y] ~P[x,z,M[0,0]] ~P[y,z,u] P[0,0,u]
389S>(388a,33a) ~P[0,x,M[0,0]] ~P[0,x,y] P[0,0,y]
390S>(389b,37a) ~P[0,1,M[0,0]] P[0,0,0]
391S>(390a,220a) P[0,0,0]

392: (23b,10b) S[x,0,0] P[MI[x],x,1] ~defined[x]
393S>(392b,5b) S[x,0,0] ~defined[x] P[MI[x],y,z] ~P[x,u,y] ~P[1,u,z]
394S>(393b,40a) S[AI[b],0,0] P[MI[AI[b]],x,y] ~P[AI[b],z,x] ~P[1,z,y]
395S>(394a,212a) P[MI[AI[b]],x,y] ~P[AI[b],z,x] ~P[1,z,y]
396S>(395b,6a) P[MI[AI[b]],x,y] ~P[1,z,y] ~P[u,v,AI[b]] ~P[v,z,w] ~P[u,w,x]
397S>(396b,33a) P[MI[AI[b]],x,0] ~P[y,z,AI[b]] ~P[z,0,u] ~P[y,u,x]
398S>(397bd) P[MI[AI[b]],AI[b],0] ~P[x,0,x] ~P[y,x,AI[b]]
399S>(398a,374a) ~P[x,0,x] ~P[y,x,AI[b]]
400: 399{|0/x} ~P[0,0,0] ~P[x,0,AI[b]]
401S>(400a,391a) ~P[x,0,AI[b]]

402S>(19a,6d) ~defined[x] P[y,z,x] ~P[1,u,y] ~P[u,z,x]
403S>(402b,22b) ~defined[x] ~P[1,y,z] ~P[y,u,x] P[u,z,x]
404S>(403b,33a) ~defined[x] ~P[0,y,x] P[y,0,x]
405S>(404b,6a) ~defined[x] P[y,0,x] ~P[z,u,0] ~P[u,y,v] ~P[z,v,x]
406S>(405a,40a) P[x,0,AI[b]] ~P[y,z,0] ~P[z,x,u] ~P[y,u,AI[b]]
407S>(406a,401a) ~P[y,z,0] ~P[z,x,u] ~P[y,u,AI[b]]
408S>(407bc) ~P[x,x,0] ~P[x,AI[b],AI[b]]
409S>(408a,251a) ~P[0,AI[b],AI[b]]

410S>(19a,1e) ~defined[x] S[y,x,z] ~S[u,1,v] ~P[v,x,z] ~P[u,x,y]
411S>(410a,40a) S[x,AI[b],y] ~S[z,1,u] ~P[u,AI[b],y] ~P[z,AI[b],x]
412S>(411a,23b) ~S[x,1,y] ~P[y,AI[b],z] ~P[x,AI[b],u] S[AI[b],u,z]
413S>(412a,18a) ~P[0,AI[b],x] ~P[AI[1],AI[b],y] S[AI[b],y,x] ~defined[1]
414S>(413d,25a) ~P[0,AI[b],x] ~P[AI[1],AI[b],y] S[AI[b],y,x]
415S>(414b,15a) ~P[0,AI[b],x] S[AI[b],M[AI[1],AI[b]],x] ~defined[AI[1]] ~defined[AI[b]]
416S>(415a,256a) S[AI[b],M[AI[1],AI[b]],0] ~defined[AI[1]] ~defined[AI[b]]
417S>(416b,259a) S[AI[b],M[AI[1],AI[b]],0] ~defined[AI[b]]
418S>(417b,40a) S[AI[b],M[AI[1],AI[b]],0]

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419: (23a,2b) ~S[x,y,z] P[z,u,v] ~P[y,u,w] ~P[x,u,t] ~S[w,t,v]
420: (419a,18a) P[0,x,y] ~P[z,x,u] ~P[AI[z],x,v] ~S[u,v,y] ~defined[z]
421S>(420b,19a) P[0,x,y] ~P[AI[1],x,z] ~S[x,z,y] ~defined[1] ~defined[x]
422S>(421d,25a) P[0,x,y] ~P[AI[1],x,z] ~S[x,z,y] ~defined[x]
423S>(422b*15a) P[0,x,y] ~S[x,M[AI[1],x],y] ~defined[x] ~defined[AI[1]]
424S>(423d,259a) P[0,x,y] ~S[x,M[AI[1],x],y] ~defined[x]
425S>(424b,8a) P[0,x,y] ~defined[x] ~S[x,z,u] ~S[z,v,M[AI[1],x]] ~S[u,v,y]
426S>(425b,40a) P[0,AI[b],x] ~S[AI[b],y,z] ~S[y,u,M[AI[1],AI[b]]] ~S[z,u,x]
427S>(426bd) P[0,AI[b],AI[b]] ~S[x,x,M[AI[1],AI[b]]] ~S[AI[b],x,AI[b]]
428S>(427a,409a) ~S[x,x,M[AI[1],AI[b]]] ~S[AI[b],x,AI[b]]
429: 428|{0/x} ~S[0,0,M[AI[1],AI[b]]] ~S[AI[b],0,AI[b]]
430S>(429b,258a) ~S[0,0,M[AI[1],AI[b]]]

431S>(23b,21a) S[x,0,x] ~defined[x]
432S>(431b,14a) S[M[x,y],0,M[x,y]] ~defined[x] ~defined[y]
433S>(432a,9d) ~defined[x] ~defined[y] S[z,u,M[x,y]] ~S[M[x,y],v,z] ~S[v,u,0]
434S>(433a,20a) ~defined[x] S[y,z,M[AI[u],x]] ~S[M[AI[u],x],v,y] ~S[v,z,0] ~defined[u]
435S>(434b,23b) ~defined[x] ~S[M[AI[y],x],z,u] ~S[z,v,0] ~defined[y] S[v,u,M[AI[y],x]]
436S>(435b,23a) ~defined[x] ~S[y,z,0] ~defined[u] S[z,v,M[AI[u],x]] ~S[y,M[AI[u],x],v]
437S>(436a,40a) ~S[x,y,0] ~defined[z] S[y,u,M[AI[z],AI[b]]] ~S[x,M[AI[z],AI[b]],u]
438S>(437b,25a) ~S[x,y,0] S[y,z,M[AI[1],AI[b]]] ~S[x,M[AI[1],AI[b]],z]
439S>(438a,42a) S[b,x,M[AI[1],AI[b]]] ~S[AI[b],M[AI[1],AI[b]],x]
440: 439|{0/x} S[b,0,M[AI[1],AI[b]]] ~S[AI[b],M[AI[1],AI[b]],0]
441S>(440b,418a) S[b,0,M[AI[1],AI[b]]]

442S>(21b,14a) S[0,M[x,y],M[x,y]] ~defined[x] ~defined[y]
443S>(442a,8d) ~defined[x] ~defined[y] S[z,u,M[x,y]] ~S[z,v,0] ~S[v,M[x,y],u]
444S>(443a,20a) ~defined[x] S[y,z,M[AI[u],x]] ~S[y,v,0] ~S[v,M[AI[u],x],z] ~defined[u]
445S>(444b,23b) ~defined[x] ~S[y,z,0] ~S[z,M[AI[u],x],v] ~defined[u] S[v,y,M[AI[u],x]]
446S>(445b,21a) ~defined[x] ~S[0,M[AI[y],x],z] ~defined[y] S[z,0,M[AI[y],x]] ~defined[0]
447S>(446e,26a) ~defined[x] ~S[0,M[AI[y],x],z] ~defined[y] S[z,0,M[AI[y],x]]
448S>(447a,40a) ~S[0,M[AI[x],AI[b]],y] ~defined[x] S[y,0,M[AI[x],AI[b]]]
449S>(448b,25a) ~S[0,M[AI[1],AI[b]],x] S[x,0,M[AI[1],AI[b]]]
450: 449|{0/x} ~S[0,M[AI[1],AI[b]],0] S[0,0,M[AI[1],AI[b]]]
451S>(450b,430a) ~S[0,M[AI[1],AI[b]],0]

452S>(28a,4b) LE[x,y] ~S[a,z,x] ~S[0,z,y]
453S>(452a,4b) ~S[a,x,y] ~S[0,x,z] LE[u,v] ~S[y,w,u] ~S[z,w,v]
454S>(453b,21a) ~S[a,x,y] LE[z,u] ~S[y,v,z] ~S[x,v,u] ~defined[x]
455S>(454a,23a) LE[x,y] ~S[z,u,x] ~S[v,u,y] ~defined[v] ~S[v,a,z]
456S>(455b,46a) LE[0,x] ~S[y,a,x] ~defined[y] ~S[y,a,AI[a]]
457S>(456b,43a) LE[0,a] ~defined[0] ~S[0,a,AI[a]]
458S>(457a,352a) ~defined[0] ~S[0,a,AI[a]]
459S>(458a,26a) ~S[0,a,AI[a]]

460S>(23b,21a) S[x,0,x] ~defined[x]
461S>(460b,44a) S[AI[a],0,AI[a]]
462S>(461a,9d) S[x,y,AI[a]] ~S[AI[a],z,x] ~S[z,y,0]
463S>(462a,9c) ~S[AI[a],x,y] ~S[x,z,0] S[u,z,v] ~S[w,y,u] ~S[w,AI[a],v]
464S>(463a,46a) ~S[a,x,0] S[y,x,z] ~S[u,0,y] ~S[u,AI[a],z]
465S>(464ad) S[x,AI[a],0] ~S[a,0,x] ~S[a,AI[a],0]
466S>(465a,382a) ~S[a,0,0] ~S[a,AI[a],0]
467S>(466b,165a) ~S[a,0,0]

468S>(23b,21a) S[x,0,x] ~defined[x]
469S>(468b,44a) S[AI[a],0,AI[a]]
470S>(469a,9d) S[x,y,AI[a]] ~S[AI[a],z,x] ~S[z,y,0]
471S>(470b,23a) S[x,y,AI[a]] ~S[z,y,0] ~S[z,AI[a],x]
472S>(471b,46a) S[x,a,AI[a]] ~S[AI[a],AI[a],x]
473S>(472a,459a) ~S[AI[a],AI[a],0]

474S>(44a,10c) P[MI[AI[a]],AI[a],1] S[0,AI[a],0]
475S>(474b,382a) P[MI[AI[a]],AI[a],1]
476S>(475a,5b) P[MI[AI[a]],x,y] ~P[AI[a],z,x] ~P[1,z,y]
477S>(476b,15a) P[MI[AI[a]],M[AI[a],x],y] ~P[1,x,y] ~defined[AI[a]] ~defined[x]
478S>(477c,44a) P[MI[AI[a]],M[AI[a],x],y] ~P[1,x,y] ~defined[x]
479S>(478b,41a) P[MI[AI[a]],M[AI[a],b],b] ~defined[b]
480S>(479b,31a) P[MI[AI[a]],M[AI[a],b],b]
481S>(480a,3d) LE[0,b] ~LE[0,MI[AI[a]]] ~LE[0,M[AI[a],b]]
482S>(481a,339a) ~LE[0,MI[AI[a]]] ~LE[0,M[AI[a],b]]
483S>(482b,7a) ~LE[0,MI[AI[a]]] LE[M[AI[a],b],0] ~defined[0] ~defined[M[AI[a],b]]
484S>(483b,331a) ~LE[0,MI[AI[a]]] ~defined[0] ~defined[M[AI[a],b]]
485S>(484b,26a) ~LE[0,MI[AI[a]]] ~defined[M[AI[a],b]]
486S>(485b,128a) ~LE[0,MI[AI[a]]]

487S>(23b,21a) S[x,0,x] ~defined[x]
488S>(487a,8d) ~defined[x] S[y,z,x] ~S[y,u,x] ~S[u,0,z]
489S>(488a,30a) S[x,y,a] ~S[x,z,a] ~S[z,0,y]
490S>(489a,9b) ~S[x,y,a] ~S[y,0,z] S[a,u,v] ~S[z,u,w] ~S[x,w,v]
491S>(490bd) ~S[x,y,a] S[a,0,z] ~S[y,0,y] ~S[x,y,z]

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492S>(491a,43a) S[a,0,x] ~S[a,0,a] ~S[0,a,x]
493S>(492a,467a) ~S[a,0,a] ~S[0,a,0]
494S>(493a,132a) ~S[0,a,0]

495S>(23b,21a) S[x,0,x] ~defined[x]
496S>(495b,44a) S[AI[a],0,AI[a]]
497S>(496a,9d) S[x,y,AI[a]] ~S[AI[a],z,x] ~S[z,y,0]
498S>(497a,8c) ~S[AI[a],x,y] ~S[x,z,0] S[u,AI[a],v] ~S[u,y,w] ~S[w,z,v]
499S>(498ad) ~S[x,y,0] S[AI[a],AI[a],z] ~S[AI[a],x,x] ~S[x,y,z]
500S>(499ad) S[AI[a],AI[a],0] ~S[AI[a],x,x] ~S[x,y,0]
501S>(500a,473a) ~S[AI[a],x,x] ~S[x,y,0]
502: 501|[0/x,0/y] ~S[AI[a],0,0] ~S[0,0,0]
503S>(502b,136a) ~S[AI[a],0,0]

504: (23b,11c) S[x,0,0] defined[MI[x]] ~defined[x]
505: (504b,20b) S[x,0,0] ~defined[x] defined[AI[MI[x]]]
506S>(505b,44a) S[AI[a],0,0] defined[AI[MI[AI[a]]]]
507S>(506a,503a) defined[AI[MI[AI[a]]]]

508S>(23b,21a) S[x,0,x] ~defined[x]
509S>(508a,9d) ~defined[x] S[y,z,x] ~S[x,u,y] ~S[u,z,0]
510S>(509a,26a) S[x,y,0] ~S[0,z,x] ~S[z,y,0]
511S>(510b,11c) S[0,x,0] ~S[y,x,0] defined[MI[y]] ~defined[y]
512S>(511b,46a) S[0,a,0] defined[MI[AI[a]]] ~defined[AI[a]]
513S>(512a,494a) defined[MI[AI[a]]] ~defined[AI[a]]
514S>(513b,44a) defined[MI[AI[a]]]

515S>(44a,10c) P[MI[AI[a]],AI[a],1] S[0,AI[a],0]
516S>(515b,382a) P[MI[AI[a]],AI[a],1]
517S>(516a,1e) S[x,1,y] ~S[z,MI[AI[a]],u] ~P[u,AI[a],y] ~P[z,AI[a],x]
518S>(517b,18a) S[x,1,y] ~P[0,AI[a],y] ~P[AI[MI[AI[a]]],AI[a],x] ~defined[MI[AI[a]]]
519S>(518d,514a) S[x,1,y] ~P[0,AI[a],y] ~P[AI[MI[AI[a]]],AI[a],x]
520S>(519a,23b) ~P[0,AI[a],x] ~P[AI[MI[AI[a]]],AI[a],y] S[1,y,x]
521S>(520b,15a) ~P[0,AI[a],x] S[1,M[AI[MI[AI[a]]],AI[a],x] ~defined[AI[MI[AI[a]]]]
~defined[AI[a]]
522S>(521c,507a) ~P[0,AI[a],x] S[1,M[AI[MI[AI[a]]],AI[a],x] ~defined[AI[a]]
523S>(522c,44a) ~P[0,AI[a],x] S[1,M[AI[MI[AI[a]]],AI[a],x]
524S>(523b,23b) ~P[0,AI[a],x] S[M[AI[MI[AI[a]]],AI[a],1,x]
525S>(524a,170a) S[M[AI[MI[AI[a]]],AI[a],1,0]

526S>(44a,11b) defined[MI[AI[a]]] S[0,AI[a],0]
527S>(526b,382a) defined[MI[AI[a]]]
528S>(527a,18b) S[AI[MI[AI[a]]],MI[AI[a]],0]
529S>(528a,23b) S[MI[AI[a]],AI[MI[AI[a]]],0]
530S>(529a,4c) LE[0,x] ~LE[MI[AI[a]],y] ~S[y,AI[MI[AI[a]]],x]
531S>(530b,7b) LE[0,x] ~S[y,AI[MI[AI[a]]],x] LE[y,MI[AI[a]]] ~defined[y] ~defined[MI[AI[a]]]
532S>(531e,527a) LE[0,x] ~S[y,AI[MI[AI[a]]],x] LE[y,MI[AI[a]]] ~defined[y]
533S>(532b,21a) LE[0,AI[MI[AI[a]]]] LE[0,MI[AI[a]]] ~defined[0] ~defined[AI[MI[AI[a]]]]
534S>(533b,486a) LE[0,AI[MI[AI[a]]]] ~defined[0] ~defined[AI[MI[AI[a]]]]
535S>(534b,26a) LE[0,AI[MI[AI[a]]]] ~defined[AI[MI[AI[a]]]]
536S>(535b,507a) LE[0,AI[MI[AI[a]]]]

537S>(44a,10c) P[MI[AI[a]],AI[a],1] S[0,AI[a],0]
538S>(537b,382a) P[MI[AI[a]],AI[a],1]
539S>(538a,1e) S[x,1,y] ~S[z,MI[AI[a]],u] ~P[u,AI[a],y] ~P[z,AI[a],x]
540S>(539b,18a) S[x,1,y] ~P[0,AI[a],y] ~P[AI[MI[AI[a]]],AI[a],x] ~defined[MI[AI[a]]]
541S>(540d,514a) S[x,1,y] ~P[0,AI[a],y] ~P[AI[MI[AI[a]]],AI[a],x]
542S>(541b,22a) S[x,1,y] ~P[AI[MI[AI[a]]],AI[a],x] ~P[AI[a],0,y]
543S>(542b,15a) S[M[AI[MI[AI[a]]],AI[a],1,x] ~P[AI[a],0,x] ~defined[AI[MI[AI[a]]]]
~defined[AI[a]]
544S>(543c,507a) S[M[AI[MI[AI[a]]],AI[a],1,x] ~P[AI[a],0,x] ~defined[AI[a]]
545S>(544c,44a) S[M[AI[MI[AI[a]]],AI[a],1,x] ~P[AI[a],0,x]
546S>(545a,8c) ~P[AI[a],0,x] S[y,x,z] ~S[y,M[AI[MI[AI[a]]],AI[a],u] ~S[u,1,z]
547S>(546b,24a) ~P[AI[a],0,0] ~S[0,M[AI[MI[AI[a]]],AI[a],x] ~S[x,1,1]
548S>(547a,146a) ~S[0,M[AI[MI[AI[a]]],AI[a],x] ~S[x,1,1]
549S>(548b,32a) ~S[0,M[AI[MI[AI[a]]],AI[a],0]

550S>(44a,15c) P[x,AI[a],M[x,AI[a]]] ~defined[x]
551S>(550b,20a) P[AI[x],AI[a],M[AI[x],AI[a]]] ~defined[x]
552S>(551a,22b) ~defined[x] P[AI[a],AI[x],M[AI[x],AI[a]]]
553S>(552b,3d) ~defined[x] LE[0,M[AI[x],AI[a]]] ~LE[0,AI[a]] ~LE[0,AI[x]]
554S>(553c,194a) ~defined[x] LE[0,M[AI[x],AI[a]]] ~LE[0,AI[x]]
555S>(554a,11a) LE[0,M[AI[MI[x]],AI[a]]] ~LE[0,AI[MI[x]]] ~defined[x] S[0,x,0]
556S>(555a,12c) ~LE[0,AI[MI[x]]] ~defined[x] S[0,x,0] S[0,M[AI[MI[x]],AI[a],0]
~LE[M[AI[MI[x]],AI[a]],0]
557S>(556b,44a) ~LE[0,AI[MI[AI[a]]]] S[0,AI[a],0] S[0,M[AI[MI[AI[a]]],AI[a],0]
~LE[M[AI[MI[AI[a]]],AI[a]],0]
558S>(557a,536a) S[0,AI[a],0] S[0,M[AI[MI[AI[a]]],AI[a],0] ~LE[M[AI[MI[AI[a]]],AI[a]],0]
559S>(558a,382a) S[0,M[AI[MI[AI[a]]],AI[a],0] ~LE[M[AI[MI[AI[a]]],AI[a]],0]
560S>(559a,549a) ~LE[M[AI[MI[AI[a]]],AI[a]],0]

```

```

561: (20a,14b) ~defined[x] defined[M[AI[x],y]] ~defined[y]
562: (561a,11a) defined[M[AI[MI[x]],y]] ~defined[y] ~defined[x] S[0,x,0]
563S>(562b,44a) defined[M[AI[MI[x]],AI[a]]] ~defined[x] S[0,x,0]
564S>(563a,7d) ~defined[x] S[0,x,0] LE[y,M[AI[MI[x]],AI[a]]] LE[M[AI[MI[x]],AI[a]],y] ~defined[y]
565S>(564a,44a) S[0,AI[a],0] LE[x,M[AI[MI[AI[a]],AI[a]]] LE[M[AI[MI[AI[a]],AI[a]],x]
~defined[x]
566S>(565a,382a) LE[x,M[AI[MI[AI[a]],AI[a]]] LE[M[AI[MI[AI[a]],AI[a]],x] ~defined[x]
567S>(566a,4b) LE[M[AI[MI[AI[a]],AI[a]],x] ~defined[x] LE[y,z] ~S[x,u,y]
~S[M[AI[MI[AI[a]],AI[a]],u,z]
568S>(567b,26a) LE[M[AI[MI[AI[a]],AI[a]],0] LE[x,y] ~S[0,z,x] ~S[M[AI[MI[AI[a]],AI[a]],z,y]
569S>(568a,560a) LE[x,y] ~S[0,z,x] ~S[M[AI[MI[AI[a]],AI[a]],z,y]
570S>(569b,32a) LE[1,x] ~S[M[AI[MI[AI[a]],AI[a]],1,x]
571: 570{|0/x} LE[1,0] ~S[M[AI[MI[AI[a]],AI[a]],1,0]
572S>(571b,525a) LE[1,0]

573: (20a,17b) ~defined[x] S[AI[x],y,A[AI[x],y]] ~defined[y]
574S>(573a,25a) S[AI[1],x,A[AI[1],x]] ~defined[x]
575S>(574a,8c) ~defined[x] S[y,A[AI[1],x],z] ~S[y,AI[1],u] ~S[u,x,z]
576S>(575a,25a) S[x,A[AI[1],1],y] ~S[x,AI[1],z] ~S[z,1,y]
577S>(576b,18a) S[AI[AI[1]],A[AI[1],1],x] ~S[0,1,x] ~defined[AI[1]]
578S>(577c,259a) S[AI[AI[1]],A[AI[1],1],x] ~S[0,1,x]
579S>(578b,12a) S[AI[AI[1]],A[AI[1],1],x] ~LE[1,x] ~LE[x,1]
580S>(579a,242a) ~LE[1,0] ~LE[0,1]
581S>(580a,572a) ~LE[0,1]

582: (20a,17b) ~defined[x] S[AI[x],y,A[AI[x],y]] ~defined[y]
583S>(582a,25a) S[AI[1],x,A[AI[1],x]] ~defined[x]
584S>(583a,9b) ~defined[x] S[A[AI[1],x],y,z] ~S[x,y,u] ~S[AI[1],u,z]
585S>(584a,26a) S[A[AI[1],0],x,y] ~S[0,x,z] ~S[AI[1],z,y]
586S>(585a,4c) ~S[0,x,y] ~S[AI[1],y,z] LE[z,u] ~LE[A[AI[1],0],v] ~S[v,x,u]
587S>(586b,18a) ~S[0,x,1] LE[0,y] ~LE[A[AI[1],0],z] ~S[z,x,y] ~defined[1]
588S>(587e,25a) ~S[0,x,1] LE[0,y] ~LE[A[AI[1],0],z] ~S[z,x,y]
589S>(588ad) LE[0,1] ~LE[A[AI[1],0],0] ~S[0,x,1]
590S>(589a,581a) ~LE[A[AI[1],0],0] ~S[0,x,1]
591S>(590b,32a) ~LE[A[AI[1],0],0]

592: (20a,17b) ~defined[x] S[AI[x],y,A[AI[x],y]] ~defined[y]
593: (592b,4c) ~defined[x] ~defined[y] LE[A[AI[x],y],z] ~LE[AI[x],u] ~S[u,y,z]
594S>(593a,25a) ~defined[x] LE[A[AI[1],x],y] ~LE[AI[1],z] ~S[z,x,y]
595S>(594a,26a) LE[A[AI[1],0],x] ~LE[AI[1],y] ~S[y,0,x]
596S>(595b,7b) LE[A[AI[1],0],x] ~S[y,0,x] LE[y,AI[1]] ~defined[y] ~defined[AI[1]]
597S>(596e,259a) LE[A[AI[1],0],x] ~S[y,0,x] LE[y,AI[1]] ~defined[y]
598S>(597b*21a) LE[A[AI[1],0],0] LE[0,AI[1]] ~defined[0]
599S>(598a,591a) LE[0,AI[1]] ~defined[0]
600S>(599b,26a) LE[0,AI[1]]

601: (15a,3d) ~defined[x] ~defined[y] LE[0,M[x,y]] ~LE[0,x] ~LE[0,y]
602S>(601b,40a) ~defined[x] LE[0,M[x,AI[b]]] ~LE[0,x] ~LE[0,AI[b]]
603S>(602d,265a) ~defined[x] LE[0,M[x,AI[b]]] ~LE[0,x]
604S>(603a,20a) LE[0,M[AI[x],AI[b]]] ~LE[0,AI[x]] ~defined[x]
605S>(604a,12c) ~LE[0,AI[x]] ~defined[x] S[0,M[AI[x],AI[b]],0] ~LE[M[AI[x],AI[b]],0]
606S>(605b,25a) ~LE[0,AI[1]] S[0,M[AI[1],AI[b]],0] ~LE[M[AI[1],AI[b]],0]
607S>(606a,600a) S[0,M[AI[1],AI[b]],0] ~LE[M[AI[1],AI[b]],0]
608S>(607a,451a) ~LE[M[AI[1],AI[b]],0]
609S>(608a,4a) ~LE[x,y] ~S[x,z,M[AI[1],AI[b]]] ~S[y,z,0]
610S>(609a,29a) ~S[b,x,M[AI[1],AI[b]]] ~S[0,x,0]
611S>(610a,441a) ~S[0,0,0]
612S>(611a,269a) []

```

```

PHASE 0: 0 s PHASE 1: 495 s PHASE 2: 2248 s Total Time: 2743 s
NOD: 74928529 RES: 178835587 FAC: 50162452 T: 9 V: 32 L: 16
CTE: 15297479 CTH: 16218992 CTF: 3400382 CSZ: 16777216
UTE: 1352458 UTH: 20278384 UTF: 62435 SBA: 0
BAS: 31 RED: 50 LEN: 14+565=579

```

## FLD073-4

Predicates: sum product less\_or\_equal defined  
Functions: additive\_identity b a uC . multiplicative\_identity : additive\_inverse add multiply  
multiplicative\_inverse  
Renamed predicates: S P LE defined  
Renamed functions: 0 b a uC . 1 : AI A M MI

Cl3: xvl 9.14.4 t30 W6 29S LE[x,0] ... ?

Axioms:

```
1: S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]
2: P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]
3: LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]
4: >LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]
5: P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]
6: P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]
7: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
8: S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]
9: S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]
10: P[MI[x],x,1] S[0,x,0] ~defined[x]
11: defined[MI[x]] ~defined[x] S[0,x,0]
12: S[0,x,y] ~LE[x,y] ~LE[y,x]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: defined[M[x,y]] ~defined[x] ~defined[y]
15: P[x,y,M[x,y]] ~defined[x] ~defined[y]
16: defined[A[x,y]] ~defined[x] ~defined[y]
17: S[x,y,A[x,y]] ~defined[x] ~defined[y]
18: >S[AI[x],x,0] ~defined[x]
19: P[1,x,x] ~defined[x]
20: defined[AI[x]] ~defined[x]
21: >S[0,x,x] ~defined[x]
22: P[x,y,z] ~P[y,x,z]
23: S[x,y,z] ~S[y,x,z]
24: ~S[0,0,1]
25: defined[1]
26: defined[0]
```

Negated conclusion:

```
27S>~LE[0,uC]
28S LE[a,0]
29S>LE[x,0] ... ?
30S P[a,b,uC]
31S>defined[uC]
32S defined[a]
33S defined[b]
```

Phase 0 clauses used in proof:

```
42S>(31a*21b) S[0,uC,uC]
45S>(31a*18b) S[AI[uC],uC,0]
```

Phases 1 and 2 clauses used in proof:

```
50S>(45a,4c) LE[0,x] ~LE[AI[uC],y] ~S[y,uC,x]
51S>(50a,27a) ~LE[AI[uC],x] ~S[x,uC,uC]
52S>(51a,29a) ~S[0,uC,uC] ... ?
53S>(52a,42a) [] ... ?
```

Inferences derived from proof before weakened clause repaired

42 45 51

```
PHASE 0: 0 s PHASE 1: 2 s PHASE 2: -NA- Total Time: 2 s
NOD: 98 RES: 284 FAC: 28 T: 9 V: 32 L: 16
CTE: 276 CTH: 10 CTF: 0 CSZ: 16777216
UTE: 121 UTH: 67 UTF: 0 SBA: 0
BAS: 33 RED: 49 LEN: 2+4=6
```

Proof found to theorem by slave 13!

Axioms:

```
1 >S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]
2 >P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]
3 >LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]
4S>LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]
5 >P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]
6 >P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]
7: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
8 >S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]
9 >S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]
10: P[MI[x],x,1] S[0,x,0] ~defined[x]
```

```

11: defined[MI[x]] ~defined[x] S[0,x,0]
12: S[0,x,y] ~LE[x,y] ~LE[y,x]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14 >defined[M[x,y]] ~defined[x] ~defined[y]
15 >P[x,y,M[x,y]] ~defined[x] ~defined[y]
16: defined[A[x,y]] ~defined[x] ~defined[y]
17 >S[x,y,A[x,y]] ~defined[x] ~defined[y]
18S>S[AI[x],x,0] ~defined[x]
19 >P[1,x,x] ~defined[x]
20 >defined[AI[x]] ~defined[x]
21S>S[0,x,x] ~defined[x]
22 >P[x,y,z] ~P[y,x,z]
23 >S[x,y,z] ~S[y,x,z]
24: ~S[0,0,1]
25 >defined[1]
26 >defined[0]

```

Negated conclusion:

```

27S ~LE[0,uC]
28S>LE[a,0]
29S>LE[b,0]
30S>P[a,b,uC]
31S>defined[uC]
32S>defined[a]
33S>defined[b]

```

-----

Phase 0 clauses used in proof:

```

34S>S[0,uC,uC]
35S>S[AI[uC],uC,0]
36S>~LE[AI[uC],x] ~S[x,uC,uC]
37S>(36b*34a) ~LE[AI[uC],0]
38S>(35a*23b) S[uC,AI[uC],0]
39S>(34a*23b) S[uC,0,uC]
40S>(33a*21b) S[0,b,b]
41S>(33a*20b) defined[AI[b]]
42S>(33a*19b) P[1,b,b]
43S>(33a*18b) S[AI[b],b,0]
44S>(32a*21b) S[0,a,a]
45S>(32a*20b) defined[AI[a]]
46S>(32a*19b) P[1,a,a]
47S>(32a*18b) S[AI[a],a,0]
48S>(31a*20b) defined[AI[uC]]
49S>(31a*19b) P[1,uC,uC]

```

Phases 1 and 2 clauses used in proof:

```

50S>(44a,1b) S[x,y,z] ~P[a,u,z] ~P[0,u,x] ~P[a,u,y]
51S>(50a,23b) ~P[a,x,y] ~P[0,x,z] ~P[a,x,u] S[u,z,y]
52S>(51b,15a) ~P[a,x,y] ~P[a,x,z] S[z,M[0,x],y] ~defined[0] ~defined[x]
53S>(52d,26a) ~P[a,x,y] ~P[a,x,z] S[z,M[0,x],y] ~defined[x]
54S>(53a,30a) ~P[a,b,x] S[x,M[0,b],uC] ~defined[b]
55S>(54a,30a) S[uC,M[0,b],uC] ~defined[b]
56S>(55b,33a) S[uC,M[0,b],uC]

57S>(33a,14c) defined[M[x,b]] ~defined[x]
58S>(57a,21b) ~defined[x] S[0,M[x,b],M[x,b]]
59S>(58b,8d) ~defined[x] S[y,z,M[x,b]] ~S[y,u,0] ~S[u,M[x,b],z]
60S>(59a,26a) S[x,y,M[0,b]] ~S[x,z,0] ~S[z,M[0,b],y]
61S>(60a,9b) ~S[x,y,0] ~S[y,M[0,b],z] S[M[0,b],u,v] ~S[z,u,w] ~S[x,w,v]
62S>(61a,35a) ~S[uC,M[0,b],x] S[M[0,b],y,z] ~S[x,y,u] ~S[AI[uC],u,z]
63S>(62ac) S[M[0,b],M[0,b],x] ~S[uC,M[0,b],uC] ~S[AI[uC],uC,x]
64S>(63b,56a) S[M[0,b],M[0,b],x] ~S[AI[uC],uC,x]
65S>(64b,35a) S[M[0,b],M[0,b],0]

66S>(25a,21b) S[0,1,1]
67S>(66a,23b) S[1,0,1]

68S>(33a,15c) P[x,b,M[x,b]] ~defined[x]
69S>(68b,26a) P[0,b,M[0,b]]
70S>(69a,2d) P[x,b,y] ~S[z,0,x] ~P[z,b,u] ~S[u,M[0,b],y]
71S>(70b,21a) P[0,b,x] ~P[0,b,y] ~S[y,M[0,b],x] ~defined[0]
72S>(71d,26a) P[0,b,x] ~P[0,b,y] ~S[y,M[0,b],x]
73S>(72a,22b) ~P[0,b,x] ~S[x,M[0,b],y] P[b,0,y]
74S>(73a,69a) ~S[M[0,b],M[0,b],x] P[b,0,x]
75S>(74a,65a) P[b,0,0]

76S>(31a,15b) P[uC,x,M[uC,x]] ~defined[x]
77S>(76a,22b) ~defined[x] P[x,uC,M[uC,x]]
78S>(77b,1e) ~defined[x] S[y,M[uC,x],z] ~S[u,x,v] ~P[v,uC,z] ~P[u,uC,y]
79S>(78b,23b) ~defined[x] ~S[y,x,z] ~P[z,uC,u] ~P[y,uC,v] S[M[uC,x],v,u]
80S>(79a,26a) ~S[x,0,y] ~P[y,uC,z] ~P[x,uC,u] S[M[uC,0],u,z]

```

```

81S>(80b,49a) ~S[x,0,1] ~P[x,uC,y] S[M[uC,0],y,uC]
82S>(81b,49a) ~S[1,0,1] S[M[uC,0],uC,uC]
83S>(82a,67a) S[M[uC,0],uC,uC]

84S>(31a,15b) P[uC,x,M[uC,x]] ~defined[x]
85S>(84a,22b) ~defined[x] P[x,uC,M[uC,x]]
86S>(85a,26a) P[0,uC,M[uC,0]]

87S>(33a,15c) P[x,b,M[x,b]] ~defined[x]
88S>(87a,22b) ~defined[x] P[b,x,M[x,b]]
89S>(88b,5c) ~defined[x] P[y,M[x,b],z] ~P[y,b,u] ~P[u,x,z]
90S>(89a,26a) P[x,M[0,b],y] ~P[x,b,z] ~P[z,0,y]
91S>(90a,22b) ~P[x,b,y] ~P[y,0,z] P[M[0,b],x,z]
92S>(91a,42a) ~P[b,0,x] P[M[0,b],1,x]
93S>(92a,75a) P[M[0,b],1,0]

94S>(34a,1b) S[x,y,z] ~P[uC,u,z] ~P[0,u,x] ~P[uC,u,y]
95S>(94b,15a) S[x,y,M[uC,z]] ~P[0,z,x] ~P[uC,z,y] ~defined[uC] ~defined[z]
96S>(95d,31a) S[x,y,M[uC,z]] ~P[0,z,x] ~P[uC,z,y] ~defined[z]
97S>(96b*15a) S[M[0,x],y,M[uC,x]] ~P[uC,x,y] ~defined[x] ~defined[0]
98S>(97d,26a) S[M[0,x],y,M[uC,x]] ~P[uC,x,y] ~defined[x]
99S>(98a,8b) ~P[uC,x,y] ~defined[x] S[M[0,x],z,u] ~S[y,v,z] ~S[M[uC,x],v,u]
100S>(99a,22a) ~defined[x] S[M[0,x],y,z] ~S[u,v,y] ~S[M[uC,x],v,z] ~P[x,uC,u]
101S>(100a,26a) S[M[0,0],x,y] ~S[z,u,x] ~S[M[uC,0],u,y] ~P[0,uC,z]
102S>(101bc) S[M[0,0],x,x] ~S[M[uC,0],y,x] ~P[0,uC,M[uC,0]]
103S>(102c,86a) S[M[0,0],x,x] ~S[M[uC,0],y,x]
104: 103|{uC/x,uC/y} S[M[0,0],uC,uC] ~S[M[uC,0],uC,uC]
105S>(104b,83a) S[M[0,0],uC,uC]

106S>(49a,6d) P[x,y,uC] ~P[1,z,x] ~P[z,y,uC]
107S>(106a,22b) ~P[1,x,y] ~P[x,z,uC] P[z,y,uC]
108S>(107a,46a) ~P[a,x,uC] P[x,a,uC]
109S>(108a,30a) P[b,a,uC]

110S>(31a,17c) S[x,uC,A[x,uC]] ~defined[x]
111S>(110b,14a) S[M[x,y],uC,A[M[x,y],uC]] ~defined[x] ~defined[y]
112S>(111b,26a) S[M[0,x],uC,A[M[0,x],uC]] ~defined[x]
113S>(112b,26a) S[M[0,0],uC,A[M[0,0],uC]]
114S>(113a,9b) S[A[M[0,0],uC],x,y] ~S[uC,x,z] ~S[M[0,0],z,y]
115S>(114b,39a) S[A[M[0,0],uC],0,x] ~S[M[0,0],uC,x]
116: 115|{uC/x} S[A[M[0,0],uC],0,uC] ~S[M[0,0],uC,uC]
117S>(116b,105a) S[A[M[0,0],uC],0,uC]

118S>(33a,15b) P[b,x,M[b,x]] ~defined[x]
119S>(118b,25a) P[b,1,M[b,1]]
120S>(119a,5c) P[x,M[b,1],y] ~P[x,b,z] ~P[z,1,y]
121S>(120b,15a) P[x,M[b,1],y] ~P[M[x,b],1,y] ~defined[x] ~defined[b]
122S>(121d,33a) P[x,M[b,1],y] ~P[M[x,b],1,y] ~defined[x]
123S>(122a,22b) ~P[M[x,b],1,y] ~defined[x] P[M[b,1],x,y]
124S>(123b,26a) ~P[M[0,b],1,x] P[M[b,1],0,x]
125S>(124a,93a) P[M[b,1],0,0]

126S>(32a,15c) P[x,a,M[x,a]] ~defined[x]
127S>(126a,1d) ~defined[x] S[M[x,a],y,z] ~S[x,u,v] ~P[v,a,z] ~P[u,a,y]
128S>(127b,23b) ~defined[x] ~S[x,y,z] ~P[z,a,u] ~P[y,a,v] S[v,M[x,a],u]
129S>(128b,40a) ~defined[0] ~P[b,a,x] ~P[b,a,y] S[y,M[0,a],x]
130S>(129a,26a) ~P[b,a,x] ~P[b,a,y] S[y,M[0,a],x]
131S>(130ab) ~P[b,a,x] S[x,M[0,a],x]
132S>(131a,109a) S[uC,M[0,a],uC]

133S>(31a,17c) S[x,uC,A[x,uC]] ~defined[x]
134S>(133b,14a) S[M[x,y],uC,A[M[x,y],uC]] ~defined[x] ~defined[y]
135S>(134b,26a) S[M[0,x],uC,A[M[0,x],uC]] ~defined[x]
136S>(135a,8b) ~defined[x] S[M[0,x],y,z] ~S[uC,u,y] ~S[A[M[0,x],uC],u,z]
137S>(136b,23b) ~defined[x] ~S[uC,y,z] ~S[A[M[0,x],uC],y,u] S[z,M[0,x],u]
138S>(137a,26a) ~S[uC,x,y] ~S[A[M[0,0],uC],x,z] S[y,M[0,0],z]
139S>(138a,39a) ~S[A[M[0,0],uC],0,x] S[uC,M[0,0],x]
140S>(139a,117a) S[uC,M[0,0],uC]

141S>(32a,14c) defined[M[x,a]] ~defined[x]
142S>(141a,21b) ~defined[x] S[0,M[x,a],M[x,a]]
143S>(142b,8d) ~defined[x] S[y,z,M[x,a]] ~S[y,u,0] ~S[u,M[x,a],z]
144S>(143a,26a) S[x,y,M[0,a]] ~S[x,z,0] ~S[z,M[0,a],y]
145S>(144a,9b) ~S[x,y,0] ~S[y,M[0,a],z] S[M[0,a],u,v] ~S[z,u,w] ~S[x,w,v]
146S>(145a,35a) ~S[uC,M[0,a],x] S[M[0,a],y,z] ~S[x,y,u] ~S[AI[uC],u,z]
147S>(146ac) S[M[0,a],M[0,a],x] ~S[uC,M[0,a],uC] ~S[AI[uC],uC,x]
148S>(147b,132a) S[M[0,a],M[0,a],x] ~S[AI[uC],uC,x]
149S>(148b,35a) S[M[0,a],M[0,a],0]

150S>(33a,15b) P[b,x,M[b,x]] ~defined[x]
151S>(150b,25a) P[b,1,M[b,1]]

```

```

152S>(151a,5b) P[b,x,y] ~P[l,z,x] ~P[M[b,1],z,y]
153S>(152b,19a) P[b,x,y] ~P[M[b,1],x,y] ~defined[x]
154S>(153a,22b) ~P[M[b,1],x,y] ~defined[x] P[x,b,y]
155S>(154b,26a) ~P[M[b,1],0,x] P[0,b,x]
156S>(155a,125a) P[0,b,0]

157: (26a,14c) defined[M[x,0]] ~defined[x]
158S>(157a,21b) ~defined[x] S[0,M[x,0],M[x,0]]
159S>(158b,8d) ~defined[x] S[y,z,M[x,0]] ~S[y,u,0] ~S[u,M[x,0],z]
160S>(159a,26a) S[x,y,M[0,0]] ~S[x,z,0] ~S[z,M[0,0],y]
161S>(160a,9b) ~S[x,y,0] ~S[y,M[0,0],z] S[M[0,0],u,v] ~S[z,u,w] ~S[x,w,v]
162S>(161a,35a) ~S[uC,M[0,0],x] S[M[0,0],y,z] ~S[x,y,u] ~S[AI[uC],u,z]
163S>(162ac) S[M[0,0],M[0,0],x] ~S[uC,M[0,0],uC] ~S[AI[uC],uC,x]
164S>(163b,140a) S[M[0,0],M[0,0],x] ~S[AI[uC],uC,x]
165S>(164b,35a) S[M[0,0],M[0,0],0]

166S>(32a,15c) P[x,a,M[x,a]] ~defined[x]
167S>(166b,26a) P[0,a,M[0,a]]
168S>(167a,2d) P[x,a,y] ~S[z,0,x] ~P[z,a,u] ~S[u,M[0,a],y]
169S>(168b,21a) P[0,a,x] ~P[0,a,y] ~S[y,M[0,a],x] ~defined[0]
170S>(169d,26a) P[0,a,x] ~P[0,a,y] ~S[y,M[0,a],x]
171S>(170a,22b) ~P[0,a,x] ~S[x,M[0,a],y] P[a,0,y]
172S>(171a,167a) ~S[M[0,a],M[0,a],x] P[a,0,x]
173S>(172a,149a) P[a,0,0]

174S>(33a,15b) P[b,x,M[b,x]] ~defined[x]
175S>(174b,20a) P[b,AI[x],M[b,AI[x]]] ~defined[x]
176S>(175a,22b) ~defined[x] P[AI[x],b,M[b,AI[x]]]
177S>(176b,1d) ~defined[x] S[M[b,AI[x]],y,z] ~S[AI[x],u,v] ~P[v,b,z] ~P[u,b,y]
178S>(177b,23b) ~defined[x] ~S[AI[x],y,z] ~P[z,b,u] ~P[y,b,v] S[v,M[b,AI[x]],u]
179S>(178b,47a) ~defined[a] ~P[0,b,x] ~P[a,b,y] S[y,M[b,AI[a]],x]
180S>(179a,32a) ~P[0,b,x] ~P[a,b,y] S[y,M[b,AI[a]],x]
181S>(180b,30a) ~P[0,b,x] S[uC,M[b,AI[a]],x]
182S>(181a,156a) S[uC,M[b,AI[a]],0]

183S>(43a,23b) S[b,AI[b],0]

184S>(45a,15c) P[x,AI[a],M[x,AI[a]]] ~defined[x]
185S>(184a,22b) ~defined[x] P[AI[a],x,M[x,AI[a]]]
186S>(185a,26a) P[AI[a],0,M[0,AI[a]]]

187S>(47a,23b) S[a,AI[a],0]

188S>(21b,14a) S[0,M[x,y],M[x,y]] ~defined[x] ~defined[y]
189S>(188a,8c) ~defined[x] ~defined[y] S[z,M[x,y],u] ~S[z,0,v] ~S[v,M[x,y],u]
190S>(189b,45a) ~defined[x] S[y,M[x,AI[a]],z] ~S[y,0,u] ~S[u,M[x,AI[a]],z]
191S>(190b,23b) ~defined[x] ~S[y,0,z] ~S[z,M[x,AI[a]],u] S[M[x,AI[a]],y,u]
192S>(191b,39a) ~defined[x] ~S[uC,M[x,AI[a]],y] S[M[x,AI[a]],uC,y]
193S>(192a,33a) ~S[uC,M[b,AI[a]],x] S[M[b,AI[a]],uC,x]
194S>(193a,182a) S[M[b,AI[a]],uC,0]

195: (26a,15c) P[x,0,M[x,0]] ~defined[x]
196: (195b,26a) P[0,0,M[0,0]]
197: (196a,2d) P[x,0,y] ~S[z,0,x] ~P[z,0,u] ~S[u,M[0,0],y]
198S>(197b,21a) P[0,0,x] ~P[0,0,y] ~S[y,M[0,0],x] ~defined[0]
199S>(198d,26a) P[0,0,x] ~P[0,0,y] ~S[y,M[0,0],x]
200S>(199a,22b) ~P[0,0,x] ~S[x,M[0,0],y] P[0,0,y]
201S>(200a,196a) ~S[M[0,0],M[0,0],x] P[0,0,x]
202S>(201a,165a) P[0,0,0]

203: (26a,14b) defined[M[0,x]] ~defined[x]
204S>(203a,21b) ~defined[x] S[0,M[0,x],M[0,x]]
205S>(204b,23b) ~defined[x] S[M[0,x],0,M[0,x]]
206S>(205b,2e) ~defined[x] P[y,z,M[0,x]] ~S[u,v,y] ~P[u,z,M[0,x]] ~P[v,z,0]
207S>(206b,22b) ~defined[x] ~S[y,z,u] ~P[y,v,M[0,x]] ~P[z,v,0] P[v,u,M[0,x]]
208S>(207a,45a) ~S[x,y,z] ~P[x,u,M[0,AI[a]]] ~P[y,u,0] P[u,z,M[0,AI[a]]]
209S>(208a,47a) ~P[AI[a],x,M[0,AI[a]]] ~P[a,x,0] P[x,0,M[0,AI[a]]]
210S>(209a,186a) ~P[a,0,0] P[0,0,M[0,AI[a]]]
211S>(210a,173a) P[0,0,M[0,AI[a]]]

212S>(41a,21b) S[0,AI[b],AI[b]]
213S>(212a,4d) LE[x,AI[b]] ~LE[y,0] ~S[y,AI[b],x]
214S>(213b,29a) LE[x,AI[b]] ~S[b,AI[b],x]
215: 214|{0/x} LE[0,AI[b]] ~S[b,AI[b],0]
216S>(215b,183a) LE[0,AI[b]]

217S>(45a,21b) S[0,AI[a],AI[a]]
218S>(217a,4d) LE[x,AI[a]] ~LE[y,0] ~S[y,AI[a],x]
219S>(218b,28a) LE[x,AI[a]] ~S[a,AI[a],x]
220: 219|{0/x} LE[0,AI[a]] ~S[a,AI[a],0]
221S>(220b,187a) LE[0,AI[a]]

```

```

222S>(48a,21b) S[0,AI[uC],AI[uC]]
223S>(45a,15c) P[x,AI[a],M[x,AI[a]]] ~defined[x]
224S>(223b,26a) P[0,AI[a],M[0,AI[a]]]
225S>(224a,6c) P[x,AI[a],y] ~P[z,0,x] ~P[z,M[0,AI[a]],y]
226S>(225b,22a) P[x,AI[a],y] ~P[z,M[0,AI[a]],y] ~P[0,z,x]
227S>(226b,5a) P[x,AI[a],y] ~P[0,z,x] ~P[z,u,v] ~P[u,w,M[0,AI[a]]] ~P[v,w,y]
228S>(227be) P[x,AI[a],x] ~P[y,z,0] ~P[z,y,M[0,AI[a]]] ~P[0,y,x]
229S>(228bd) P[0,AI[a],0] ~P[0,0,M[0,AI[a]]] ~P[0,0,0]
230S>(229b,211a) P[0,AI[a],0] ~P[0,0,0]
231S>(230b,202a) P[0,AI[a],0]

232S>(45a,14c) defined[M[x,AI[a]]] ~defined[x]
233S>(232b,33a) defined[M[b,AI[a]]]
234S>(233a,21b) S[0,M[b,AI[a]],M[b,AI[a]]]
235S>(234a,9b) S[M[b,AI[a]],x,y] ~S[M[b,AI[a]],x,z] ~S[0,z,y]
236S>(235b,8a) S[M[b,AI[a]],x,y] ~S[0,z,y] ~S[M[b,AI[a]],u,v] ~S[u,w,x] ~S[v,w,z]
237S>(236a,23b) ~S[0,x,y] ~S[M[b,AI[a]],z,u] ~S[z,v,w] ~S[u,v,x] S[w,M[b,AI[a]],y]
238S>(237ad) ~S[M[b,AI[a]],x,0] ~S[x,y,z] ~S[0,y,y] S[z,M[b,AI[a]],y]
239S>(238b,38a) ~S[M[b,AI[a]],uC,0] ~S[0,AI[uC],AI[uC]] S[0,M[b,AI[a]],AI[uC]]
240S>(239a,194a) ~S[0,AI[uC],AI[uC]] S[0,M[b,AI[a]],AI[uC]]
241S>(240a,222a) S[0,M[b,AI[a]],AI[uC]]

242S>(41a,15b) P[AI[b],x,M[AI[b],x]] ~defined[x]
243S>(242a,3d) ~defined[x] LE[0,M[AI[b],x]] ~LE[0,AI[b]] ~LE[0,x]
244S>(243c,216a) ~defined[x] LE[0,M[AI[b],x]] ~LE[0,x]
245S>(244a,45a) LE[0,M[AI[b],AI[a]]] ~LE[0,AI[a]]
246S>(245b,221a) LE[0,M[AI[b],AI[a]]]

247S>(43a,1b) S[x,y,z] ~P[0,u,z] ~P[AI[b],u,x] ~P[b,u,y]
248S>(247b,22a) S[x,y,z] ~P[AI[b],u,x] ~P[b,u,y] ~P[u,0,z]
249S>(248b,15a) S[M[AI[b],x],y,z] ~P[b,x,y] ~P[x,0,z] ~defined[AI[b]] ~defined[x]
250S>(249d,41a) S[M[AI[b],x],y,z] ~P[b,x,y] ~P[x,0,z] ~defined[x]
251S>(250b*15a) S[M[AI[b],x],M[b,x],y] ~P[x,0,y] ~defined[x] ~defined[b]
252S>(251d,33a) S[M[AI[b],x],M[b,x],y] ~P[x,0,y] ~defined[x]
253S>(252b,22a) S[M[AI[b],x],M[b,x],y] ~defined[x] ~P[0,x,y]
254S>(253a,4d) ~defined[x] ~P[0,x,y] LE[z,y] ~LE[u,M[AI[b],x]] ~S[u,M[b,x],z]
255S>(254a,45a) ~P[0,AI[a],x] LE[y,x] ~LE[z,M[AI[b],AI[a]]] ~S[z,M[b,AI[a]],y]
256S>(255b,37a) ~P[0,AI[a],0] ~LE[x,M[AI[b],AI[a]]] ~S[x,M[b,AI[a]],AI[uC]]
257S>(256a,231a) ~LE[x,M[AI[b],AI[a]]] ~S[x,M[b,AI[a]],AI[uC]]
258S>(257a,246a) ~S[0,M[b,AI[a]],AI[uC]]
259S>(258a,241a) []

```

```

PHASE 0: 0 s PHASE 1: 807 s PHASE 2: 2862 s Total Time: 3671 s
NOD: 30628447 RES: 88902878 FAC: 19813074 T: 9 v: 32 L: 16
CTE: 12118776 CTH: 7510311 CTF: 149793 CSZ: 16777216
UTE: 2371323 UTH: 12638842 UTF: 1666 SBA: 0
BAS: 33 RED: 49 LEN: 16+210=226

```

## FLD079-4

Predicates: sum product less\_or\_equal defined  
Functions: additive\_identity a multiplicative\_identity b uC : . additive\_inverse add multiply  
multiplicative\_inverse  
Renamed predicates: S P LE defined  
Renamed functions: 0 a l b uC : . AI A M MI

C37: xvl 9.14.4 t3 W18 24S ~S[x,0,1] ... ?

Axioms:

```
1: S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]
2: P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]
3: LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]
4: LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]
5: P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]
6: P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]
7: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
8: S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]
9: S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]
10: P[MI[x],x,1] S[0,x,0] ~defined[x]
11: defined[MI[x]] ~defined[x] S[0,x,0]
12: S[0,x,y] ~LE[x,y] ~LE[y,x]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: defined[M[x,y]] ~defined[x] ~defined[y]
15: P[x,y,M[x,y]] ~defined[x] ~defined[y]
16: defined[A[x,y]] ~defined[x] ~defined[y]
17: S[x,y,A[x,y]] ~defined[x] ~defined[y]
18: S[AI[x],x,0] ~defined[x]
19: P[1,x,x] ~defined[x]
20: defined[AI[x]] ~defined[x]
21 >S[0,x,x] ~defined[x]
22: P[x,y,z] ~P[y,x,z]
23 >S[x,y,z] ~S[y,x,z]
24S>~S[x,0,1] ... ?
25 >defined[1]
26: defined[0]
```

Negated conclusion:

```
27S LE[uC,1]
28S ~S[0,b,0]
29S LE[0,b]
30S P[a,MI[b],uC]
31S ~LE[a,b]
32S defined[uC]
33S defined[a]
34S defined[b]
```

Phases 1 and 2 clauses used in proof:

52: (25a,21b) S[0,1,1]

53S>(24a,23a) ~S[0,x,1] ... ?

54S>(53a,52a) [] ... ?

Inferences derived when weakened clause replaced with given clause

53

```
PHASE 0: 0 s PHASE 1: 1 s PHASE 2: 1 s Total Time: 2 s
NOD: 7296 RES: 30613 FAC: 1911 T: 9 V: 32 L: 16
CTE: 7673 CTH: 345 CTF: 0 CSZ: 16777216
UTE: 8425 UTH: 9218 UTF: 0 SBA: 0
BAS: 34 RED: 51 LEN: 0+3=3
```

Proof found to theorem by slave 37!

Axioms:

```
1 >S[x,y,z] ~S[u,v,w] ~P[w,t,z] ~P[u,t,x] ~P[v,t,y]
2: P[x,y,z] ~S[u,v,x] ~P[u,y,w] ~P[v,y,t] ~S[w,t,z]
3 >LE[0,x] ~LE[0,y] ~LE[0,z] ~P[y,z,x]
4 >LE[x,y] ~LE[z,u] ~S[z,v,x] ~S[u,v,y]
5 >P[x,y,z] ~P[x,u,v] ~P[u,w,y] ~P[v,w,z]
6 >P[x,y,z] ~P[u,v,x] ~P[v,y,w] ~P[u,w,z]
7: LE[x,y] LE[y,x] ~defined[x] ~defined[y]
8 >S[x,y,z] ~S[x,u,v] ~S[u,w,y] ~S[v,w,z]
9 >S[x,y,z] ~S[u,v,x] ~S[v,y,w] ~S[u,w,z]
10 >P[MI[x],x,1] S[0,x,0] ~defined[x]
11: defined[MI[x]] ~defined[x] S[0,x,0]
12: S[0,x,y] ~LE[x,y] ~LE[y,x]
13: LE[x,y] ~LE[x,z] ~LE[z,y]
14: defined[M[x,y]] ~defined[x] ~defined[y]
```

```

15 >P[x,y,M[x,y]] ~defined[x] ~defined[y]
16 >defined[A[x,y]] ~defined[x] ~defined[y]
17 >S[x,y,A[x,y]] ~defined[x] ~defined[y]
18 >S[AI[x],x,0] ~defined[x]
19 >P[1,x,x] ~defined[x]
20 >defined[AI[x]] ~defined[x]
21S>S[0,x,x] ~defined[x]
22 >P[x,y,z] ~P[y,x,z]
23S>S[x,y,z] ~S[y,x,z]
24S ~S[0,0,1]
25S>defined[1]
26 >defined[0]

```

Negated conclusion:

```

27S>LE[uC,1]
28S>~S[0,b,0]
29S>LE[0,b]
30S>P[a,MI[b],uC]
31S>~LE[a,b]
32S>defined[uC]
33S>defined[a]
34S>defined[b]

```

-----  
Phase 0 clauses used in proof:

```

35S>S[0,1,1]
36S>(35a*23b) S[1,0,1]
37S>(34a*21b) S[0,b,b]
39S>(34a*19b) P[1,b,b]
41S>(33a*21b) S[0,a,a]
43S>(33a*19b) P[1,a,a]
46S>(32a*20b) defined[AI[uC]]
47S>(32a*19b) P[1,uC,uC]
48S>(32a*18b) S[AI[uC],uC,0]
49S>(30a*22b) P[MI[b],a,uC]
51S>(26a*21b) S[0,0,0]

```

Phases 1 and 2 clauses used in proof:

```

52S>(51a,8d) S[x,y,0] ~S[x,z,0] ~S[z,0,y]
53S>(52b,51a) S[0,x,0] ~S[0,0,x]
54S>(53a,28a) ~S[0,0,b]

55S>(51a,8b) S[0,x,y] ~S[0,z,x] ~S[0,z,y]
56S>(55b,10b) S[0,0,x] ~S[0,y,x] P[MI[y],y,1] ~defined[y]
57S>(56b,37a) S[0,0,b] P[MI[b],b,1] ~defined[b]
58S>(57a,54a) P[MI[b],b,1] ~defined[b]
59S>(58b,34a) P[MI[b],b,1]

60S>(49a,5c) P[x,uC,y] ~P[x,MI[b],z] ~P[z,a,y]
61S>(60b,22a) P[x,uC,y] ~P[z,a,y] ~P[MI[b],x,z]
62S>(61b,43a) P[x,uC,a] ~P[MI[b],x,1]
63: 62|{b/x} P[b,uC,a] ~P[MI[b],b,1]
64S>(63b,59a) P[b,uC,a]

65S>(46a,16c) defined[A[x,AI[uC]]] ~defined[x]
66S>(65b,25a) defined[A[1,AI[uC]]]

67S>(47a,6b) P[uC,x,y] ~P[uC,x,z] ~P[1,z,y]
68S>(67b,22a) P[uC,x,y] ~P[1,z,y] ~P[x,uC,z]
69S>(68b,43a) P[uC,x,a] ~P[x,uC,a]
70: 69|{b/x} P[uC,b,a] ~P[b,uC,a]
71S>(70b,64a) P[uC,b,a]

72S>(48a,9c) S[x,uC,y] ~S[z,AI[uC],x] ~S[z,0,y]
73S>(72b,17a) S[A[x,AI[uC]],uC,y] ~S[x,0,y] ~defined[x] ~defined[AI[uC]]
74S>(73d,46a) S[A[x,AI[uC]],uC,y] ~S[x,0,y] ~defined[x]
75S>(74b,36a) S[A[1,AI[uC]],uC,1] ~defined[1]
76S>(75b,25a) S[A[1,AI[uC]],uC,1]
77S>(76a,1b) S[x,y,z] ~P[1,u,z] ~P[A[1,AI[uC]],u,x] ~P[uC,u,y]
78S>(77b,39a) S[x,y,b] ~P[A[1,AI[uC]],b,x] ~P[uC,b,y]
79S>(78b,15a) S[M[A[1,AI[uC]],b],x,b] ~P[uC,b,x] ~defined[A[1,AI[uC]]] ~defined[b]
80S>(79c,66a) S[M[A[1,AI[uC]],b],x,b] ~P[uC,b,x] ~defined[b]
81S>(80c,34a) S[M[A[1,AI[uC]],b],x,b] ~P[uC,b,x]
82: 81|{a/x} S[M[A[1,AI[uC]],b],a,b] ~P[uC,b,a]
83S>(82b,71a) S[M[A[1,AI[uC]],b],a,b]

84S>(27a,4b) LE[x,y] ~S[uC,z,x] ~S[1,z,y]
85S>(84b,23a) LE[x,y] ~S[1,z,y] ~S[z,uC,x]
86S>(85b,17a) LE[x,A[1,y]] ~S[y,uC,x] ~defined[1] ~defined[y]
87S>(86c,25a) LE[x,A[1,y]] ~S[y,uC,x] ~defined[y]
88S>(87b,48a) LE[0,A[1,AI[uC]]] ~defined[AI[uC]]

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89S>(88b,46a) LE[0,A[1,AI[uC]]]

90S>(34a,15c) P[x,b,M[x,b]] ~defined[x]
91S>(90b,16a) P[A[x,y],b,M[A[x,y],b]] ~defined[x] ~defined[y]
92S>(91b,25a) P[A[1,x],b,M[A[1,x],b]] ~defined[x]
93S>(92a,3d) ~defined[x] LE[0,M[A[1,x],b]] ~LE[0,A[1,x]] ~LE[0,b]
94S>(93d,29a) ~defined[x] LE[0,M[A[1,x],b]] ~LE[0,A[1,x]]
95S>(94a,46a) LE[0,M[A[1,AI[uC]],b]] ~LE[0,A[1,AI[uC]]]
96S>(95b,89a) LE[0,M[A[1,AI[uC]],b]]
97S>(96a,4b) LE[x,y] ~S[0,z,x] ~S[M[A[1,AI[uC]],b],z,y]
98S>(97a,31a) ~S[0,x,a] ~S[M[A[1,AI[uC]],b],x,b]
99S>(98a,41a) ~S[M[A[1,AI[uC]],b],a,b]
100S>(99a,83a) []

PHASE 0: 0 s PHASE 1: 385 s PHASE 2: 204 s Total Time: 591 s
NOD: 4578525 RES: 17656314 FAC: 1712781 T: 9 V: 32 L: 16
CTE: 4375326 CTH: 3111865 CTF: 2 CSZ: 16777216
UTE: 2536786 UTH: 11335554 UTF: 0 SBA: 0
BAS: 34 RED: 52 LEN: 11+49=60

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## NUM044-1

Predicates: operation compatible equal homomorphism member function subclass connected well\_ordering  
single\_valued\_class inductive section irreflexive asymmetric transitive maps one\_to\_one

Functions: yC xC null\_class identity\_relation . add\_relation ordinal\_numbers singleton\_relation  
subset\_relation omega rest\_relation domain\_relation composition\_function choice limit\_ordinals  
element\_relation kind\_l\_ordinals universal\_class successor\_relation application\_function  
union\_of\_range\_map : flip rotate image singleton compose\_class successor least inverse sum\_class  
restrict segment complement unordered\_pair intersection not\_well\_ordering range\_of  
symmetric\_difference not\_subclass\_element union symmetrization\_of  
ordinal\_multiply cross\_product first second single\_valued1 single\_valued2 single\_valued3 recursion  
power\_class regular ordinal\_add diagonalise domain range integer\_of recursion\_equation\_functions  
cantor rest\_of compose domain\_of not\_homomorphism2 not\_homomorphism1 ordered\_pair apply

Renamed predicates: operation compatible E. HOM MBR function subclass connected well\_ordering  
single\_valued\_class inductive section irreflexive asymmetric transitive maps one\_to\_one

Renamed functions: yC xC NULC identity\_relation . add\_relation ordinal\_numbers singleton\_relation  
SUBR omega rest\_relation DOMR CMPF choice limit\_ordinals  
element\_relation kind\_l\_ordinals UCLS SUCR application\_function  
union\_of\_range\_map : flip rotate image singleton compose\_class SUC least INV sum\_class  
restrict segment complement UNPR INT not\_well\_ordering range\_of  
symmetric\_difference not\_subclass\_element U symmetrization\_of  
ordinal\_multiply CXP first second single\_valued1 single\_valued2 single\_valued3 recursion  
power\_class regular ordinal\_add diagonalise DOMN range integer\_of recursion\_equation\_functions  
cantor rest\_of COMP domain\_of not\_homomorphism2 not\_homomorphism1 ORP APY

C86: xv1 40.20.3 t14 W43 234S E.[INT[INV[SUBR],SUBR],x] ... ?

Axioms:

1: ~operation[x] ~operation[y] ~compatible[z,x,y]  
~E.[APY[y,ORP[APY[z,not\_homomorphism1[z,x,y]],APY[z,not\_homomorphism2[z,x,y]]],APY[z,  
APY[x,ORP[not\_homomorphism1[z,x,y],not\_homomorphism2[z,x,y]]]] HOM[z,x,y]  
2: ~operation[x] ~operation[y] ~compatible[z,x,y]  
MBR[ORP[not\_homomorphism1[z,x,y],not\_homomorphism2[z,x,y]],domain\_of[x]] HOM[z,x,y]  
3: ~function[x] ~function[y] ~MBR[domain\_of[y],ordinal\_numbers] ~E.[COMP[x,rest\_of[y]],y]  
MBR[y,recursion\_equation\_functions[x]]  
4: ~function[x] ~E.[CXP[domain\_of[domain\_of[x]],domain\_of[domain\_of[x]],domain\_of[x]]  
~subclass[range\_of[x],domain\_of[domain\_of[x]]] operation[x]  
5: ~function[x] ~E.[domain\_of[domain\_of[y]],domain\_of[x]]  
~subclass[range\_of[x],domain\_of[domain\_of[z]]] compatible[x,y,z]  
6: ~MBR[x,not\_well\_ordering[y,z]] ~E.[segment[y,not\_well\_ordering[y,z],x],NULC] ~connected[y,z]  
well\_ordering[y,z]  
7: ~well\_ordering[element\_relation,x] ~subclass[sum\_class[x],x] ~MBR[x,UCLS] MBR[x,ordinal\_numbers]  
8: ~well\_ordering[element\_relation,x] ~subclass[sum\_class[x],x] MBR[x,ordinal\_numbers]  
E.[x,ordinal\_numbers]  
9: ~well\_ordering[x,y] ~subclass[z,y] ~MBR[u,z] ~MBR[ORP[u,least[x,z]],x]  
10: ~well\_ordering[x,y] ~subclass[z,y] E.[z,NULC] MBR[least[x,z],z]  
11: ~well\_ordering[x,y] ~subclass[z,y] ~MBR[u,z] MBR[least[x,z],z]  
12: ~MBR[ORP[x,ORP[y,z]],CXP[UCLS,CXP[UCLS,UCLS]]] ~MBR[y,domain\_of[x]]  
MBR[ORP[x,ORP[y,APY[x,y]]],application\_function]  
13: ~MBR[ORP[ORP[x,y],z],u] ~MBR[ORP[ORP[z,x],y],CXP[CXP[UCLS,UCLS],UCLS]]  
MBR[ORP[ORP[z,x],y],rotate[u]]  
14: ~MBR[ORP[ORP[x,y],z],u] ~MBR[ORP[ORP[y,x],z],CXP[CXP[UCLS,UCLS],UCLS]]  
MBR[ORP[ORP[y,x],z],flip[u]]  
15: ~HOM[x,y,z] ~MBR[ORP[u,v],domain\_of[y]] E.[APY[z,ORP[APY[x,u],APY[x,v]]],APY[x,APY[y,ORP[u,v]]]]  
16: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] ~E.[sum\_class[range\_of[x]],y] MBR[ORP[x,y],union\_of\_range\_map]  
17: ~MBR[x,image[y,image[z, singleton[u]]]] ~MBR[ORP[u,x],CXP[UCLS,UCLS]] MBR[ORP[u,x],COMP[y,z]]  
18: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] ~E.[COMP[z,x],y] MBR[ORP[x,y],compose\_class[z]]  
19: ~E.[SUC[x],y] ~MBR[ORP[x,y],CXP[UCLS,UCLS]] MBR[ORP[x,y],SUCR]  
20: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] ~MBR[x,y] MBR[ORP[x,y],element\_relation]  
21: ~subclass[x,CXP[UCLS,UCLS]] ~subclass[COMP[x,INV[x]],identity\_relation] function[x]  
22: ~MBR[x,UCLS] E.[restrict[y, singleton[x],UCLS],NULC] MBR[x,domain\_of[y]]  
23: ~MBR[x,domain\_of[y]] ~E.[restrict[y,x,UCLS],z] MBR[ORP[x,z],rest\_of[y]]  
24: ~E.[x,y] ~single\_valued\_class[x] single\_valued\_class[y]  
25: ~well\_ordering[x,y] ~subclass[z,y] E.[segment[x,z,least[x,z]],NULC]  
26: ~connected[x,y] ~E.[not\_well\_ordering[x,y],NULC] well\_ordering[x,y]  
27: ~function[x] ~MBR[y,UCLS] MBR[image[x,y],UCLS]  
28: ~MBR[x,y] ~MBR[z,u] MBR[ORP[x,z],CXP[y,u]]  
29: ~MBR[NULC,x] ~subclass[image[SUCR,x],x] inductive[x]  
30: ~connected[x,y] subclass[not\_well\_ordering[x,y],y] well\_ordering[x,y]  
31: ~E.[x,y] ~HOM[x,z,u] HOM[y,z,u]  
32: ~E.[x,y] ~HOM[z,x,u] HOM[z,y,u]  
33: ~E.[x,y] ~HOM[z,u,x] HOM[z,u,y]  
34: ~MBR[x,UCLS] E.[x,NULC] MBR[APY[choice,x],x]  
35: ~E.[x,y] ~well\_ordering[x,z] well\_ordering[y,z]  
36: ~E.[x,y] ~well\_ordering[z,x] well\_ordering[z,y]  
37: ~E.[x,y] ~compatible[x,z,u] compatible[y,z,u]  
38: ~E.[x,y] ~compatible[z,x,u] compatible[z,y,u]

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39: ~E.[x,y] ~compatible[z,u,x] compatible[z,u,y]
40: ~subclass[x,y] ~subclass[domain_of[restrict[z,y,x]],x] section[z,x,y]
41: ~E.[x,y] ~irreflexive[x,z] irreflexive[y,z]
42: ~E.[x,y] ~irreflexive[z,x] irreflexive[z,y]
43: ~MBR[x,UCLS] MBR[x,complement[y]] MBR[x,y]
44: ~MBR[x,UNPR[y,z]] E.[x,y] E.[x,z]
45: ~E.[x,y] ~asymmetric[x,z] asymmetric[y,z]
46: ~E.[x,y] ~asymmetric[z,x] asymmetric[z,y]
47: ~E.[x,y] ~transitive[x,z] transitive[y,z]
48: ~E.[x,y] ~transitive[z,x] transitive[z,y]
49: ~E.[x,y] ~connected[x,z] connected[y,z]
50: ~E.[x,y] ~connected[z,x] connected[z,y]
51: ~E.[x,y] ~section[x,z,u] section[y,z,u]
52: ~E.[x,y] ~section[z,x,u] section[z,y,u]
53: ~E.[x,y] ~section[z,u,x] section[z,u,y]
54: ~function[x] ~subclass[range_of[x],y] maps[x,domain_of[x],y]
55: ~E.[x,y] ~one_to_one[x] one_to_one[y]
56: ~E.[x,y] ~subclass[x,z] subclass[y,z]
57: ~E.[x,y] ~subclass[z,x] subclass[z,y]
58: ~MBR[x,y] ~MBR[x,z] MBR[x,INT[y,z]]
59: ~E.[x,y] ~inductive[x] inductive[y]
60: ~E.[x,y] ~operation[x] operation[y]
61: ~subclass[x,y] ~subclass[y,x] E.[x,y]
62: ~function[INV[x]] ~function[x] one_to_one[x]
63: ~E.[x,y] ~function[x] function[y]
64: ~E.[x,y] ~MBR[x,z] MBR[y,z]
65: >~E.[x,y] ~MBR[z,x] MBR[z,y]
66: ~E.[x,y] ~maps[x,z,u] maps[y,z,u]
67: ~E.[x,y] ~maps[z,x,u] maps[z,y,u]
68: ~E.[x,y] ~maps[z,u,x] maps[z,u,y]
69: ~subclass[x,y] ~MBR[z,x] MBR[z,y]
70: ~E.[x,y] ~E.[y,z] E.[x,z]
71: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] MBR[ORP[x,ORP[y,COMP[x,y]]],CMPF]
72: ~E.[x,y] E.[recursion_equation_functions[x],recursion_equation_functions[y]]
73: ~MBR[ORP[x,ORP[y,z]],CMPF] E.[COMP[x,y],z]
74: ~MBR[ORP[x,ORP[y,z]],application_function] MBR[y,domain_of[x]]
75: ~operation[x] E.[CXP[domain_of[domain_of[x]],domain_of[domain_of[x]]],domain_of[x]]
76: ~MBR[x,UCLS] MBR[ORP[x,domain_of[x]],DOMR]
77: ~E.[x,y] E.[symmetric_difference[x,z],symmetric_difference[y,z]]
78: ~E.[x,y] E.[symmetric_difference[z,x],symmetric_difference[z,y]]
79: ~E.[x,y] E.[not_subclass_element[x,z],not_subclass_element[y,z]]
80: ~E.[x,y] E.[not_subclass_element[z,x],not_subclass_element[z,y]]
81: ~MBR[ORP[x,ORP[y,z]],application_function] E.[APY[x,y],z]
82: ~MBR[x,recursion_equation_functions[y]] MBR[domain_of[x],ordinal_numbers]
83: ~E.[x,y] E.[not_homomorphism1[x,z,u],not_homomorphism1[y,z,u]]
84: ~E.[x,y] E.[not_homomorphism1[z,x,u],not_homomorphism1[z,y,u]]
85: ~E.[x,y] E.[not_homomorphism1[z,u,x],not_homomorphism1[z,u,y]]
86: ~E.[x,y] E.[not_homomorphism2[x,z,u],not_homomorphism2[y,z,u]]
87: ~E.[x,y] E.[not_homomorphism2[z,x,u],not_homomorphism2[z,y,u]]
88: ~E.[x,y] E.[not_homomorphism2[z,u,x],not_homomorphism2[z,u,y]]
89: ~MBR[ORP[ORP[x,y],z],rotate[u]] MBR[ORP[ORP[y,z],x],u]
90: ~connected[x,y] subclass[CXP[y,y],U[identity_relation,symmetrization_of[x]]]
91: ~subclass[CXP[x,x],U[identity_relation,symmetrization_of[y]]] connected[y,x]
92: ~MBR[x,recursion_equation_functions[y]] E.[COMP[y,rest_of[x]],x]
93: ~MBR[ORP[ORP[x,y],z],flip[u]] MBR[ORP[ORP[y,x],z],u]
94: ~transitive[x,y] subclass[COMP[restrict[x,y,y],restrict[x,y,y]],restrict[x,y,y]]
95: ~subclass[COMP[restrict[x,y,y],restrict[x,y,y]],restrict[x,y,y]] transitive[x,y]
96: ~E.[x,y] E.[ordinal_multiply[z,x],ordinal_multiply[z,y]]
97: ~MBR[ORP[x,y],union_of_range_map] E.[sum_class[range_of[x]],y]
98: ~MBR[ORP[x,y],DOMR] E.[domain_of[x],y]
99: ~E.[x,y] E.[not_well_ordering[x,z],not_well_ordering[y,z]]
100: ~E.[x,y] E.[not_well_ordering[z,x],not_well_ordering[z,y]]
101: ~E.[x,y] E.[symmetrization_of[x],symmetrization_of[y]]
102: ~single_valued_class[x] subclass[COMP[x,INV[x]],identity_relation]
103: ~subclass[COMP[x,INV[x]],identity_relation] single_valued_class[x]
104: ~MBR[ORP[x,y],COMP[z,u]] MBR[y,image[z,image[u, singleton[x]]]]
105: ~E.[x,y] E.[UNPR[x,z],UNPR[y,z]]
106: ~E.[x,y] E.[UNPR[z,x],UNPR[z,y]]
107: ~E.[x,y] E.[ordinal_multiply[x,z],ordinal_multiply[y,z]]
108: ~E.[restrict[x, singleton[y],UCLS],NULC] ~MBR[y,domain_of[x]]
109: ~MBR[x,UCLS] MBR[ORP[x,rest_of[x]],rest_relation]
110: ~MBR[ORP[x,y],compose_class[z]] E.[COMP[z,x],y]
111: ~asymmetric[x,y] E.[restrict[INT[x,INV[x]],y,y],NULC]
112: ~E.[restrict[INT[x,INV[x]],y,y],NULC] asymmetric[x,y]
113: ~MBR[x,CXP[y,z]] E.[ORP[first[x],second[x]],x]
114: ~MBR[ORP[x,y],SUCR] E.[SUC[x],y]
115: ~E.[x,y] E.[INT[x,z],INT[y,z]]
116: ~E.[x,y] E.[INT[z,x],INT[z,y]]
117: ~E.[x,y] E.[ORP[x,z],ORP[y,z]]
118: ~E.[x,y] E.[ORP[z,x],ORP[z,y]]
119: ~E.[x,y] E.[single_valued1[x],single_valued1[y]]

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120: ~E.[x,y] E.[single_valued2[x],single_valued2[y]]
121: ~E.[x,y] E.[single_valued3[x],single_valued3[y]]
122: ~irreflexive[x,y] subclass[restrict[x,y,y],complement[identity_relation]]
123: ~subclass[restrict[x,y,y],complement[identity_relation]] irreflexive[x,y]
124: ~MBR[ORP[x,y],rest_of[z]] E.[restrict[z,x,UCLS],y]
125: ~function[x] subclass[x,CXP[UCLS,UCLS]]
126: ~compatible[x,y,z] subclass[range_of[x],domain_of[domain_of[z]]]
127: ~E.[x,y] E.[CXP[x,z],CXP[y,z]]
128: ~E.[x,y] E.[CXP[z,x],CXP[z,y]]
129: ~MBR[x,recursion_equation_functions[y]] function[y]
130: ~MBR[x,recursion_equation_functions[y]] function[x]
131: ~E.[x,y] E.[recursion[x,z,u],recursion[y,z,u]]
132: ~E.[x,y] E.[recursion[z,x,u],recursion[z,y,u]]
133: ~E.[x,y] E.[recursion[z,u,x],recursion[z,u,y]]
134: ~compatible[x,y,z] E.[domain_of[domain_of[y]],domain_of[x]]
135: ~E.[x,y] E.[restrict[x,z,u],restrict[y,z,u]]
136: ~E.[x,y] E.[restrict[z,x,u],restrict[z,y,u]]
137: ~E.[x,y] E.[restrict[z,u,x],restrict[z,u,y]]
138: ~MBR[x,UCLS] MBR[power_class[x],UCLS]
139: ~E.[x,y] E.[compose_class[x],compose_class[y]]
140: E.[x,NULC] E.[INT[x,regular[x]],NULC]
141: ~MBR[x,ordinal_numbers] well_ordering[element_relation,x]
142: ~E.[x,y] E.[power_class[x],power_class[y]]
143: ~E.[x,y] E.[ordinal_add[x,z],ordinal_add[y,z]]
144: ~E.[x,y] E.[ordinal_add[z,x],ordinal_add[z,y]]
145: ~E.[x,y] E.[segment[x,z,u],segment[y,z,u]]
146: ~E.[x,y] E.[segment[z,x,u],segment[z,y,u]]
147: ~E.[x,y] E.[segment[z,u,x],segment[z,u,y]]
148: ~MBR[ORP[x,y],CXP[z,u]] MBR[x,z]
149: ~MBR[ORP[x,y],CXP[z,u]] MBR[y,u]
150: ~MBR[x,UCLS] MBR[sum_class[x],UCLS]
151: ~function[x] subclass[COMP[x,INV[x]],identity_relation]
152: ~E.[x,y] E.[diagonalise[x],diagonalise[y]]
153: ~MBR[ORP[x,y],rest_relation] E.[rest_of[x],y]
154: ~E.[x,y] E.[DOMN[x,z,u],DOMN[y,z,u]]
155: ~E.[x,y] E.[DOMN[z,x,u],DOMN[z,y,u]]
156: ~E.[x,y] E.[DOMN[z,u,x],DOMN[z,u,y]]
157: ~MBR[ORP[x,y],element_relation] MBR[x,y]
158: ~MBR[x,UCLS] MBR[x,UNPR[x,y]]
159: ~MBR[x,UCLS] MBR[x,UNPR[y,x]]
160: ~operation[x] subclass[range_of[x],domain_of[domain_of[x]]]
161: MBR[not_subclass_element[x,y],x] subclass[x,y]
162: ~MBR[not_subclass_element[x,y],y] subclass[x,y]
163: ~E.[x,y] E.[domain_of[x],domain_of[y]]
164: ~E.[x,y] E.[range[x,z,u],range[y,z,u]]
165: ~E.[x,y] E.[range[z,x,u],range[z,y,u]]
166: ~E.[x,y] E.[range[z,u,x],range[z,u,y]]
167: ~E.[x,y] E.[singleton[x],singleton[y]]
168: ~E.[x,y] E.[SUC[x],SUC[y]]
169: ~E.[x,y] E.[sum_class[x],sum_class[y]]
170: ~E.[x,y] E.[complement[x],complement[y]]
171: ~E.[x,y] E.[integer_of[x],integer_of[y]]
172: ~E.[x,y] E.[range_of[x],range_of[y]]
173: ~MBR[x,ordinal_numbers] subclass[sum_class[x],x]
174: ~MBR[ORP[x,y],rest_of[z]] MBR[x,domain_of[z]]
175: MBR[x,omega] E.[integer_of[x],NULC]
176: ~E.[x,y] E.[COMP[x,z],COMP[y,z]]
177: ~E.[x,y] E.[COMP[z,x],COMP[z,y]]
178: ~E.[x,y] E.[image[x,z],image[y,z]]
179: ~E.[x,y] E.[image[z,x],image[z,y]]
180: ~E.[x,y] E.[INV[x],INV[y]]
181: ~E.[x,y] E.[regular[x],regular[y]]
182: ~E.[x,y] E.[U[x,z],U[y,z]]
183: ~E.[x,y] E.[U[z,x],U[z,y]]
184: ~E.[x,y] E.[rest_of[x],rest_of[y]]
185: ~inductive[x] subclass[image[SUCR,x],x]
186: ~section[x,y,z] subclass[domain_of[restrict[x,z,y]],y]
187: ~E.[x,y] E.[rotate[x],rotate[y]]
188: ~E.[x,y] E.[second[x],second[y]]
189: ~HOM[x,y,z] compatible[x,y,z]
190: ~E.[x,y] E.[APY[x,z],APY[y,z]]
191: ~E.[x,y] E.[APY[z,x],APY[z,y]]
192: ~E.[x,y] E.[first[x],first[y]]
193: ~E.[x,y] E.[least[x,z],least[y,z]]
194: ~E.[x,y] E.[least[z,x],least[z,y]]
195: ~E.[x,y] E.[cantor[x],cantor[y]]
196: ~MBR[x,omega] E.[integer_of[x],x]
197: ~E.[x,y] E.[flip[x],flip[y]]
198: ~MBR[x,INT[y,z]] MBR[x,y]
199: ~MBR[x,INT[y,z]] MBR[x,z]
200 >E.[x,NULC] MBR[regular[x],x]

```

```

201: ~HOM[x,y,z] operation[y]
202: ~HOM[x,y,z] operation[z]
203: ~inductive[x] subclass[omega,x]
204: ~E.[x,y] subclass[x,y]
205: ~E.[x,y] subclass[y,x]
206: ~one_to_one[x] function[INV[x]]
207: ~well_ordering[x,y] connected[x,y]
208 >~MBR[x,complement[y]] ~MBR[x,y]
209: ~compatible[x,y,z] function[x]
210: ~inductive[x] MBR[NULC,x]
211: ~maps[x,y,z] subclass[range_of[x],z]
212: ~maps[x,y,z] E.[domain_of[x],y]
213: ~one_to_one[x] function[x]
214: ~section[x,y,z] subclass[y,z]
215: ~operation[x] function[x]
216: ~maps[x,y,z] function[x]
217 >~E.[x,y] E.[y,x]
218: E.[INT[CXP[UCLS,UCLS]],INT[CXP[UCLS,UCLS]],complement[COMP[complement[element_relation],
INV[element_relation]]]],SUBR]
219: E.[INT[complement[COMP[element_relation,complement[identity_relation]]],element_relation],
singleton_relation]
220: E.[INT[complement[INT[x,y]],complement[INT[complement[x],complement[y]]]],
symmetric_difference[x,y]]
221: subclass[CMPP[CXP[UCLS,CXP[UCLS,UCLS]]]]
222: subclass[application_function,CXP[UCLS,CXP[UCLS,UCLS]]]
223: E.[DOMN[x,image[INV[x]],singleton[single_valued1[x]],single_valued2[x]],single_valued3[x]]
224: E.[second[not_subclass_element[COMP[x,INV[x]],identity_relation]],single_valued2[x]]
225: E.[first[not_subclass_element[COMP[x,INV[x]],identity_relation]],single_valued1[x]]
226: E.[recursion[NULC,APY[add_relation,x],union_of_range_map],ordinal_multiply[x,y]]
227: E.[U[singleton[NULC],image[SUCR,ordinal_numbers]],kind_1_ordinals]
228: subclass[rotate[x],CXP[CXP[UCLS,UCLS],UCLS]]
229: E.[INT[domain_of[x],diagonalise[COMP[INV[element_relation],x]]],cantor[x]]
230: subclass[DOMR,CXP[UCLS,UCLS]]
231: subclass[flip[x],CXP[CXP[UCLS,UCLS],UCLS]]
232: E.[APY[recursion[x,SUCR,union_of_range_map],y],ordinal_add[x,y]]
233: subclass[compose_class[x],CXP[UCLS,UCLS]]
234S>E.[INT[INV[SUBR],SUBR],x] ... ?
235: E.[domain_of[restrict[element_relation,UCLS,x]],sum_class[x]]
236: subclass[union_of_range_map,CXP[UCLS,UCLS]]
237: E.[UNPR[singleton[x],UNPR[x,singleton[y]]],ORP[x,y]]
238: subclass[SUCR,CXP[UCLS,UCLS]]
239: E.[complement[domain_of[INT[x,identity_relation]]],diagonalise[x]]
240: E.[INT[complement[kind_1_ordinals],ordinal_numbers],limit_ordinals]
241: E.[first[not_subclass_element[restrict[x,y,singleton[z]],NULC]],DOMN[x,y,z]]
242: E.[complement[image[element_relation,complement[x]]],power_class[x]]
243: subclass[element_relation,CXP[UCLS,UCLS]]
244: E.[second[not_subclass_element[restrict[x,singleton[y],z],NULC]],range[x,y,z]]
245: subclass[rest_relation,CXP[UCLS,UCLS]]
246: subclass[COMP[x,y],CXP[UCLS,UCLS]]
247: E.[complement[INT[complement[x],complement[y]]],U[x,y]]
248: subclass[rest_of[x],CXP[UCLS,UCLS]]
249: E.[domain_of[flip[CXP[x,UCLS]]],INV[x]]
250: MBR[UNPR[x,y],UCLS]
251: E.[INT[x,CXP[y,z]],restrict[x,y,z]]
252: E.[INT[CXP[x,y],z],restrict[z,x,y]]
253: E.[segment[x,y,z],domain_of[restrict[x,y,singleton[z]]]]
254: E.[range_of[restrict[x,y,UCLS]],image[x,y]]
255: E.[U[x,INV[x]],symmetrization_of[x]]
256: E.[sum_class[image[x,singleton[y]]],APY[x,y]]
257: E.[UNPR[x,x],singleton[x]]
258: subclass[x,UCLS]
259: E.[U[x,singleton[x]],SUC[x]]
260: MBR[omega,UCLS]
261: E.[domain_of[INV[x]],range_of[x]]
262: inductive[omega]
263: function[choice]
264: E.[x,x]

Negated conclusion:
265S>~E.[restrict[INT[xC,identity_relation],yC,yC],NULC]
266S irreflexive[xC,yC]

-----
Phase 0 clauses used in proof:
273S>(265a*200a) MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],restrict[INT[xC,
identity_relation],yC,yC]]

Phases 1 and 2 clauses used in proof:
287S>(273a,208b) ~MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],complement[restrict[INT[xC,
identity_relation],yC,yC]]]

```

```

288S>(287a,65c) ~E.[x,complement[restrict[INT[xC,identity_relation],yC,yC]]]
~MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],x]
289S>(288a,234a) ~MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],INT[INV[SUBR],SUBR]] ... ?

290S>(273a,65b) ~E.[restrict[INT[xC,identity_relation],yC,yC],x]
MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],x]
291S>(290a,217b) MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],x]
~E.[x,restrict[INT[xC,identity_relation],yC,yC]]
292S>(291b,234a) MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],INT[INV[SUBR],SUBR]] ... ?
293S>(292a,289a) [] ... ?

```

Inferences derived from proof before weakened clause repaired  
273 288 292 298

Inferences derived when weakened clause replaced with given clause  
272

```

PHASE 0: 1 s PHASE 1: 1 s PHASE 2: 0 s Total Time: 2 s
NOD: 338 RES: 1854 FAC: 26 T: 40 V: 32 L: 16
CTE: 1246 CTH: 9 CTF: 0 CSZ: 16777216
UTE: 17546 UTH: 18027 UTF: 0 SBA: 43
BAS: 266 RED: 279 LEN: 1+7=8

```

Proof found to theorem by slave 86!

Axioms:

```

58 >~MBR[x,y] ~MBR[x,z] MBR[x,INT[y,z]]
65S>~E.[x,y] ~MBR[z,x] MBR[z,y]
69 >~subclass[x,y] ~MBR[z,x] MBR[z,y]
122 >~irreflexive[x,y] subclass[restrict[x,y,y],complement[identity_relation]]
198 >~MBR[x,INT[y,z]] MBR[x,y]
199 >~MBR[x,INT[y,z]] MBR[x,z]
200S E.[x,NULC] MBR[regular[x],x]
208S>~MBR[x,complement[y]] ~MBR[x,y]
217S ~E.[x,y] E.[y,x]
234S E.[INT[INV[SUBR],SUBR],identity_relation]
251 >E.[INT[x,CXP[y,z]],restrict[x,y,z]]

```

Negated conclusion:

```

265S ~E.[restrict[INT[xC,identity_relation],yC,yC],NULC]
266S>irreflexive[xC,yC]

```

-----

Phase 0 clauses used in proof:

```

269S>MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],x]
~E.[x,restrict[INT[xC,identity_relation],yC,yC]]
276S>(269b*251a) MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],INT[INT[xC,
identity_relation],CXP[yC,yC]]]
279S>(266a*122a) subclass[restrict[xC,yC,yC],complement[identity_relation]]

```

Phases 1 and 2 clauses used in proof:

```

287S>(276a,198a) MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],INT[xC,identity_relation]]
288S>(287a,198a) MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],xC]

289S>(276a,199a) MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],CXP[yC,yC]]
290S>(289a,58b) ~MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],x]

    MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],INT[x,CXP[yC,yC]]]
291S>(290b,65b) ~MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],x] ~E.[INT[x,CXP[yC,yC]],y]

    MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],y]
292S>(291b,251a) ~MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],x]

    MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],restrict[x,yC,yC]]
293S>(292a,288a) MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],restrict[xC,yC,yC]]

294S>(208b,199b) ~MBR[x,complement[y]] ~MBR[x,INT[z,y]]
295S>(294b,198b) ~MBR[x,complement[y]] ~MBR[x,INT[INT[z,y],u]]
296S>(295b,276a) ~MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],
complement[identity_relation]]

297S>(276a,58b) ~MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],x]
MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],INT[x,INT[INT[xC,identity_relation],
CXP[yC,yC]]]]
298S>(297a,69c) MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],INT[x,INT[INT[xC,
identity_relation],CXP[yC,yC]]]] ~subclass[y,x]
~MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],y]
299S>(298a,198a) ~subclass[x,y] ~MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],x]
    MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],y]
300S>(299a,279a) ~MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],restrict[xC,yC,yC]]
    MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],complement[identity_relation]]

```

```
301S>(300a,293a) MBR[regular[restrict[INT[xC,identity_relation],yC,yC]],complement
[identity_relation]]
302S>(301a,296a) []
```

```
PHASE 0: 1 s PHASE 1: 20 s PHASE 2: 25 s Total Time: 48 s
NOD: 29484 RES: 326463 FAC: 3564 T: 40 V: 32 L: 16
CTE: 149990 CTH: 6953 CTF: 0 CSZ: 16777216
UTE: 556772 UTH: 546091 UTF: 0 SBA: 68
BAS: 266 RED: 279 LEN: 3+16=19
```

## SET020-3

Predicates: member little\_set equal single\_valued\_set ordered\_pair\_predicate homomorphism closed maps  
function subset one\_to\_one\_function proper\_subset disjoint relation  
Functions: b a . universal\_set infinity f25 empty\_set estin identity\_relation : apply\_to\_two\_arguments  
f32 f33 cross\_product range\_of domain\_of converse apply\_non\_ordered\_pair second first f26 image  
intersection f1 complement powerset sigma f10 f11 f9 f12 f14 f13 singleton\_set f2 f3 f29 f31 f30 f22  
restrict f19 f20 f21 successor union f6 f7 f28 f4 f5 f27 f8 flip\_range\_of f16 f17 f23 f18 f24  
rotate\_right ordered\_pair compose

Renamed predicates: MBR LST E. SVS OPP HOM closed maps function SUS one\_to\_one\_function PRS disjoint  
relation

Renamed functions: b a . universal\_set IY f25 empty\_set estin identity\_relation :  
apply\_to\_two\_arguments f32 f33 CXP range\_of domain\_of converse APY NOP second first f26 image INT f1  
complement powerset sigma f10 f11 f9 f12 f14 f13 singleton\_set f2 f3 f29 f31 f30 f22 restrict f19 f20  
f21 SUC U f6 f7 f28 f4 f5 f27 f8 flip\_range\_of f16 f17 f23 f18 f24 rotate\_right ORP COMP

C74: Spawned: 150 xvl 16.28.7 W36 249S ~maps[x,y,z] SUS[u,z] ... ?

Axioms:

```
1: MBR[x,COMP[y,z]] ~LST[x] ~LST[u] ~LST[v] ~LST[w] ~E.[x,ORP[u,v]] ~MBR[ORP[u,w],y]
~MBR[ORP[w,v],z]
2: MBR[x,rotate_right[y]] ~LST[x] ~LST[z] ~LST[u] ~LST[v] ~E.[x,ORP[z,ORP[u,v]]]
~MBR[ORP[u,ORP[v,z]],y]
3: MBR[x,flip_range_of[y]] ~LST[x] ~LST[z] ~LST[u] ~LST[v] ~E.[x,ORP[z,ORP[u,v]]]
~MBR[ORP[z,ORP[v,u]],y]
4: ~SVS[x] ~LST[y] ~LST[z] ~LST[u] ~MBR[ORP[y,z],x] ~MBR[ORP[y,u],x] E.[z,u]
5: ~LST[x] ~LST[y] ~LST[z] ~LST[u] ~E.[ORP[x,y],ORP[z,u]] E.[y,u]
6: MBR[x,image[y,z]] ~LST[x] ~OPP[u] ~MBR[u,z] ~MBR[first[u],y] ~E.[second[u],x]
7: HOM[x,y,z,u,v] ~closed[y,z] ~closed[u,v] ~maps[x,y,u]
~E.[APY[x,apply_to_two_arguments[z,f32[x,y,z,u,v],f33[x,y,z,u,v]],apply_to_two_arguments[v,APY[x,f32[
x,y,z,u,v]],APY[x,f33[x,y,z,u,v]]]
8: HOM[x,y,z,u,v] ~closed[y,z] ~closed[u,v] ~maps[x,y,u] MBR[f32[x,y,z,u,v],y]
9: HOM[x,y,z,u,v] ~closed[y,z] ~closed[u,v] ~maps[x,y,u] MBR[f33[x,y,z,u,v],y]
10: MBR[x,CXP[y,z]] ~LST[x] ~OPP[x] ~MBR[first[x],y] ~MBR[second[x],z]
11: MBR[x,range_of[y]] ~LST[x] ~OPP[z] ~MBR[z,y] ~E.[x,second[z]]
12: MBR[x,APY[y,z]] ~OPP[u] ~MBR[u,y] ~E.[first[u],z] ~MBR[x,second[u]]
13: MBR[x,domain_of[y]] ~LST[x] ~OPP[z] ~MBR[z,y] ~E.[x,first[z]]
14: MBR[x,second[y]] ~LST[z] ~LST[u] ~E.[y,ORP[z,u]] ~MBR[x,u]
15: MBR[x,first[y]] ~LST[z] ~LST[u] ~E.[y,ORP[z,u]] ~MBR[x,z]
16: ~HOM[x,y,z,u,v] ~MBR[w,y] ~MBR[t,y]
E.[APY[x,apply_to_two_arguments[z,w,t]],apply_to_two_arguments[v,APY[x,w],APY[x,t]]]
17: MBR[x,converse[y]] ~LST[x] ~OPP[x] ~MBR[ORP[second[x],first[x]],y]
18: MBR[x,identity_relation] ~LST[x] ~OPP[x] ~E.[first[x],second[x]]
19: ~LST[x] ~LST[y] ~E.[ORP[x,z],ORP[y,u]] E.[x,y]
20: ~LST[x] ~LST[y] ~E.[NOP[z,x],NOP[z,y]] E.[x,y]
21: OPP[x] ~LST[y] ~LST[z] ~E.[x,ORP[y,z]]
22: MBR[x,estin] ~LST[x] ~OPP[x] ~MBR[first[x],second[x]]
23: closed[x,y] ~LST[x] ~LST[y] ~maps[y,CXP[x,x],x]
24: >maps[x,y,z] ~function[x] ~E.[domain_of[x],y] ~SUS[range_of[x],z]
25: ~E.[x,y] ~HOM[x,z,u,v,w] HOM[y,z,u,v,w]
26: ~E.[x,y] ~HOM[z,x,u,v,w] HOM[z,y,u,v,w]
27: ~E.[x,y] ~HOM[z,u,x,v,w] HOM[z,u,y,v,w]
28: ~E.[x,y] ~HOM[z,u,v,x,w] HOM[z,u,v,y,w]
29: ~E.[x,y] ~HOM[z,u,v,w,x] HOM[z,u,v,w,y]
30: ~E.[x,y] ~OPP[x] OPP[y]
31: ~E.[x,y] ~one_to_one_function[x] one_to_one_function[y]
32: ~E.[x,y] ~SVS[x] SVS[y]
33: ~E.[x,y] ~PRS[x,z] PRS[y,z]
34: ~E.[x,y] ~PRS[z,x] PRS[z,y]
35: one_to_one_function[x] ~function[x] ~function[converse[x]]
36: ~LST[x] E.[x,empty_set] MBR[ORP[x,f26[x]],f25]
37: MBR[x,NOP[y,z]] ~LST[x] ~E.[x,y]
38: MBR[x,NOP[y,z]] ~LST[x] ~E.[x,z]
39: ~LST[x] ~function[y] LST[image[x,y]]
40: ~E.[x,y] ~disjoint[x,z] disjoint[y,z]
41: ~E.[x,y] ~disjoint[z,x] disjoint[z,y]
42: ~MBR[x,NOP[y,z]] E.[x,y] E.[x,z]
43: ~E.[x,y] ~LST[x] LST[y]
44: ~E.[x,y] ~maps[x,z,u] maps[y,z,u]
45: ~E.[x,y] ~maps[z,x,u] maps[z,y,u]
46: ~E.[x,y] ~maps[z,u,x] maps[z,u,y]
47: ~E.[x,y] ~closed[x,z] closed[y,z]
48: ~E.[x,y] ~closed[z,x] closed[z,y]
49: ~E.[x,y] ~MBR[x,z] MBR[y,z]
50: ~E.[x,y] ~MBR[z,x] MBR[z,y]
51: ~E.[x,y] ~SUS[x,z] SUS[y,z]
52: ~E.[x,y] ~SUS[z,x] SUS[z,y]
53: MBR[x,INT[y,z]] ~MBR[x,y] ~MBR[x,z]
```

```

54: ~E.[x,y] ~function[x] function[y]
55: ~E.[x,y] ~relation[x] relation[y]
56: MBR[f1[x,y],x] MBR[f1[x,y],y] E.[x,y]
57: ~MBR[f1[x,y],x] ~MBR[f1[x,y],y] E.[x,y]
58: MBR[x,complement[y]] ~LST[x] MBR[x,y]
59: ~relation[x] ~MBR[y,x] OPP[y]
60: ~LST[x] E.[x,empty_set] MBR[f26[x],x]
61: MBR[x,powerset[y]] ~LST[x] ~SUS[x,y]
62: >function[x] ~relation[x] ~SVS[x]
63: PRS[x,y] ~SUS[x,y] E.[x,y]
64: MBR[x,sigma[y]] ~MBR[z,y] ~MBR[x,z]
65: ~SUS[x,y] ~SUS[y,x] E.[x,y]
66: ~disjoint[x,y] ~MBR[z,x] ~MBR[z,y]
67: ~E.[x,y] ~E.[y,z] E.[x,z]
68: >~SUS[x,y] ~MBR[z,x] MBR[z,y]
69: ~E.[x,y] E.[apply_to_two_arguments[x,z,u],apply_to_two_arguments[y,z,u]]
70: ~E.[x,y] E.[apply_to_two_arguments[z,x,u],apply_to_two_arguments[z,y,u]]
71: ~E.[x,y] E.[apply_to_two_arguments[z,u,x],apply_to_two_arguments[z,u,y]]
72: ~MBR[x,rotate_right[y]] MBR[ORP[f10[x,y],ORP[f11[x,y],f9[x,y]]],y]
73: ~MBR[x,flip_range_of[y]] MBR[ORP[f12[x,y],ORP[f14[x,y],f13[x,y]]],y]
74: ~E.[x,y] E.[NOP[x,z],NOP[y,z]]
75: ~E.[x,y] E.[NOP[z,x],NOP[z,y]]
76: ~MBR[x,rotate_right[y]] E.[x,ORP[f9[x,y],ORP[f10[x,y],f11[x,y]]]]
77: ~MBR[x,flip_range_of[y]] E.[x,ORP[f12[x,y],ORP[f13[x,y],f14[x,y]]]]
78: ~E.[x,y] E.[INT[x,z],INT[y,z]]
79: ~E.[x,y] E.[INT[z,x],INT[z,y]]
80: ~E.[x,y] E.[ORP[x,z],ORP[y,z]]
81: ~E.[x,y] E.[ORP[z,x],ORP[z,y]]
82: ~E.[x,y] E.[CXP[x,z],CXP[y,z]]
83: ~E.[x,y] E.[CXP[z,x],CXP[z,y]]
84: ~E.[x,y] E.[singleton_set[x],singleton_set[y]]
85: ~OPP[x] E.[x,ORP[f2[x],f3[x]]]
86: ~MBR[x,COMP[y,z]] MBR[ORP[f29[x,y,z],f31[x,y,z]],y]
87: ~MBR[x,COMP[y,z]] MBR[ORP[f31[x,y,z],f30[x,y,z]],z]
88: ~E.[x,y] E.[flip_range_of[x],flip_range_of[y]]
89: ~E.[x,y] E.[rotate_right[x],rotate_right[y]]
90: ~MBR[x,COMP[y,z]] E.[x,ORP[f29[x,y,z],f30[x,y,z]]]
91: ~MBR[x,image[y,z]] OPP[f22[x,y,z]]
92: ~E.[x,y] E.[restrict[x,z],restrict[y,z]]
93: ~E.[x,y] E.[restrict[z,x],restrict[z,y]]
94: SVS[x] MBR[ORP[f19[x],f20[x]],x]
95: >SVS[x] MBR[ORP[f19[x],f21[x]],x]
96: ~E.[x,y] E.[f32[x,z,u,v,w],f32[y,z,u,v,w]]
97: ~E.[x,y] E.[f32[z,x,u,v,w],f32[z,y,u,v,w]]
98: ~E.[x,y] E.[f32[z,u,x,v,w],f32[z,u,y,v,w]]
99: ~E.[x,y] E.[f32[z,u,v,x,w],f32[z,u,v,y,w]]
100: ~E.[x,y] E.[f32[z,u,v,w,x],f32[z,u,v,w,y]]
101: ~E.[x,y] E.[f33[x,z,u,v,w],f33[y,z,u,v,w]]
102: ~E.[x,y] E.[f33[z,x,u,v,w],f33[z,y,u,v,w]]
103: ~E.[x,y] E.[f33[z,u,x,v,w],f33[z,u,y,v,w]]
104: ~E.[x,y] E.[f33[z,u,v,x,w],f33[z,u,v,y,w]]
105: ~E.[x,y] E.[f33[z,u,v,w,x],f33[z,u,v,w,y]]
106: ~E.[x,y] E.[SUC[x],SUC[y]]
107: ~MBR[x,converse[y]] MBR[ORP[second[x],first[x]],y]
108: ~MBR[x,image[y,z]] MBR[first[f22[x,y,z]],y]
109: ~MBR[x,image[y,z]] E.[second[f22[x,y,z]],x]
110: ~MBR[x,identity_relation] OPP[x]
111: ~MBR[x,identity_relation] E.[first[x],second[x]]
112: ~E.[x,y] E.[powerset[x],powerset[y]]
113: ~E.[x,y] E.[image[x,z],image[y,z]]
114: ~E.[x,y] E.[image[z,x],image[z,y]]
115: ~E.[x,y] E.[U[x,z],U[y,z]]
116: ~E.[x,y] E.[U[z,x],U[z,y]]
117: ~E.[x,y] E.[complement[x],complement[y]]
118: ~MBR[x,CXP[y,z]] OPP[x]
119: ~E.[x,y] E.[range_of[x],range_of[y]]
120: ~MBR[x,image[y,z]] MBR[f22[x,y,z],z]
121: ~E.[x,y] E.[converse[x],converse[y]]
122: ~E.[x,y] E.[domain_of[x],domain_of[y]]
123: ~OPP[x] LST[f2[x]]
124: ~OPP[x] LST[f3[x]]
125: ~one_to_one_function[x] function[converse[x]]
126: ~MBR[x,second[y]] E.[y,ORP[f6[x,y],f7[x,y]]]
127: ~E.[x,y] E.[second[x],second[y]]
128: ~E.[x,y] E.[f22[x,z,u],f22[y,z,u]]
129: ~E.[x,y] E.[f22[z,x,u],f22[z,y,u]]
130: ~E.[x,y] E.[f22[z,u,x],f22[z,u,y]]
131: ~E.[x,y] E.[f28[x,z,u],f28[y,z,u]]
132: ~E.[x,y] E.[f28[z,x,u],f28[z,y,u]]
133: ~E.[x,y] E.[f28[z,u,x],f28[z,u,y]]
134: ~E.[x,y] E.[f29[x,z,u],f29[y,z,u]]

```

135: ~E.[x,y] E.[f29[z,x,u],f29[z,y,u]]  
136: ~E.[x,y] E.[f29[z,u,x],f29[z,u,y]]  
137: ~E.[x,y] E.[f30[x,z,u],f30[y,z,u]]  
138: ~E.[x,y] E.[f30[z,x,u],f30[z,y,u]]  
139: ~E.[x,y] E.[f30[z,u,x],f30[z,u,y]]  
140: ~E.[x,y] E.[f31[x,z,u],f31[y,z,u]]  
141: ~E.[x,y] E.[f31[z,x,u],f31[z,y,u]]  
142: ~E.[x,y] E.[f31[z,u,x],f31[z,u,y]]  
143: ~E.[x,y] E.[COMP[x,z],COMP[y,z]]  
144: ~E.[x,y] E.[COMP[z,x],COMP[z,y]]  
145: ~MBR[x,first[y]] E.[y,ORP[f4[x,y],f5[x,y]]]  
146: ~MBR[x,range\_of[y]] OPP[f27[x,y]]  
147: ~MBR[x,APY[y,z]] OPP[f28[x,y,z]]  
148: ~MBR[x,CXP[y,z]] MBR[second[x],z]  
149: ~HOM[x,y,z,u,v] maps[x,y,u]  
150: ~E.[x,y] E.[first[x],first[y]]  
151: ~E.[x,y] E.[sigma[x],sigma[y]]  
152: ~MBR[x,domain\_of[y]] OPP[f8[x,y]]  
153: ~MBR[x,CXP[y,z]] MBR[first[x],y]  
154: ~HOM[x,y,z,u,v] closed[y,z]  
155: ~HOM[x,y,z,u,v] closed[u,v]  
156: ~MBR[x,estin] MBR[first[x],second[x]]  
157: ~MBR[x,estin] OPP[x]  
158: ~MBR[x,rotate\_right[y]] LST[f10[x,y]]  
159: ~MBR[x,rotate\_right[y]] LST[f11[x,y]]  
160: ~MBR[x,IY] MBR[SUC[x],IY]  
161: SVS[x] ~E.[f20[x],f21[x]]  
162: ~MBR[x,rotate\_right[y]] LST[f9[x,y]]  
163: ~MBR[x,flip\_range\_of[y]] LST[f12[x,y]]  
164: ~MBR[x,flip\_range\_of[y]] LST[f13[x,y]]  
165: ~MBR[x,flip\_range\_of[y]] LST[f14[x,y]]  
166: ~E.[x,y] E.[f13[z,x],f13[z,y]]  
167: ~E.[x,y] E.[f14[x,z],f14[y,z]]  
168: ~E.[x,y] E.[f14[z,x],f14[z,y]]  
169: ~E.[x,y] E.[f16[x,z],f16[y,z]]  
170: ~E.[x,y] E.[f16[z,x],f16[z,y]]  
171: ~E.[x,y] E.[f17[x,z],f17[y,z]]  
172: ~E.[x,y] E.[f17[z,x],f17[z,y]]  
173: ~E.[x,y] E.[f23[x,z],f23[y,z]]  
174: ~E.[x,y] E.[f23[z,x],f23[z,y]]  
175: ~E.[x,y] E.[f27[x,z],f27[y,z]]  
176: ~E.[x,y] E.[f27[z,x],f27[z,y]]  
177: ~E.[x,y] E.[APY[x,z],APY[y,z]]  
178: ~E.[x,y] E.[APY[z,x],APY[z,y]]  
179: SVS[x] LST[f19[x]]  
180: SVS[x] LST[f20[x]]  
181: SVS[x] LST[f21[x]]  
182: ~MBR[x,APY[y,z]] MBR[x,second[f28[x,y,z]]]  
183: ~MBR[x,COMP[y,z]] LST[f29[x,y,z]]  
184: ~MBR[x,COMP[y,z]] LST[f30[x,y,z]]  
185: ~MBR[x,COMP[y,z]] LST[f31[x,y,z]]  
186: ~MBR[x,converse[y]] OPP[x]  
187: ~MBR[x,range\_of[y]] E.[x,second[f27[x,y]]]  
188: ~one\_to\_one\_function[x] function[x]  
189: ~MBR[x,APY[y,z]] E.[first[f28[x,y,z]],z]  
190: ~E.[x,y] E.[f4[x,z],f4[y,z]]  
191: ~E.[x,y] E.[f4[z,x],f4[z,y]]  
192: ~E.[x,y] E.[f5[x,z],f5[y,z]]  
193: ~E.[x,y] E.[f5[z,x],f5[z,y]]  
194: ~E.[x,y] E.[f6[x,z],f6[y,z]]  
195: ~E.[x,y] E.[f6[z,x],f6[z,y]]  
196: ~E.[x,y] E.[f7[x,z],f7[y,z]]  
197: ~E.[x,y] E.[f7[z,x],f7[z,y]]  
198: ~E.[x,y] E.[f8[x,z],f8[y,z]]  
199: ~E.[x,y] E.[f8[z,x],f8[z,y]]  
200: ~E.[x,y] E.[f9[x,z],f9[y,z]]  
201: ~E.[x,y] E.[f9[z,x],f9[z,y]]  
202: ~E.[x,y] E.[f13[x,z],f13[y,z]]  
203: ~MBR[x,domain\_of[y]] E.[x,first[f8[x,y]]]  
204: ~closed[x,y] maps[y,CXP[x,x],x]  
205: ~MBR[x,INT[y,z]] MBR[x,y]  
206: ~MBR[x,INT[y,z]] MBR[x,z]  
207: ~E.[x,y] E.[f10[x,z],f10[y,z]]  
208: ~E.[x,y] E.[f10[z,x],f10[z,y]]  
209: ~E.[x,y] E.[f11[x,z],f11[y,z]]  
210: ~E.[x,y] E.[f11[z,x],f11[z,y]]  
211: ~E.[x,y] E.[f12[x,z],f12[y,z]]  
212: ~E.[x,y] E.[f12[z,x],f12[z,y]]  
213: ~E.[x,y] E.[f18[x],f18[y]]  
214: ~E.[x,y] E.[f19[x],f19[y]]  
215: ~E.[x,y] E.[f20[x],f20[y]]

```

216: ~E.[x,y] E.[f21[x],f21[y]]
217: ~E.[x,y] E.[f24[x],f24[y]]
218: ~E.[x,y] E.[f26[x],f26[y]]
219 >MBR[x,universal_set] ~LST[x]
220: relation[x] ~OPP[f18[x]]
221 >~MBR[x,APY[y,z]] MBR[f28[x,y,z],y]
222: E.[x,empty_set] disjoint[f24[x],x]
223 >~MBR[x,range_of[y]] MBR[f27[x,y],y]
224: ~E.[x,y] E.[f1[x,z],f1[y,z]]
225: ~E.[x,y] E.[f1[z,x],f1[z,y]]
226: ~E.[x,y] E.[f2[x],f2[y]]
227: ~E.[x,y] E.[f3[x],f3[y]]
228 >~MBR[x,domain_of[y]] MBR[f8[x,y],y]
229 >E.[x,empty_set] MBR[f24[x],x]
230: ~MBR[x,second[y]] LST[f6[x,y]]
231: ~MBR[x,second[y]] LST[f7[x,y]]
232: ~MBR[x,complement[y]] ~MBR[x,y]
233: ~LST[x] LST[powerset[x]]
234: LST[f1[x,y]] E.[x,y]
235: ~MBR[x,first[y]] LST[f4[x,y]]
236: ~MBR[x,first[y]] LST[f5[x,y]]
237: ~MBR[x,second[y]] MBR[x,f7[x,y]]
238: ~PRS[x,y] SUS[x,y]
239: ~MBR[x,sigma[y]] MBR[f16[x,y],y]
240: ~MBR[x,sigma[y]] MBR[x,f16[x,y]]
241: ~PRS[x,y] ~E.[x,y]
242: ~function[x] SVS[x]
243: ~MBR[x,first[y]] MBR[x,f4[x,y]]
244: ~MBR[x,powerset[y]] SUS[x,y]
245: disjoint[x,y] MBR[f23[x,y],x]
246: disjoint[x,y] MBR[f23[x,y],y]
247: ~LST[x] LST[sigma[x]]
248: ~maps[x,y,z] E.[domain_of[x],y]
249S>~maps[x,y,z] SUS[u,z] ... ?
250 >SUS[x,y] MBR[f17[x,y],x]
251: SUS[x,y] ~MBR[f17[x,y],y]
252 >relation[x] MBR[f18[x],x]
253: ~closed[x,y] LST[x]
254: ~closed[x,y] LST[y]
255: ~function[x] relation[x]
256: ~maps[x,y,z] function[x]
257: ~E.[x,y] E.[y,x]
258: ~MBR[x,y] LST[x]
259: E.[ORP[x,y],NOP[singleton_set[x],NOP[x,y]]]
260: E.[apply_to_two_arguments[x,y,z],APY[x,ORP[y,z]]]
261: E.[U[x,y],complement[INT[complement[x],complement[y]]]]
262: E.[restrict[x,y],INT[x,CXP[y,universal_set]]]
263: E.[singleton_set[x],NOP[x,x]]
264: E.[SUC[x],U[x,singleton_set[x]]]
265: LST[NOP[x,y]]
266: MBR[empty_set,IY]
267: LST[IY]
268 >~MBR[x,empty_set]
269 >E.[x,x]
270: function[f25]

```

Negated conclusion:

```

271S>LST[a]
272S LST[b]
273S ~E.[first[ORP[a,b]],a]
-----

```

Phase 0 clauses used in proof:

```

283S>(271a*219b) MBR[a,universal_set]
286 >(268a*252b) relation[empty_set]
291 >(268a*228b) ~MBR[x,domain_of[empty_set]]
292 >(268a*223b) ~MBR[x,range_of[empty_set]]
293 >(268a*221b) ~MBR[x,APY[empty_set],y]

```

Phases 1 and 2 clauses used in proof:

```

294: (293a,229b) E.[APY[empty_set,x],empty_set]

```

```

295: (293a,95b) SVS[APY[empty_set,x]]
296: [295a,294a] SVS[empty_set]

```

```

297: (293a,229b) E.[APY[empty_set,x],empty_set]

```

```

298S>(283a,68b) ~SUS[universal_set,x] MBR[a,x]
299S>(298a,249b) MBR[a,x] ~maps[y,z,x] ... ?
300S>(299a,293a) ~maps[x,y,APY[empty_set,z]] ... ?
301S>(300a,297a) ~maps[x,y,empty_set] ... ?

```

```

302: (286a,62b) function[empty_set] ~SVS[empty_set]
303: (302b,296a) function[empty_set]

304: (291a,229b) E.[domain_of[empty_set],empty_set]

305: (250a,24d) MBR[f17[range_of[x],y],range_of[x]] maps[x,z,y] ~function[x] ~E.[domain_of[x],z]
306: (305a,292a) maps[empty_set,x,y] ~function[empty_set] ~E.[domain_of[empty_set],x]
307: [306c,304a] maps[empty_set,x,y] ~function[empty_set] ~E.[empty_set,x]
308: [307b,303a] maps[empty_set,x,y] ~E.[empty_set,x]
309: 308|{empty_set/y} maps[empty_set,x,empty_set] ~E.[empty_set,x]
310S>(309a,301a) ~E.[empty_set,x] ... ?
311S>(310a,269a) [] ... ?

```

Inferences derived from proof before variable replaced by constant  
283 286 291 292 293 294 296 297 298 303 304 308

```

PHASE 0: 1 s PHASE 1: 41 s PHASE 2: 0 s Total Time: 42 s
NOD: 221678 RES: 653046 FAC: 66088 T: 16 V: 32 L: 16
CTE: 769725 CTH: 3258 CTF: 0 CSZ: 16777216
UTE: 36583 UTH: 47470 UTF: 0 SBA: 9
BAS: 273 RED: 292 LEN: 5+18=23

```

Proof found to theorem by slave 74!

Axioms:

```

15 >MBR[x,first[y]] ~LST[z] ~LST[u] ~E.[y,ORP[z,u]] ~MBR[x,z]
19 >~LST[x] ~LST[y] ~E.[ORP[x,z],ORP[y,u]] E.[x,y]
24S maps[x,y,z] ~function[x] ~E.[domain_of[x],y] ~SUS[range_of[x],z]
33 >~E.[x,y] ~PRS[x,z] PRS[y,z]
56 >MBR[f1[x,y],x] MBR[f1[x,y],y] E.[x,y]
57 >~MBR[f1[x,y],x] ~MBR[f1[x,y],y] E.[x,y]
62S function[x] ~relation[x] ~SVS[x]
63 >PRS[x,y] ~SUS[x,y] E.[x,y]
68S>~SUS[x,y] ~MBR[z,x] MBR[z,y]
95S SVS[x] MBR[ORP[f19[x],f21[x]],x]
145 >~MBR[x,first[y]] E.[y,ORP[f4[x,y],f5[x,y]]]
219S MBR[x,universal_set] ~LST[x]
221S ~MBR[x,APY[y,z]] MBR[f28[x,y,z],y]
223S ~MBR[x,range_of[y]] MBR[f27[x,y],y]
228S ~MBR[x,domain_of[y]] MBR[f8[x,y],y]
229S E.[x,empty_set] MBR[f24[x],x]
235 >~MBR[x,first[y]] LST[f4[x,y]]
238 >~PRS[x,y] SUS[x,y]
243 >~MBR[x,first[y]] MBR[x,f4[x,y]]
249S ~maps[x,y,z] SUS[range_of[x],z]
250S>SUS[x,y] MBR[f17[x,y],x]
251 >SUS[x,y] ~MBR[f17[x,y],y]
252S relation[x] MBR[f18[x],x]
268S ~MBR[x,empty_set]
269S>E.[x,x]

```

Negated conclusion:

```

271S>LST[a]
272S>LST[b]
273S>~E.[first[ORP[a,b]],a]

```

-----

Phases 1 and 2 clauses used in proof:

```

294S>(273a,57c) ~MBR[f1[first[ORP[a,b]],a],first[ORP[a,b]]] ~MBR[f1[first[ORP[a,b]],a],a]
295S>(294a,15a) ~MBR[f1[first[ORP[a,b]],a],a] ~LST[x] ~LST[y] ~E.[ORP[a,b],ORP[x,y]]
~MBR[f1[first[ORP[a,b]],a],x]
296S>(295ae) ~LST[a] ~LST[x] ~E.[ORP[a,b],ORP[a,x]] ~MBR[f1[first[ORP[a,b]],a],a]
297S>(296a,271a) ~LST[x] ~E.[ORP[a,b],ORP[a,x]] ~MBR[f1[first[ORP[a,b]],a],a]
298S>(297b,269a) ~LST[b] ~MBR[f1[first[ORP[a,b]],a],a]
299S>(298a,272a) ~MBR[f1[first[ORP[a,b]],a],a]

300S>(273a,56c) MBR[f1[first[ORP[a,b]],a],first[ORP[a,b]]] MBR[f1[first[ORP[a,b]],a],a]
301S>(300b,299a) MBR[f1[first[ORP[a,b]],a],first[ORP[a,b]]]
302S>(301a,68b) ~SUS[first[ORP[a,b]],x] MBR[f1[first[ORP[a,b]],a],x]
303S>(302a,238b) MBR[f1[first[ORP[a,b]],a],x] ~PRS[first[ORP[a,b]],x]
304S>(303a,299a) ~PRS[first[ORP[a,b]],a]

305S>(273a,56c) MBR[f1[first[ORP[a,b]],a],first[ORP[a,b]]] MBR[f1[first[ORP[a,b]],a],a]
306S>(305b,299a) MBR[f1[first[ORP[a,b]],a],first[ORP[a,b]]]
307S>(306a,68b) ~SUS[first[ORP[a,b]],x] MBR[f1[first[ORP[a,b]],a],x]
308S>(307a,251a) MBR[f1[first[ORP[a,b]],a],x] ~MBR[f17[first[ORP[a,b]],x],x]
309S>(308a,299a) ~MBR[f17[first[ORP[a,b]],a],a]

310S>(273a,63c) PRS[first[ORP[a,b]],a] ~SUS[first[ORP[a,b]],a]
311S>(310b,250a) PRS[first[ORP[a,b]],a] MBR[f17[first[ORP[a,b]],a],first[ORP[a,b]]]
312S>(311a,304a) MBR[f17[first[ORP[a,b]],a],first[ORP[a,b]]]

```

```

313S>(312a,235a) LST[f4[f17[first[ORP[a,b]],a],ORP[a,b]]]
314S>(250b,145a) SUS[first[x],y] E.[x,ORP[f4[f17[first[x],y],x],f5[f17[first[x],y],x]]]
315S>(314a,63b) E.[x,ORP[f4[f17[first[x],y],x],f5[f17[first[x],y],x]]] PRS[first[x],y] E.[first[x],y]
316S>(315a,19c) PRS[first[ORP[x,y]],z] E.[first[ORP[x,y]],z] ~LST[x]
~LST[f4[f17[first[ORP[x,y]],z],ORP[x,y]]]
E.[x,f4[f17[first[ORP[x,y]],z],ORP[x,y]]]
317S>(316b,273a) PRS[first[ORP[a,b]],a] ~LST[a]
~LST[f4[f17[first[ORP[a,b]],a],ORP[a,b]]] E.[a,f4[f17[first[ORP[a,b]],a],ORP[a,b]]]
318S>(317a,304a) ~LST[a] ~LST[f4[f17[first[ORP[a,b]],a],ORP[a,b]]]
E.[a,f4[f17[first[ORP[a,b]],a],ORP[a,b]]]
319S>(318a,271a) ~LST[f4[f17[first[ORP[a,b]],a],ORP[a,b]]]
E.[a,f4[f17[first[ORP[a,b]],a],ORP[a,b]]]
320S>(319a,313a) E.[a,f4[f17[first[ORP[a,b]],a],ORP[a,b]]]

321S>(250b,243a) SUS[first[x],y] MBR[f17[first[x],y],f4[f17[first[x],y],x]]
322S>(321a,63b) MBR[f17[first[x],y],f4[f17[first[x],y],x]] PRS[first[x],y] E.[first[x],y]
323S>(322b,33b) MBR[f17[first[x],y],f4[f17[first[x],y],x]] E.[first[x],y] ~E.[first[x],z] PRS[z,y]
324S>(323b,273a) MBR[f17[first[ORP[a,b]],a],f4[f17[first[ORP[a,b]],a],ORP[a,b]]]
~E.[first[ORP[a,b]],x] PRS[x,a]
325S>[324a,320a] MBR[f17[first[ORP[a,b]],a],a] ~E.[first[ORP[a,b]],x] PRS[x,a]
326S>[325a,309a] ~E.[first[ORP[a,b]],x] PRS[x,a]
327S>(326a,269a) PRS[first[ORP[a,b]],a]
328S>(327a,304a) []

```

```

PHASE 0: 0 s PHASE 1: 190 s PHASE 2: 228 s Total Time: 460 s
NOD: 2784722 RES: 7958778 FAC: 978571 T: 16 V: 32 L: 16
CTE: 4242095 CTH: 379875 CTF: 0 CSZ: 16777216
UTE: 1741699 UTH: 3458640 UTF: 0 SBA: 116
BAS: 273 RED: 293 LEN: 0+35=35

```

## SET021-3

Predicates: member little\_set equal single\_valued\_set ordered\_pair\_predicate homomorphism closed maps  
function subset one\_to\_one\_function proper\_subset disjoint relation  
Functions: b a . universal\_set infinity f25 empty\_set estin identity\_relation : apply\_to\_two\_arguments  
f32 f33 cross\_product range\_of domain\_of converse apply non\_ordered\_pair second first f26 image  
intersection f1 complement powerset sigma f10 f11 f9 f12 f14 f13 singleton\_set f2 f3 f29 f31 f30 f22  
restrict f19 f20 f21 successor union f6 f7 f28 f4 f5 f27 f8 flip\_range\_of f16 f17 f23 f18 f24  
rotate\_right ordered\_pair compose

Renamed predicates: MBR LST E. SVS OPP HOM closed maps function SUS one\_to\_one\_function PRS disjoint  
relation

Renamed functions: b a . universal\_set IY f25 empty\_set estin identity\_relation :  
apply\_to\_two\_arguments f32 f33 CXP range\_of domain\_of converse APY NOP second first f26 image INT f1  
complement powerset sigma f10 f11 f9 f12 f14 f13 singleton\_set f2 f3 f29 f31 f30 f22 restrict f19 f20  
f21 SUC U f6 f7 f28 f4 f5 f27 f8 flip\_range\_of f16 f17 f23 f18 f24 rotate\_right ORP COMP

C74: Spawned: 150 xvl 16.28.7 W36 250S ~maps[x,y,z] SUS[u,z] ... ?

Axioms:

```
1: MBR[x,COMP[y,z]] ~LST[x] ~LST[u] ~LST[v] ~LST[w] ~E.[x,ORP[u,v]] ~MBR[ORP[u,w],y]
~MBR[ORP[w,v],z]
2: MBR[x,rotate_right[y]] ~LST[x] ~LST[z] ~LST[u] ~LST[v] ~E.[x,ORP[z,ORP[u,v]]]
~MBR[ORP[u,ORP[v,z]],y]
3: MBR[x,flip_range_of[y]] ~LST[x] ~LST[z] ~LST[u] ~LST[v] ~E.[x,ORP[z,ORP[u,v]]]
~MBR[ORP[z,ORP[v,u]],y]
4: ~SVS[x] ~LST[y] ~LST[z] ~LST[u] ~MBR[ORP[y,z],x] ~MBR[ORP[y,u],x] E.[z,u]
5: ~LST[x] ~LST[y] ~LST[z] ~LST[u] ~E.[ORP[x,y],ORP[z,u]] E.[y,u]
6: MBR[x,image[y,z]] ~LST[x] ~OPP[u] ~MBR[u,z] ~MBR[first[u],y] ~E.[second[u],x]
7: HOM[x,y,z,u,v] ~closed[y,z] ~closed[u,v] ~maps[x,y,u]
~E.[APY[x,apply_to_two_arguments[z,f32[x,y,z,u,v],
f33[x,y,z,u,v]],apply_to_two_arguments[v,APY[x,f32[x,y,z,u,v],APY[x,f33[x,y,z,u,v]]]]
8: HOM[x,y,z,u,v] ~closed[y,z] ~closed[u,v] ~maps[x,y,u] MBR[f32[x,y,z,u,v],y]
9: HOM[x,y,z,u,v] ~closed[y,z] ~closed[u,v] ~maps[x,y,u] MBR[f33[x,y,z,u,v],y]
10: MBR[x,CXP[y,z]] ~LST[x] ~OPP[x] ~MBR[first[x],y] ~MBR[second[x],z]
11: MBR[x,range_of[y]] ~LST[x] ~OPP[z] ~MBR[z,y] ~E.[x,second[z]]
12: MBR[x,APY[y,z]] ~OPP[u] ~MBR[u,y] ~E.[first[u],z] ~MBR[x,second[u]]
13: MBR[x,domain_of[y]] ~LST[x] ~OPP[z] ~MBR[z,y] ~E.[x,first[z]]
14: MBR[x,second[y]] ~LST[z] ~LST[u] ~E.[y,ORP[z,u]] ~MBR[x,u]
15: MBR[x,first[y]] ~LST[z] ~LST[u] ~E.[y,ORP[z,u]] ~MBR[x,z]
16: ~HOM[x,y,z,u,v] ~MBR[w,y] ~MBR[t,y]
E.[APY[x,apply_to_two_arguments[z,w,t]],apply_to_two_arguments[v,APY[x,w],APY[x,t]]]
17: MBR[x,converse[y]] ~LST[x] ~OPP[x] ~MBR[ORP[second[x],first[x]],y]
18: MBR[x,identity_relation] ~LST[x] ~OPP[x] ~E.[first[x],second[x]]
19: ~LST[x] ~LST[y] ~E.[ORP[x,z],ORP[y,u]] E.[x,y]
20: ~LST[x] ~LST[y] ~E.[NOP[z,x],NOP[z,y]] E.[x,y]
21: OPP[x] ~LST[y] ~LST[z] ~E.[x,ORP[y,z]]
22: MBR[x,estin] ~LST[x] ~OPP[x] ~MBR[first[x],second[x]]
23: closed[x,y] ~LST[x] ~LST[y] ~maps[y,CXP[x,x],x]
24: >maps[x,y,z] ~function[x] ~E.[domain_of[x],y] ~SUS[range_of[x],z]
25: ~E.[x,y] ~HOM[x,z,u,v,w] HOM[y,z,u,v,w]
26: ~E.[x,y] ~HOM[z,x,u,v,w] HOM[z,y,u,v,w]
27: ~E.[x,y] ~HOM[z,u,x,v,w] HOM[z,u,y,v,w]
28: ~E.[x,y] ~HOM[z,u,v,x,w] HOM[z,u,v,y,w]
29: ~E.[x,y] ~HOM[z,u,v,w,x] HOM[z,u,v,w,y]
30: ~E.[x,y] ~OPP[x] OPP[y]
31: ~E.[x,y] ~one_to_one_function[x] one_to_one_function[y]
32: ~E.[x,y] ~SVS[x] SVS[y]
33: ~E.[x,y] ~PRS[x,z] PRS[y,z]
34: ~E.[x,y] ~PRS[z,x] PRS[z,y]
35: one_to_one_function[x] ~function[x] ~function[converse[x]]
36: ~LST[x] ~LST[y] E.[first[ORP[x,y]],x]
37: ~LST[x] E.[x,empty_set] MBR[ORP[x,f26[x]],f25]
38: MBR[x,NOP[y,z]] ~LST[x] ~E.[x,y]
39: MBR[x,NOP[y,z]] ~LST[x] ~E.[x,z]
40: ~LST[x] ~function[y] LST[image[x,y]]
41: ~E.[x,y] ~disjoint[x,z] disjoint[y,z]
42: ~E.[x,y] ~disjoint[z,x] disjoint[z,y]
43: ~MBR[x,NOP[y,z]] E.[x,y] E.[x,z]
44: ~E.[x,y] ~LST[x] LST[y]
45: ~E.[x,y] ~maps[x,z,u] maps[y,z,u]
46: ~E.[x,y] ~maps[z,x,u] maps[z,y,u]
47: ~E.[x,y] ~maps[z,u,x] maps[z,u,y]
48: ~E.[x,y] ~closed[x,z] closed[y,z]
49: ~E.[x,y] ~closed[z,x] closed[z,y]
50: ~E.[x,y] ~MBR[x,z] MBR[y,z]
51: ~E.[x,y] ~MBR[z,x] MBR[z,y]
52: ~E.[x,y] ~SUS[x,z] SUS[y,z]
53: ~E.[x,y] ~SUS[z,x] SUS[z,y]
```

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54: MBR[x,INT[y,z]] ~MBR[x,y] ~MBR[x,z]
55: ~E.[x,y] ~function[x] function[y]
56: ~E.[x,y] ~relation[x] relation[y]
57: MBR[f1[x,y],x] MBR[f1[x,y],y] E.[x,y]
58: ~MBR[f1[x,y],x] ~MBR[f1[x,y],y] E.[x,y]
59: MBR[x,complement[y]] ~LST[x] MBR[x,y]
60: ~relation[x] ~MBR[y,x] OPP[y]
61: ~LST[x] E.[x,empty_set] MBR[f26[x],x]
62: MBR[x,powerset[y]] ~LST[x] ~SUS[x,y]
63: >function[x] ~relation[x] ~SVS[x]
64: PRS[x,y] ~SUS[x,y] E.[x,y]
65: MBR[x,sigma[y]] ~MBR[z,y] ~MBR[x,z]
66: ~SUS[x,y] ~SUS[y,x] E.[x,y]
67: ~disjoint[x,y] ~MBR[z,x] ~MBR[z,y]
68: ~E.[x,y] ~E.[y,z] E.[x,z]
69: >~SUS[x,y] ~MBR[z,x] MBR[z,y]
70: ~E.[x,y] E.[apply_to_two_arguments[x,z,u],apply_to_two_arguments[y,z,u]]
71: ~E.[x,y] E.[apply_to_two_arguments[z,x,u],apply_to_two_arguments[z,y,u]]
72: ~E.[x,y] E.[apply_to_two_arguments[z,u,x],apply_to_two_arguments[z,u,y]]
73: ~MBR[x,rotate_right[y]] MBR[ORP[f10[x,y],ORP[f11[x,y],f9[x,y]]],y]
74: ~MBR[x,flip_range_of[y]] MBR[ORP[f12[x,y],ORP[f14[x,y],f13[x,y]]],y]
75: ~E.[x,y] E.[NOP[x,z],NOP[y,z]]
76: ~E.[x,y] E.[NOP[z,x],NOP[z,y]]
77: ~MBR[x,rotate_right[y]] E.[x,ORP[f9[x,y],ORP[f10[x,y],f11[x,y]]]
78: ~MBR[x,flip_range_of[y]] E.[x,ORP[f12[x,y],ORP[f13[x,y],f14[x,y]]]
79: ~E.[x,y] E.[INT[x,z],INT[y,z]]
80: ~E.[x,y] E.[INT[z,x],INT[z,y]]
81: ~E.[x,y] E.[ORP[x,z],ORP[y,z]]
82: ~E.[x,y] E.[ORP[z,x],ORP[z,y]]
83: ~E.[x,y] E.[CXP[x,z],CXP[y,z]]
84: ~E.[x,y] E.[CXP[z,x],CXP[z,y]]
85: ~E.[x,y] E.[singleton_set[x],singleton_set[y]]
86: ~OPP[x] E.[x,ORP[f2[x],f3[x]]]
87: ~MBR[x,COMP[y,z]] MBR[ORP[f29[x,y,z],f31[x,y,z]],y]
88: ~MBR[x,COMP[y,z]] MBR[ORP[f31[x,y,z],f30[x,y,z]],z]
89: ~E.[x,y] E.[flip_range_of[x],flip_range_of[y]]
90: ~E.[x,y] E.[rotate_right[x],rotate_right[y]]
91: ~MBR[x,COMP[y,z]] E.[x,ORP[f29[x,y,z],f30[x,y,z]]]
92: ~MBR[x,image[y,z]] OPP[f22[x,y,z]]
93: ~E.[x,y] E.[restrict[x,z],restrict[y,z]]
94: ~E.[x,y] E.[restrict[z,x],restrict[z,y]]
95: SVS[x] MBR[ORP[f19[x],f20[x]],x]
96: >SVS[x] MBR[ORP[f19[x],f21[x]],x]
97: ~E.[x,y] E.[f32[x,z,u,v,w],f32[y,z,u,v,w]]
98: ~E.[x,y] E.[f32[z,x,u,v,w],f32[z,y,u,v,w]]
99: ~E.[x,y] E.[f32[z,u,x,v,w],f32[z,u,y,v,w]]
100: ~E.[x,y] E.[f32[z,u,v,x,w],f32[z,u,v,y,w]]
101: ~E.[x,y] E.[f32[z,u,v,w,x],f32[z,u,v,w,y]]
102: ~E.[x,y] E.[f33[x,z,u,v,w],f33[y,z,u,v,w]]
103: ~E.[x,y] E.[f33[z,x,u,v,w],f33[z,y,u,v,w]]
104: ~E.[x,y] E.[f33[z,u,x,v,w],f33[z,u,y,v,w]]
105: ~E.[x,y] E.[f33[z,u,v,x,w],f33[z,u,v,y,w]]
106: ~E.[x,y] E.[f33[z,u,v,w,x],f33[z,u,v,w,y]]
107: ~E.[x,y] E.[SUC[x],SUC[y]]
108: ~MBR[x,converse[y]] MBR[ORP[second[x],first[x]],y]
109: ~MBR[x,image[y,z]] MBR[first[f22[x,y,z]],y]
110: ~MBR[x,image[y,z]] E.[second[f22[x,y,z]],x]
111: ~MBR[x,identity_relation] OPP[x]
112: ~MBR[x,identity_relation] E.[first[x],second[x]]
113: ~E.[x,y] E.[powerset[x],powerset[y]]
114: ~E.[x,y] E.[image[x,z],image[y,z]]
115: ~E.[x,y] E.[image[z,x],image[z,y]]
116: ~E.[x,y] E.[U[x,z],U[y,z]]
117: ~E.[x,y] E.[U[z,x],U[z,y]]
118: ~E.[x,y] E.[complement[x],complement[y]]
119: ~MBR[x,CXP[y,z]] OPP[x]
120: ~E.[x,y] E.[range_of[x],range_of[y]]
121: ~MBR[x,image[y,z]] MBR[f22[x,y,z],z]
122: ~E.[x,y] E.[converse[x],converse[y]]
123: ~E.[x,y] E.[domain_of[x],domain_of[y]]
124: ~OPP[x] LST[f2[x]]
125: ~OPP[x] LST[f3[x]]
126: ~one_to_one_function[x] function[converse[x]]
127: ~MBR[x,second[y]] E.[y,ORP[f6[x,y],f7[x,y]]]
128: ~E.[x,y] E.[second[x],second[y]]
129: ~E.[x,y] E.[f22[x,z,u],f22[y,z,u]]
130: ~E.[x,y] E.[f22[z,x,u],f22[z,y,u]]
131: ~E.[x,y] E.[f22[z,u,x],f22[z,u,y]]
132: ~E.[x,y] E.[f28[x,z,u],f28[y,z,u]]
133: ~E.[x,y] E.[f28[z,x,u],f28[z,y,u]]
134: ~E.[x,y] E.[f28[z,u,x],f28[z,u,y]]

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135: ~E.[x,y] E.[f29[x,z,u],f29[y,z,u]]  
136: ~E.[x,y] E.[f29[z,x,u],f29[z,y,u]]  
137: ~E.[x,y] E.[f29[z,u,x],f29[z,u,y]]  
138: ~E.[x,y] E.[f30[x,z,u],f30[y,z,u]]  
139: ~E.[x,y] E.[f30[z,x,u],f30[z,y,u]]  
140: ~E.[x,y] E.[f30[z,u,x],f30[z,u,y]]  
141: ~E.[x,y] E.[f31[x,z,u],f31[y,z,u]]  
142: ~E.[x,y] E.[f31[z,x,u],f31[z,y,u]]  
143: ~E.[x,y] E.[f31[z,u,x],f31[z,u,y]]  
144: ~E.[x,y] E.[COMP[x,z],COMP[y,z]]  
145: ~E.[x,y] E.[COMP[z,x],COMP[z,y]]  
146: ~MBR[x,first[y]] E.[y,ORP[f4[x,y],f5[x,y]]]  
147: ~MBR[x,range\_of[y]] OPP[f27[x,y]]  
148: ~MBR[x,APY[y,z]] OPP[f28[x,y,z]]  
149: ~MBR[x,CXP[y,z]] MBR[second[x],z]  
150: ~HOM[x,y,z,u,v] maps[x,y,u]  
151: ~E.[x,y] E.[first[x],first[y]]  
152: ~E.[x,y] E.[sigma[x],sigma[y]]  
153: ~MBR[x,domain\_of[y]] OPP[f8[x,y]]  
154: ~MBR[x,CXP[y,z]] MBR[first[x],y]  
155: ~HOM[x,y,z,u,v] closed[y,z]  
156: ~HOM[x,y,z,u,v] closed[u,v]  
157: ~MBR[x,estin] MBR[first[x],second[x]]  
158: ~MBR[x,estin] OPP[x]  
159: ~MBR[x,rotate\_right[y]] LST[f10[x,y]]  
160: ~MBR[x,rotate\_right[y]] LST[f11[x,y]]  
161: ~MBR[x,IY] MBR[SUC[x],IY]  
162: SVS[x] ~E.[f20[x],f21[x]]  
163: ~MBR[x,rotate\_right[y]] LST[f9[x,y]]  
164: ~MBR[x,flip\_range\_of[y]] LST[f12[x,y]]  
165: ~MBR[x,flip\_range\_of[y]] LST[f13[x,y]]  
166: ~MBR[x,flip\_range\_of[y]] LST[f14[x,y]]  
167: ~E.[x,y] E.[f13[z,x],f13[z,y]]  
168: ~E.[x,y] E.[f14[x,z],f14[y,z]]  
169: ~E.[x,y] E.[f14[z,x],f14[z,y]]  
170: ~E.[x,y] E.[f16[x,z],f16[y,z]]  
171: ~E.[x,y] E.[f16[z,x],f16[z,y]]  
172: ~E.[x,y] E.[f17[x,z],f17[y,z]]  
173: ~E.[x,y] E.[f17[z,x],f17[z,y]]  
174: ~E.[x,y] E.[f23[x,z],f23[y,z]]  
175: ~E.[x,y] E.[f23[z,x],f23[z,y]]  
176: ~E.[x,y] E.[f27[x,z],f27[y,z]]  
177: ~E.[x,y] E.[f27[z,x],f27[z,y]]  
178: ~E.[x,y] E.[APY[x,z],APY[y,z]]  
179: ~E.[x,y] E.[APY[z,x],APY[z,y]]  
180: SVS[x] LST[f19[x]]  
181: SVS[x] LST[f20[x]]  
182: SVS[x] LST[f21[x]]  
183: ~MBR[x,APY[y,z]] MBR[x,second[f28[x,y,z]]]  
184: ~MBR[x,COMP[y,z]] LST[f29[x,y,z]]  
185: ~MBR[x,COMP[y,z]] LST[f30[x,y,z]]  
186: ~MBR[x,COMP[y,z]] LST[f31[x,y,z]]  
187: ~MBR[x,converse[y]] OPP[x]  
188: ~MBR[x,range\_of[y]] E.[x,second[f27[x,y]]]  
189: ~one\_to\_one\_function[x] function[x]  
190: ~MBR[x,APY[y,z]] E.[first[f28[x,y,z]],z]  
191: ~E.[x,y] E.[f4[x,z],f4[y,z]]  
192: ~E.[x,y] E.[f4[z,x],f4[z,y]]  
193: ~E.[x,y] E.[f5[x,z],f5[y,z]]  
194: ~E.[x,y] E.[f5[z,x],f5[z,y]]  
195: ~E.[x,y] E.[f6[x,z],f6[y,z]]  
196: ~E.[x,y] E.[f6[z,x],f6[z,y]]  
197: ~E.[x,y] E.[f7[x,z],f7[y,z]]  
198: ~E.[x,y] E.[f7[z,x],f7[z,y]]  
199: ~E.[x,y] E.[f8[x,z],f8[y,z]]  
200: ~E.[x,y] E.[f8[z,x],f8[z,y]]  
201: ~E.[x,y] E.[f9[x,z],f9[y,z]]  
202: ~E.[x,y] E.[f9[z,x],f9[z,y]]  
203: ~E.[x,y] E.[f13[x,z],f13[y,z]]  
204: ~MBR[x,domain\_of[y]] E.[x,first[f8[x,y]]]  
205: ~closed[x,y] maps[y,CXP[x,x],x]  
206: ~MBR[x,INT[y,z]] MBR[x,y]  
207: ~MBR[x,INT[y,z]] MBR[x,z]  
208: ~E.[x,y] E.[f10[x,z],f10[y,z]]  
209: ~E.[x,y] E.[f10[z,x],f10[z,y]]  
210: ~E.[x,y] E.[f11[x,z],f11[y,z]]  
211: ~E.[x,y] E.[f11[z,x],f11[z,y]]  
212: ~E.[x,y] E.[f12[x,z],f12[y,z]]  
213: ~E.[x,y] E.[f12[z,x],f12[z,y]]  
214: ~E.[x,y] E.[f18[x],f18[y]]  
215: ~E.[x,y] E.[f19[x],f19[y]]

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216: ~E.[x,y] E.[f20[x],f20[y]]
217: ~E.[x,y] E.[f21[x],f21[y]]
218: ~E.[x,y] E.[f24[x],f24[y]]
219: ~E.[x,y] E.[f26[x],f26[y]]
220 >MBR[x,universal_set] ~LST[x]
221: relation[x] ~OPP[f18[x]]
222 >~MBR[x,APY[y,z]] MBR[f28[x,y,z],y]
223: E.[x,empty_set] disjoint[f24[x],x]
224 >~MBR[x,range_of[y]] MBR[f27[x,y],y]
225: ~E.[x,y] E.[f1[x,z],f1[y,z]]
226: ~E.[x,y] E.[f1[z,x],f1[z,y]]
227: ~E.[x,y] E.[f2[x],f2[y]]
228: ~E.[x,y] E.[f3[x],f3[y]]
229 >~MBR[x,domain_of[y]] MBR[f8[x,y],y]
230 >E.[x,empty_set] MBR[f24[x],x]
231: ~MBR[x,second[y]] LST[f6[x,y]]
232: ~MBR[x,second[y]] LST[f7[x,y]]
233: ~MBR[x,complement[y]] ~MBR[x,y]
234: ~LST[x] LST[powerset[x]]
235: LST[f1[x,y]] E.[x,y]
236: ~MBR[x,first[y]] LST[f4[x,y]]
237: ~MBR[x,first[y]] LST[f5[x,y]]
238: ~MBR[x,second[y]] MBR[x,f7[x,y]]
239: ~PRS[x,y] SUS[x,y]
240: ~MBR[x,sigma[y]] MBR[f16[x,y],y]
241: ~MBR[x,sigma[y]] MBR[x,f16[x,y]]
242: ~PRS[x,y] ~E.[x,y]
243: ~function[x] SVS[x]
244: ~MBR[x,first[y]] MBR[x,f4[x,y]]
245: ~MBR[x,powerset[y]] SUS[x,y]
246: disjoint[x,y] MBR[f23[x,y],x]
247: disjoint[x,y] MBR[f23[x,y],y]
248: ~LST[x] LST[sigma[x]]
249: ~maps[x,y,z] E.[domain_of[x],y]
250S>~maps[x,y,z] SUS[u,z] ... ?
251 >SUS[x,y] MBR[f17[x,y],x]
252: SUS[x,y] ~MBR[f17[x,y],y]
253 >relation[x] MBR[f18[x],x]
254: ~closed[x,y] LST[x]
255: ~closed[x,y] LST[y]
256: ~function[x] relation[x]
257: ~maps[x,y,z] function[x]
258: ~E.[x,y] E.[y,x]
259: ~MBR[x,y] LST[x]
260: E.[ORP[x,y],NOP[singleton_set[x],NOP[x,y]]]
261: E.[apply_to_two_arguments[x,y,z],APY[x,ORP[y,z]]]
262: E.[U[x,y],complement[INT[complement[x],complement[y]]]]
263: E.[restrict[x,y],INT[x,CXP[y,universal_set]]]
264: E.[singleton_set[x],NOP[x,x]]
265: E.[SUC[x],U[x,singleton_set[x]]]
266: LST[NOP[x,y]]
267: MBR[empty_set,IY]
268: LST[IY]
269 >~MBR[x,empty_set]
270 >E.[x,x]
271: function[f25]

```

Negated conclusion:

```

272S>LST[a]
273S LST[b]
274S ~E.[second[ORP[a,b]],b]

```

-----

Phase 0 clauses used in proof:

```

284S>(272a*220b) MBR[a,universal_set]
287 >(269a*253b) relation[empty_set]
292 >(269a*229b) ~MBR[x,domain_of[empty_set]]
293 >(269a*224b) ~MBR[x,range_of[empty_set]]
294 >(269a*222b) ~MBR[x,APY[empty_set,y]]

```

Phases 1 and 2 clauses used in proof:

```

295: (294a,230b) E.[APY[empty_set,x],empty_set]

296: (294a,96b) SVS[APY[empty_set,x]]
297: [296a,295a] SVS[empty_set]

298: (294a,230b) E.[APY[empty_set,x],empty_set]

299S>(284a,69b) ~SUS[universal_set,x] MBR[a,x]
300S>(299a,250b) MBR[a,x] ~maps[y,z,x] ... ?
301S>(300a,294a) ~maps[x,y,APY[empty_set,z]] ... ?

```

```

302S>[301a,298a] ~maps[x,y,empty_set] ... ?
303: (287a,63b) function[empty_set] ~SVS[empty_set]
304: (303b,297a) function[empty_set]
305: (292a,230b) E.[domain_of[empty_set],empty_set]
306: (251a,24d) MBR[f17[range_of[x],y],range_of[x]] maps[x,z,y] ~function[x] ~E.[domain_of[x],z]
307: (306a,293a) maps[empty_set,x,y] ~function[empty_set] ~E.[domain_of[empty_set],x]
308: [307c,305a] maps[empty_set,x,y] ~function[empty_set] ~E.[empty_set,x]
309: [308b,304a] maps[empty_set,x,y] ~E.[empty_set,x]
310: 309|{empty_set/y} maps[empty_set,x,empty_set] ~E.[empty_set,x]
311S>(310a,302a) ~E.[empty_set,x] ... ?
312S>(311a,270a) [] ... ?

```

Inferences derived from proof before variable replaced by constant  
284 287 292 293 294 295 297 298 299 304 305 309

```

PHASE 0: 0 s PHASE 1: 40 s PHASE 2: 1 s Total Time: 41 s
NOD: 223301 RES: 662053 FAC: 65926 T: 16 V: 32 L: 16
CTE: 773167 CTH: 3619 CTF: 0 CSZ: 16777216
UTE: 142240 UTH: 160882 UTF: 0 SBA: 163
BAS: 274 RED: 293 LEN: 5+18=23

```

Proof found to theorem by slave 74!

Axioms:

```

5 >~LST[x] ~LST[y] ~LST[z] ~LST[u] ~E.[ORP[x,y],ORP[z,u]] E.[y,u]
14 >MBR[x,second[y]] ~LST[z] ~LST[u] ~E.[y,ORP[z,u]] ~MBR[x,u]
19 >~LST[x] ~LST[y] ~E.[ORP[x,z],ORP[y,u]] E.[x,y]
24S maps[x,y,z] ~function[x] ~E.[domain_of[x],y] ~SUS[range_of[x],z]
33 >~E.[x,y] ~PRS[x,z] PRS[y,z]
57 >MBR[f1[x,y],x] MBR[f1[x,y],y] E.[x,y]
58 >~MBR[f1[x,y],x] ~MBR[f1[x,y],y] E.[x,y]
63S function[x] ~relation[x] ~SVS[x]
64 >PRS[x,y] ~SUS[x,y] E.[x,y]
69S>~SUS[x,y] ~MBR[z,x] MBR[z,y]
96S SVS[x] MBR[ORP[f19[x],f21[x]],x]
127 >~MBR[x,second[y]] E.[y,ORP[f6[x,y],f7[x,y]]]
220S MBR[x,universal_set] ~LST[x]
222S ~MBR[x,APY[y,z]] MBR[f28[x,y,z],y]
224S ~MBR[x,range_of[y]] MBR[f27[x,y],y]
229S ~MBR[x,domain_of[y]] MBR[f8[x,y],y]
230S E.[x,empty_set] MBR[f24[x],x]
231 >~MBR[x,second[y]] LST[f6[x,y]]
232 >~MBR[x,second[y]] LST[f7[x,y]]
238 >~MBR[x,second[y]] MBR[x,f7[x,y]]
239 >~PRS[x,y] SUS[x,y]
250S ~maps[x,y,z] SUS[range_of[x],z]
251S>SUS[x,y] MBR[f17[x,y],x]
252 >SUS[x,y] ~MBR[f17[x,y],y]
253S relation[x] MBR[f18[x],x]
269S ~MBR[x,empty_set]
270S>E.[x,x]

```

Negated conclusion:

```

272S>LST[a]
273S>LST[b]
274S>~E.[second[ORP[a,b]],b]

```

-----  
Phases 1 and 2 clauses used in proof:

```

295S>(274a,58c) ~MBR[f1[second[ORP[a,b]],b],second[ORP[a,b]]] ~MBR[f1[second[ORP[a,b]],b],b]
296S>(295a,14a) ~MBR[f1[second[ORP[a,b]],b],b] ~LST[x] ~LST[y] ~E.[ORP[a,b],ORP[x,y]]
~MBR[f1[second[ORP[a,b]],b],y]
297S>(296ae) ~LST[x] ~LST[b] ~E.[ORP[a,b],ORP[x,b]] ~MBR[f1[second[ORP[a,b]],b],b]
298S>(297b,273a) ~LST[x] ~E.[ORP[a,b],ORP[x,b]] ~MBR[f1[second[ORP[a,b]],b],b]
299S>(298b,270a) ~LST[a] ~MBR[f1[second[ORP[a,b]],b],b]
300S>(299a,272a) ~MBR[f1[second[ORP[a,b]],b],b]
301S>(274a,57c) MBR[f1[second[ORP[a,b]],b],second[ORP[a,b]]] MBR[f1[second[ORP[a,b]],b],b]
302S>(301b,300a) MBR[f1[second[ORP[a,b]],b],second[ORP[a,b]]]
303S>(302a,69b) ~SUS[second[ORP[a,b]],x] MBR[f1[second[ORP[a,b]],b],x]
304S>(303a,239b) MBR[f1[second[ORP[a,b]],b],x] ~PRS[second[ORP[a,b]],x]
305S>(304a,300a) ~PRS[second[ORP[a,b]],b]
306S>(274a,57c) MBR[f1[second[ORP[a,b]],b],second[ORP[a,b]]] MBR[f1[second[ORP[a,b]],b],b]
307S>(306b,300a) MBR[f1[second[ORP[a,b]],b],second[ORP[a,b]]]
308S>(307a,69b) ~SUS[second[ORP[a,b]],x] MBR[f1[second[ORP[a,b]],b],x]
309S>(308a,252a) MBR[f1[second[ORP[a,b]],b],x] ~MBR[f17[second[ORP[a,b]],x],x]
310S>(309a,300a) ~MBR[f17[second[ORP[a,b]],b],b]

```

```

311S>(274a,64c) PRS[second[ORP[a,b]],b] ~SUS[second[ORP[a,b]],b]
312S>(311b,251a) PRS[second[ORP[a,b]],b] MBR[f17[second[ORP[a,b]],b],second[ORP[a,b]]]
313S>(312a,305a) MBR[f17[second[ORP[a,b]],b],second[ORP[a,b]]]
314S>(313a,231a) LST[f6[f17[second[ORP[a,b]],b],ORP[a,b]]]

315S>(251b,127a) SUS[second[x],y] E.[x,ORP[f6[f17[second[x],y],x],f7[f17[second[x],y],x]]]
316S>(315a,64b) E.[x,ORP[f6[f17[second[x],y],x],f7[f17[second[x],y],x]]] PRS[second[x],y]
E.[second[x],y]
317S>(316a,19c) PRS[second[ORP[x,y]],z] E.[second[ORP[x,y]],z] ~LST[x]
~LST[f6[f17[second[ORP[x,y]],z],ORP[x,y]]] E.[x,f6[f17[second[ORP[x,y]],z],ORP[x,y]]]
318S>(317b,274a) PRS[second[ORP[a,b]],b] ~LST[a] ~LST[f6[f17[second[ORP[a,b]],b],ORP[a,b]]]
E.[a,f6[f17[second[ORP[a,b]],b],ORP[a,b]]]
319S>(318a,305a) ~LST[a] ~LST[f6[f17[second[ORP[a,b]],b],ORP[a,b]]]
E.[a,f6[f17[second[ORP[a,b]],b],ORP[a,b]]]
320S>(319a,272a) ~LST[f6[f17[second[ORP[a,b]],b],ORP[a,b]]]
E.[a,f6[f17[second[ORP[a,b]],b],ORP[a,b]]]
321S>(320a,314a) E.[a,f6[f17[second[ORP[a,b]],b],ORP[a,b]]]

322S>(274a,64c) PRS[second[ORP[a,b]],b] ~SUS[second[ORP[a,b]],b]
323S>(322b,251a) PRS[second[ORP[a,b]],b] MBR[f17[second[ORP[a,b]],b],second[ORP[a,b]]]
324S>(323a,305a) MBR[f17[second[ORP[a,b]],b],second[ORP[a,b]]]
325S>(324a,232a) LST[f7[f17[second[ORP[a,b]],b],ORP[a,b]]]

326S>(251b,127a) SUS[second[x],y] E.[x,ORP[f6[f17[second[x],y],x],f7[f17[second[x],y],x]]]
327S>(326a,64b) E.[x,ORP[f6[f17[second[x],y],x],f7[f17[second[x],y],x]]] PRS[second[x],y]
E.[second[x],y]
328S>(327a,5e) PRS[second[ORP[x,y]],z] E.[second[ORP[x,y]],z] ~LST[x] ~LST[y]
~LST[f6[f17[second[ORP[x,y]],z],ORP[x,y]]]
~LST[f7[f17[second[ORP[x,y]],z],ORP[x,y]]]
E.[y,f7[f17[second[ORP[x,y]],z],ORP[x,y]]]
329S>(328b,274a) PRS[second[ORP[a,b]],b] ~LST[a] ~LST[b] ~LST[f6[f17[second[ORP[a,b]],b],ORP[a,b]]]
~LST[f7[f17[second[ORP[a,b]],b],ORP[a,b]]] E.[b,f7[f17[second[ORP[a,b]],b],ORP[a,b]]]
330S>(329d,321a) PRS[second[ORP[a,b]],b] ~LST[a] ~LST[b] ~LST[a]
~LST[f7[f17[second[ORP[a,b]],b],ORP[a,b]]] E.[b,f7[f17[second[ORP[a,b]],b],ORP[a,b]]]
331S>(330a,305a) ~LST[a] ~LST[b] ~LST[a] ~LST[f7[f17[second[ORP[a,b]],b],ORP[a,b]]]
E.[b,f7[f17[second[ORP[a,b]],b],ORP[a,b]]]
332S>(331a,272a) ~LST[b] ~LST[a] ~LST[f7[f17[second[ORP[a,b]],b],ORP[a,b]]]
E.[b,f7[f17[second[ORP[a,b]],b],ORP[a,b]]]
333S>(332a,273a) ~LST[a] ~LST[f7[f17[second[ORP[a,b]],b],ORP[a,b]]]
E.[b,f7[f17[second[ORP[a,b]],b],ORP[a,b]]]
334S>(333a,272a) ~LST[f7[f17[second[ORP[a,b]],b],ORP[a,b]]]
E.[b,f7[f17[second[ORP[a,b]],b],ORP[a,b]]]
335S>(334a,325a) E.[b,f7[f17[second[ORP[a,b]],b],ORP[a,b]]]

336S>(251b,238a) SUS[second[x],y] MBR[f17[second[x],y],f7[f17[second[x],y],x]]
337S>(336a,64b) MBR[f17[second[x],y],f7[f17[second[x],y],x]] PRS[second[x],y] E.[second[x],y]
338S>(337b,33b) MBR[f17[second[x],y],f7[f17[second[x],y],x]] E.[second[x],y] ~E.[second[x],z]
PRS[z,y]
339S>(338b,274a) MBR[f17[second[ORP[a,b]],b],f7[f17[second[ORP[a,b]],b],ORP[a,b]]]
~E.[second[ORP[a,b]],x] PRS[x,b]
340S>(339a,335a) MBR[f17[second[ORP[a,b]],b],b] ~E.[second[ORP[a,b]],x] PRS[x,b]
341S>(340a,310a) ~E.[second[ORP[a,b]],x] PRS[x,b]
342S>(341a,270a) PRS[second[ORP[a,b]],b]
343S>(342a,305a) []

```

```

PHASE 0: 0 s PHASE 1: 192 s PHASE 2: 407 s Total Time: 640 s
NOD: 4026121 RES: 11454983 FAC: 1304262 T: 16 V: 32 L: 16
CTE: 4284742 CTH: 278196 CTF: 3 CSZ: 16777216
UTE: 2898042 UTH: 4766745 UTF: 0 SBA: 941
BAS: 274 RED: 294 LEN: 0+49=49

```

## SET200-6

Predicates: operation compatible equal homomorphism member function subclass single\_valued\_class  
inductive maps one\_to\_one  
Functions: wC yC zC xC . singleton\_relation subset\_relation omega universal\_class application\_function  
domain\_relation composition\_function choice null\_class identity\_relation element\_relation  
successor\_relation : successor compose\_class inverse compose restrict singleton image complement  
unordered\_pair intersection flip rotate symmetric\_difference not\_subclass\_element first second  
single\_valued1 single\_valued2 single\_valued3 power\_class regular\_sum\_class diagonalise domain\_range  
union cantor range\_of cross\_product domain\_of not\_homomorphism2 not\_homomorphism1 ordered\_pair apply  
Renamed predicates: operation compatible E. HOM MBR function subclass single\_valued\_class inductive  
maps one\_to\_one  
Renamed functions: wC yC zC xC . singleton\_relation SUBR omega UCLS application\_function DOMR CMPF  
choice NULC identity\_relation element\_relation SUCR : SUC compose\_class INV COMP restrict singleton  
image complement UNPR INT flip rotate symmetric\_difference not\_subclass\_element first second  
single\_valued1 single\_valued2 single\_valued3 power\_class regular\_sum\_class diagonalise DOMN range U  
cantor range\_of CXP domain\_of not\_homomorphism2 not\_homomorphism1 ORP APY

C16: Spawned: 60 xvl 40.20.3 W7 188S ~subclass[U[xC,zC],U[yC,x]] ... ?

Axioms:

```
1: ~operation[x] ~operation[y] ~compatible[z,x,y]
~E.[APY[y,ORP[APY[z,not_homomorphism1[z,x,y]],APY[z,not_homomorphism2[z,x,y]]],APY[z,APY[x,ORP[not_ho
momorphism1[z,x,y],not_homomorphism2[z,x,y]]]]] HOM[z,x,y]
2: ~operation[x] ~operation[y] ~compatible[z,x,y]
MBR[ORP[not_homomorphism1[z,x,y],not_homomorphism2[z,x,y]],domain_of[x]] HOM[z,x,y]
3: ~function[x] ~E.[CXP[domain_of[domain_of[x]],domain_of[domain_of[x]]],domain_of[x]]
~subclass[range_of[x],domain_of[domain_of[x]]] operation[x]
4: ~function[x] ~E.[domain_of[domain_of[y]],domain_of[x]]
~subclass[range_of[x],domain_of[domain_of[z]]] compatible[x,y,z]
5: ~MBR[ORP[x,ORP[y,z]],CXP[UCLS,CXP[UCLS,UCLS]]] ~MBR[y,domain_of[x]]
MBR[ORP[x,ORP[y,APY[x,y]]],application_function]
6: ~MBR[ORP[ORP[x,y],z],u] ~MBR[ORP[ORP[z,x],y],CXP[CXP[UCLS,UCLS],UCLS]]
MBR[ORP[ORP[z,x],y],rotate[u]]
7: ~MBR[ORP[ORP[x,y],z],u] ~MBR[ORP[ORP[y,x],z],CXP[CXP[UCLS,UCLS],UCLS]]
MBR[ORP[ORP[y,x],z],flip[u]]
8: ~HOM[x,y,z] ~MBR[ORP[u,v],domain_of[y]] E.[APY[z,ORP[APY[x,u],APY[x,v]]],APY[x,APY[y,ORP[u,v]]]]
9: ~MBR[x,image[y,image[z,singleton[u]]] ~MBR[ORP[u,x],CXP[UCLS,UCLS]] MBR[ORP[u,x],COMP[y,z]]
10: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] ~E.[COMP[z,x],y] MBR[ORP[x,y],compose_class[z]]
11: ~E.[SUC[x],y] ~MBR[ORP[x,y],CXP[UCLS,UCLS]] MBR[ORP[x,y],SUCR]
12: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] ~MBR[x,y] MBR[ORP[x,y],element_relation]
13: ~subclass[x,CXP[UCLS,UCLS]] ~subclass[COMP[x,INV[x]],identity_relation] function[x]
14: ~MBR[x,UCLS] E.[restrict[y,singleton[x],UCLS],NULC] MBR[x,domain_of[y]]
15: ~E.[x,y] ~single_valued_class[x] single_valued_class[y]
16: ~function[x] ~MBR[y,UCLS] MBR[image[x,y],UCLS]
17: ~MBR[x,y] ~MBR[z,u] MBR[ORP[x,z],CXP[y,u]]
18: ~MBR[NULC,x] ~subclass[image[SUCR,x],x] inductive[x]
19: ~E.[x,y] ~HOM[x,z,u] HOM[y,z,u]
20: ~E.[x,y] ~HOM[z,x,u] HOM[z,y,u]
21: ~E.[x,y] ~HOM[z,u,x] HOM[z,u,y]
22: ~MBR[x,UCLS] E.[x,NULC] MBR[APY[choice,x],x]
23: ~E.[x,y] ~compatible[x,z,u] compatible[y,z,u]
24: ~E.[x,y] ~compatible[z,x,u] compatible[z,y,u]
25: ~E.[x,y] ~compatible[z,u,x] compatible[z,u,y]
26 >~MBR[x,UCLS] MBR[x,complement[y]] MBR[x,y]
27: ~MBR[x,UNPR[y,z]] E.[x,y] E.[x,z]
28: ~function[x] ~subclass[range_of[x],y] maps[x,domain_of[x],y]
29: ~E.[x,y] ~one_to_one[x] one_to_one[y]
30: ~E.[x,y] ~subclass[x,z] subclass[y,z]
31: ~E.[x,y] ~subclass[z,x] subclass[z,y]
32 >~MBR[x,y] ~MBR[x,z] MBR[x,INT[y,z]]
33: ~E.[x,y] ~inductive[x] inductive[y]
34: ~E.[x,y] ~operation[x] operation[y]
35: ~subclass[x,y] ~subclass[y,x] E.[x,y]
36: ~function[INV[x]] ~function[x] one_to_one[x]
37: ~E.[x,y] ~function[x] function[y]
38: ~E.[x,y] ~MBR[x,z] MBR[y,z]
39 >~E.[x,y] ~MBR[z,x] MBR[z,y]
40: ~E.[x,y] ~maps[x,z,u] maps[y,z,u]
41: ~E.[x,y] ~maps[z,x,u] maps[z,y,u]
42: ~E.[x,y] ~maps[z,u,x] maps[z,u,y]
43 >~subclass[x,y] ~MBR[z,x] MBR[z,y]
44: ~E.[x,y] ~E.[y,z] E.[x,z]
45: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] MBR[ORP[x,ORP[y,COMP[x,y]]],CMPF]
46: ~MBR[ORP[x,ORP[y,z]],CMPF] E.[COMP[x,y],z]
47: ~MBR[ORP[x,ORP[y,z]],application_function] MBR[y,domain_of[x]]
48: ~operation[x] E.[CXP[domain_of[domain_of[x]],domain_of[domain_of[x]]],domain_of[x]]
49: ~MBR[x,UCLS] MBR[ORP[x,domain_of[x]],DOMR]
50: ~E.[x,y] E.[symmetric_difference[x,z],symmetric_difference[y,z]]
```

```

51: ~E.[x,y] E.[symmetric_difference[z,x],symmetric_difference[z,y]]
52: ~E.[x,y] E.[not_subclass_element[x,z],not_subclass_element[y,z]]
53: ~E.[x,y] E.[not_subclass_element[z,x],not_subclass_element[z,y]]
54: ~MBR[ORP[x,ORP[y,z]],application_function] E.[APY[x,y],z]
55: ~E.[x,y] E.[not_homomorphism1[x,z,u],not_homomorphism1[y,z,u]]
56: ~E.[x,y] E.[not_homomorphism1[z,x,u],not_homomorphism1[z,y,u]]
57: ~E.[x,y] E.[not_homomorphism1[z,u,x],not_homomorphism1[z,u,y]]
58: ~E.[x,y] E.[not_homomorphism2[x,z,u],not_homomorphism2[y,z,u]]
59: ~E.[x,y] E.[not_homomorphism2[z,x,u],not_homomorphism2[z,y,u]]
60: ~E.[x,y] E.[not_homomorphism2[z,u,x],not_homomorphism2[z,u,y]]
61: ~MBR[ORP[ORP[x,y],z],rotate[u]] MBR[ORP[ORP[y,z],x],u]
62: ~MBR[ORP[ORP[x,y],z],flip[u]] MBR[ORP[ORP[y,x],z],u]
63: ~MBR[ORP[x,y],DOMR] E.[domain_of[x],y]
64: ~single_valued_class[x] subclass[COMP[x,INV[x]],identity_relation]
65: ~subclass[COMP[x,INV[x]],identity_relation] single_valued_class[x]
66: ~MBR[ORP[x,y],COMP[z,u]] MBR[y,image[z,image[u, singleton[x]]]]
67: ~E.[x,y] E.[UNPR[x,z],UNPR[y,z]]
68: ~E.[x,y] E.[UNPR[z,x],UNPR[z,y]]
69: ~E.[restrict[x, singleton[y],UCLS],NULC] ~MBR[y,domain_of[x]]
70: ~MBR[ORP[x,y],compose_class[z]] E.[COMP[z,x],y]
71: ~MBR[x,CXP[y,z]] E.[ORP[first[x],second[x]],x]
72: ~MBR[ORP[x,y],SUCR] E.[SUC[x],y]
73: ~E.[x,y] E.[INT[x,z],INT[y,z]]
74: ~E.[x,y] E.[INT[z,x],INT[z,y]]
75: ~E.[x,y] E.[ORP[x,z],ORP[y,z]]
76: ~E.[x,y] E.[ORP[z,x],ORP[z,y]]
77: ~E.[x,y] E.[single_valued1[x],single_valued1[y]]
78: ~E.[x,y] E.[single_valued2[x],single_valued2[y]]
79: ~E.[x,y] E.[single_valued3[x],single_valued3[y]]
80: ~function[x] subclass[x,CXP[UCLS,UCLS]]
81: ~compatible[x,y,z] subclass[range_of[x],domain_of[domain_of[z]]]
82: ~E.[x,y] E.[CXP[x,z],CXP[y,z]]
83: ~E.[x,y] E.[CXP[z,x],CXP[z,y]]
84: ~compatible[x,y,z] E.[domain_of[domain_of[y]],domain_of[x]]
85: ~E.[x,y] E.[restrict[x,z,u],restrict[y,z,u]]
86: ~E.[x,y] E.[restrict[z,x,u],restrict[z,y,u]]
87: ~E.[x,y] E.[restrict[z,u,x],restrict[z,u,y]]
88: ~MBR[x,UCLS] MBR[power_class[x],UCLS]
89: ~E.[x,y] E.[compose_class[x],compose_class[y]]
90: E.[x,NULC] E.[INT[x,regular[x]],NULC]
91: ~E.[x,y] E.[power_class[x],power_class[y]]
92: ~MBR[ORP[x,y],CXP[z,u]] MBR[x,z]
93: ~MBR[ORP[x,y],CXP[z,u]] MBR[y,u]
94: ~MBR[x,UCLS] MBR[sum_class[x],UCLS]
95: ~function[x] subclass[COMP[x,INV[x]],identity_relation]
96: ~E.[x,y] E.[diagonalise[x],diagonalise[y]]
97: ~E.[x,y] E.[DOMN[x,z,u],DOMN[y,z,u]]
98: ~E.[x,y] E.[DOMN[z,x,u],DOMN[z,y,u]]
99: ~E.[x,y] E.[DOMN[z,u,x],DOMN[z,u,y]]
100: ~MBR[ORP[x,y],element_relation] MBR[x,y]
101: ~MBR[x,UCLS] MBR[x,UNPR[x,y]]
102: ~MBR[x,UCLS] MBR[x,UNPR[y,x]]
103: ~operation[x] subclass[range_of[x],domain_of[domain_of[x]]]
104: >MBR[not_subclass_element[x,y],x] subclass[x,y]
105: >>MBR[not_subclass_element[x,y],y] subclass[x,y]
106: ~E.[x,y] E.[domain_of[x],domain_of[y]]
107: ~E.[x,y] E.[range[x,z,u],range[y,z,u]]
108: ~E.[x,y] E.[range[z,x,u],range[z,y,u]]
109: ~E.[x,y] E.[range[z,u,x],range[z,u,y]]
110: ~E.[x,y] E.[singleton[x],singleton[y]]
111: ~E.[x,y] E.[SUC[x],SUC[y]]
112: ~E.[x,y] E.[sum_class[x],sum_class[y]]
113: ~E.[x,y] E.[complement[x],complement[y]]
114: ~E.[x,y] E.[range_of[x],range_of[y]]
115: ~E.[x,y] E.[COMP[x,z],COMP[y,z]]
116: ~E.[x,y] E.[COMP[z,x],COMP[z,y]]
117: ~E.[x,y] E.[image[x,z],image[y,z]]
118: ~E.[x,y] E.[image[z,x],image[z,y]]
119: ~E.[x,y] E.[INV[x],INV[y]]
120: ~E.[x,y] E.[regular[x],regular[y]]
121: ~E.[x,y] E.[U[x,z],U[y,z]]
122: ~E.[x,y] E.[U[z,x],U[z,y]]
123: ~inductive[x] subclass[image[SUCR,x],x]
124: ~E.[x,y] E.[rotate[x],rotate[y]]
125: ~E.[x,y] E.[second[x],second[y]]
126: ~HOM[x,y,z] compatible[x,y,z]
127: ~E.[x,y] E.[APY[x,z],APY[y,z]]
128: ~E.[x,y] E.[APY[z,x],APY[z,y]]
129: ~E.[x,y] E.[first[x],first[y]]
130: ~E.[x,y] E.[cantor[x],cantor[y]]
131: ~E.[x,y] E.[flip[x],flip[y]]

```

```

132: ~MBR[x,INT[y,z]] MBR[x,y]
133: >~MBR[x,INT[y,z]] MBR[x,z]
134: E.[x,NULC] MBR[regular[x],x]
135: ~HOM[x,y,z] operation[y]
136: ~HOM[x,y,z] operation[z]
137: ~inductive[x] subclass[omega,x]
138: ~E.[x,y] subclass[x,y]
139: ~E.[x,y] subclass[y,x]
140: ~one_to_one[x] function[INV[x]]
141: >~MBR[x,complement[y]] ~MBR[x,y]
142: ~compatible[x,y,z] function[x]
143: ~inductive[x] MBR[NULC,x]
144: ~maps[x,y,z] subclass[range_of[x],z]
145: ~maps[x,y,z] E.[domain_of[x],y]
146: ~one_to_one[x] function[x]
147: ~operation[x] function[x]
148: ~maps[x,y,z] function[x]
149: ~E.[x,y] E.[y,x]
150: E.[INT[CXP[UCLS,UCLS],INT[CXP[UCLS,UCLS],complement[COMP[complement[elemet_relation],
INV[element_relation]]]]],SUBR]
151: E.[INT[complement[COMP[element_relation,complement[identity_relation]]],element_relation],
singleton_relation]
152: E.[INT[complement[INT[x,y]],complement[INT[complement[x],complement[y]]]],
symmetric_difference[x,y]]
153: subclass[CMFP,CXP[UCLS,CXP[UCLS,UCLS]]]
154: subclass[application_function,CXP[UCLS,CXP[UCLS,UCLS]]]
155: E.[DOMN[x,image[INV[x],singleton[single_valued1[x]]],single_valued2[x],single_valued3[x]]]
156: E.[second[not_subclass_element[COMP[x,INV[x]],identity_relation]],single_valued2[x]]
157: E.[first[not_subclass_element[COMP[x,INV[x]],identity_relation]],single_valued1[x]]
158: subclass[rotate[x],CXP[CXP[UCLS,UCLS],UCLS]]
159: E.[INT[domain_of[x],diagonalise[COMP[INV[element_relation],x]]],cantor[x]]
160: subclass[DOMR,CXP[UCLS,UCLS]]
161: subclass[flip[x],CXP[CXP[UCLS,UCLS],UCLS]]
162: subclass[compose_class[x],CXP[UCLS,UCLS]]
163: E.[INT[INV[SUBR],SUBR],identity_relation]
164: E.[domain_of[restrict[element_relation,UCLS,x]],sum_class[x]]
165: E.[UNPR[singleton[x],UNPR[x,singleton[y]]],ORP[x,y]]
166: subclass[SUCR,CXP[UCLS,UCLS]]
167: E.[complement[domain_of[INT[x,identity_relation]]],diagonalise[x]]
168: E.[first[not_subclass_element[restrict[x,y,singleton[z]],NULC]],DOMN[x,y,z]]
169: E.[complement[image[element_relation,complement[x]]],power_class[x]]
170: subclass[element_relation,CXP[UCLS,UCLS]]
171: E.[second[not_subclass_element[restrict[x,singleton[y],z],NULC]],range[x,y,z]]
172: subclass[COMP[x,y],CXP[UCLS,UCLS]]
173: >E.[complement[INT[complement[x],complement[y]]],U[x,y]]
174: E.[domain_of[flip[x,UCLS]],INV[x]]
175: MBR[UNPR[x,y],UCLS]
176: E.[INT[x,CXP[y,z]],restrict[x,y,z]]
177: E.[INT[CXP[x,y],z],restrict[z,x,y]]
178: E.[range_of[restrict[x,y,UCLS]],image[x,y]]
179: E.[sum_class[image[x,singleton[y]]],APY[x,y]]
180: E.[UNPR[x,x],singleton[x]]
181: >subclass[x,UCLS]
182: E.[U[x,singleton[x]],SUC[x]]
183: MBR[omega,UCLS]
184: E.[domain_of[INV[x]],range_of[x]]
185: inductive[omega]
186: function[choice]
187: E.[x,x]

```

Negated conclusion:

```

188S>~subclass[U[xC,zC],U[yC,x]] ... ?
189S subclass[xC,yC]
190S subclass[zC,wC]

```

-----

Phase 0 clauses used in proof:

```

198S>(188a*105b) ~MBR[not_subclass_element[U[xC,zC],U[yC,x]],U[yC,x]] ... ?
199S>(188a*104b) MBR[not_subclass_element[U[xC,zC],U[yC,x]],U[xC,zC]] ... ?

```

Phases 1 and 2 clauses used in proof:

```

211S>(199a,43b) ~subclass[U[xC,zC],x] MBR[not_subclass_element[U[xC,zC],U[yC,y]],x] ... ?
212S>(211b,141b) ~subclass[U[xC,zC],x]
~MBR[not_subclass_element[U[xC,zC],U[yC,y]],complement[x]] ... ?
213S>(212a,181a) ~MBR[not_subclass_element[U[xC,zC],U[yC,x]],complement[UCLS]] ... ?

214S>(199a,32b) ~MBR[not_subclass_element[U[xC,zC],U[yC,x]],y]
MBR[not_subclass_element[U[xC,zC],U[yC,x]],INT[y,U[xC,zC]]] ... ?
215S>(214b,43b) ~MBR[not_subclass_element[U[xC,zC],U[yC,x]],y] ~subclass[INT[y,U[xC,zC]],z]
MBR[not_subclass_element[U[xC,zC],U[yC,x]],z] ... ?

```

```

216S>(215b,181a) ~MBR[not_subclass_element[U[xC,zC],U[yC,x]],y]
MBR[not_subclass_element[U[xC,zC],U[yC,x]],UCLS] ... ?
217S>(216a,199a) MBR[not_subclass_element[U[xC,zC],U[yC,x]],UCLS] ... ?

218S>(198a,39c) ~E.[x,U[yC,y]] ~MBR[not_subclass_element[U[xC,zC],U[yC,y]],x] ... ?
219S>(218b,26b) ~E.[complement[x],U[yC,y]] ~MBR[not_subclass_element[U[xC,zC],U[yC,y]],UCLS]
MBR[not_subclass_element[U[xC,zC],U[yC,y]],x] ... ?
220S>(219b,217a) ~E.[complement[x],U[yC,y]] MBR[not_subclass_element[U[xC,zC],U[yC,y]],x] ... ?
221S>(220b,133a) ~E.[complement[INT[x,y]],U[yC,z]]
MBR[not_subclass_element[U[xC,zC],U[yC,z]],y] ... ?
222S>(221a,173a) MBR[not_subclass_element[U[xC,zC],U[yC,x]],complement[x]] ... ?
223S>(222a,213a) [] ... ?

```

Inferences derived when variable is replaced by original constant  
211 212 213 214 215 216 217 218 219 220 221 222

```

PHASE 0: 0 s PHASE 1: 4 s PHASE 2: 1 s Total Time: 5 s
NOD: 7122 RES: 45055 FAC: 956 T: 40 V: 32 L: 16
CTE: 32858 CTH: 2256 CTF: 0 CSZ: 16777216
UTE: 389565 UTH: 699066 UTF: 0 SBA: 23
BAS: 190 RED: 203 LEN: 2+13=15

```

Proof found to theorem by slave 16!

Axioms:

```

26S>~MBR[x,UCLS] MBR[x,complement[y]] MBR[x,y]
32S>~MBR[x,y] ~MBR[x,z] MBR[x,INT[y,z]]
43S>~subclass[x,y] ~MBR[z,x] MBR[z,y]
104S MBR[not_subclass_element[x,y],x] subclass[x,y]
105S ~MBR[not_subclass_element[x,y],y] subclass[x,y]
132 >~MBR[x,INT[y,z]] MBR[x,y]
133S ~MBR[x,INT[y,z]] MBR[x,z]
139 >~E.[x,y] subclass[y,x]
141S>~MBR[x,complement[y]] ~MBR[x,y]
173S>E.[complement[INT[complement[x],complement[y]]],U[x,y]]
181S subclass[x,UCLS]

```

Negated conclusion:

```

188S ~subclass[U[xC,zC],U[yC,wC]]
189S>subclass[xC,yC]
190S>subclass[zC,wC]

```

Phase 0 clauses used in proof:

```

191S>~subclass[U[xC,zC],x] MBR[not_subclass_element[U[xC,zC],U[yC,wC]],x]
197S>MBR[not_subclass_element[U[xC,zC],U[yC,wC]],UCLS]
200S>~E.[complement[x],U[yC,wC]] MBR[not_subclass_element[U[xC,zC],U[yC,wC]],x]
202S>MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[wC]]

```

Phases 1 and 2 clauses used in proof:

```

211S>(200b,132a) ~E.[complement[INT[x,y]],U[yC,wC]] MBR[not_subclass_element[U[xC,zC],U[yC,wC]],x]
212S>(211b,141a) ~E.[complement[INT[complement[x],y]],U[yC,wC]]
~MBR[not_subclass_element[U[xC,zC],U[yC,wC]],x]
213S>(212a,173a) ~MBR[not_subclass_element[U[xC,zC],U[yC,wC]],yC]

214S>(202a,141a) ~MBR[not_subclass_element[U[xC,zC],U[yC,wC]],wC]

215S>(197a,26a) MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[x]]
MBR[not_subclass_element[U[xC,zC],U[yC,wC]],x]
216S>(215b,43b) MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[x]] ~subclass[x,y]
MBR[not_subclass_element[U[xC,zC],U[yC,wC]],y]
217S>(216b,189a) MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[xC]]
MBR[not_subclass_element[U[xC,zC],U[yC,wC]],yC]
218S>(217b,213a) MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[xC]]

219S>(197a,26a) MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[x]]
MBR[not_subclass_element[U[xC,zC],U[yC,wC]],x]
220S>(219b,43b) MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[x]] ~subclass[x,y]
MBR[not_subclass_element[U[xC,zC],U[yC,wC]],y]
221S>(220b,190a) MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[zC]]
MBR[not_subclass_element[U[xC,zC],U[yC,wC]],wC]
222S>(221b,214a) MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[zC]]

223S>(191a,139b) MBR[not_subclass_element[U[xC,zC],U[yC,wC]],x] ~E.[x,U[xC,zC]]
224S>(223a,141a) ~E.[complement[x],U[xC,zC]] ~MBR[not_subclass_element[U[xC,zC],U[yC,wC]],x]
225S>(224b,32c) ~E.[complement[INT[x,y]],U[xC,zC]] ~MBR[not_subclass_element[U[xC,zC],U[yC,wC]],x]
~MBR[not_subclass_element[U[xC,zC],U[yC,wC]],y]
226S>(225a,173a) ~MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[xC]]
~MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[zC]]
227S>(226a,218a) ~MBR[not_subclass_element[U[xC,zC],U[yC,wC]],complement[zC]]
228S>(227a,222a) []

```

PHASE 0: 0 s PHASE 1: 21 s PHASE 2: 2 s Total Time: 28 s  
NOD: 53646 RES: 384160 FAC: 6852 T: 40 V: 32 L: 16  
CTE: 328571 CTH: 27191 CTF: 0 CSZ: 16777216  
UTE: 671606 UTH: 1301561 UTF: 0 SBA: 23  
BAS: 190 RED: 201 LEN: 4+18=22

## SET222-6

Predicates: operation compatible equal homomorphism member function subclass single\_valued\_class  
inductive maps one\_to\_one  
Functions: zC yC xC wC . singleton\_relation subset\_relation omega universal\_class application\_function  
domain\_relation composition\_function choice null\_class identity\_relation element\_relation  
successor\_relation : successor compose\_class inverse compose restrict singleton image complement  
unordered\_pair intersection flip rotate symmetric\_difference not\_subclass\_element first second  
single\_valued1 single\_valued2 single\_valued3 power\_class regular sum\_class diagonalise domain range  
union cantor range\_of cross\_product domain\_of not\_homomorphism2 not\_homomorphism1 ordered\_pair apply  
Renamed predicates: operation compatible E. HOM MBR function subclass single\_valued\_class inductive  
maps one\_to\_one

Renamed functions: zC yC xC wC . singleton\_relation SUBR omega UCLS application\_function DOMR CMPF  
choice NULC identity\_relation element\_relation SUCR : SUC compose\_class INV COMP restrict singleton  
image complement UNPR INT flip rotate symmetric\_difference not\_subclass\_element first second  
single\_valued1 single\_valued2 single\_valued3 power\_class regular sum\_class diagonalise DOMN range U  
cantor range\_of CXP domain\_of not\_homomorphism2 not\_homomorphism1 ORP APY

C6: Spawned: 60 xv1 40.20.3 W2 188S ~subclass[INT[CXP[wC,xC],CXP[x,zC]],CXP[wC,zC]] ... ?

Axioms:

```
1: ~operation[x] ~operation[y] ~compatible[z,x,y]
~E.[APY[y,ORP[APY[z,not_homomorphism1[z,x,y]],APY[z,not_homomorphism2[z,x,y]]],APY[z,APY[x,ORP[not_ho
morphism1[z,x,y],not_homomorphism2[z,x,y]]]]] HOM[z,x,y]
2: ~operation[x] ~operation[y] ~compatible[z,x,y]
MBR[ORP[not_homomorphism1[z,x,y],not_homomorphism2[z,x,y]],domain_of[x]] HOM[z,x,y]
3: ~function[x] ~E.[CXP[domain_of[domain_of[x]],domain_of[domain_of[x]]],domain_of[x]]
~subclass[range_of[x],domain_of[domain_of[x]]] operation[x]
4: ~function[x] ~E.[domain_of[domain_of[y]],domain_of[x]]
~subclass[range_of[x],domain_of[domain_of[z]]] compatible[x,y,z]
5: ~MBR[ORP[x,ORP[y,z]],CXP[UCLS,CXP[UCLS,UCLS]]] ~MBR[y,domain_of[x]]
MBR[ORP[x,ORP[y,APY[x,y]]],application_function]
6: ~MBR[ORP[ORP[x,y],z],u] ~MBR[ORP[ORP[z,x],y],CXP[CXP[UCLS,UCLS],UCLS]]
MBR[ORP[ORP[z,x],y],rotate[u]]
7: ~MBR[ORP[ORP[x,y],z],u] ~MBR[ORP[ORP[y,x],z],CXP[CXP[UCLS,UCLS],UCLS]]
MBR[ORP[ORP[y,x],z],flip[u]]
8: ~HOM[x,y,z] ~MBR[ORP[u,v],domain_of[y]] E.[APY[z,ORP[APY[x,u],APY[x,v]]],APY[x,APY[y,ORP[u,v]]]]
9: ~MBR[x,image[y,image[z,singleton[u]]] ~MBR[ORP[u,x],CXP[UCLS,UCLS]] MBR[ORP[u,x],COMP[y,z]]
10: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] ~E.[COMP[z,x],y] MBR[ORP[x,y],compose_class[z]]
11: ~E.[SUC[x],y] ~MBR[ORP[x,y],CXP[UCLS,UCLS]] MBR[ORP[x,y],SUCR]
12: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] ~MBR[x,y] MBR[ORP[x,y],element_relation]
13: ~subclass[x,CXP[UCLS,UCLS]] ~subclass[COMP[x,INV[x]],identity_relation] function[x]
14: ~MBR[x,UCLS] E.[restrict[y,singleton[x],UCLS],NULC] MBR[x,domain_of[y]]
15: ~E.[x,y] ~single_valued_class[x] single_valued_class[y]
16: ~function[x] ~MBR[y,UCLS] MBR[image[x,y],UCLS]
17: ~MBR[x,y] ~MBR[z,u] MBR[ORP[x,z],CXP[y,u]]
18: ~MBR[NULC,x] ~subclass[image[SUCR,x],x] inductive[x]
19: ~E.[x,y] ~HOM[x,z,u] HOM[y,z,u]
20: ~E.[x,y] ~HOM[z,x,u] HOM[z,y,u]
21: ~E.[x,y] ~HOM[z,u,x] HOM[z,u,y]
22: ~MBR[x,UCLS] E.[x,NULC] MBR[APY[choice,x],x]
23: ~E.[x,y] ~compatible[x,z,u] compatible[y,z,u]
24: ~E.[x,y] ~compatible[z,x,u] compatible[z,y,u]
25: ~E.[x,y] ~compatible[z,u,x] compatible[z,u,y]
26: ~MBR[x,UCLS] MBR[x,complement[y]] MBR[x,y]
27: ~MBR[x,UNPR[y,z]] E.[x,y] E.[x,z]
28: ~function[x] ~subclass[range_of[x],y] maps[x,domain_of[x],y]
29: ~E.[x,y] ~one_to_one[x] one_to_one[y]
30: ~E.[x,y] ~subclass[x,z] subclass[y,z]
31: ~E.[x,y] ~subclass[z,x] subclass[z,y]
32: ~MBR[x,y] ~MBR[x,z] MBR[x,INT[y,z]]
33: ~E.[x,y] ~inductive[x] inductive[y]
34: ~E.[x,y] ~operation[x] operation[y]
35: ~subclass[x,y] ~subclass[y,x] E.[x,y]
36: ~function[INV[x]] ~function[x] one_to_one[x]
37: ~E.[x,y] ~function[x] function[y]
38: ~E.[x,y] ~MBR[x,z] MBR[y,z]
39: ~E.[x,y] ~MBR[z,x] MBR[z,y]
40: ~E.[x,y] ~maps[x,z,u] maps[y,z,u]
41: ~E.[x,y] ~maps[z,x,u] maps[z,y,u]
42: ~E.[x,y] ~maps[z,u,x] maps[z,u,y]
43: ~subclass[x,y] ~MBR[z,x] MBR[z,y]
44: ~E.[x,y] ~E.[y,z] E.[x,z]
45: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] MBR[ORP[x,ORP[y,COMP[x,y]]],CMPF]
46: ~MBR[ORP[x,ORP[y,z]],CMPF] E.[COMP[x,y],z]
47: ~MBR[ORP[x,ORP[y,z]],application_function] MBR[y,domain_of[x]]
48: ~operation[x] E.[CXP[domain_of[domain_of[x]],domain_of[domain_of[x]]],domain_of[x]]
49: ~MBR[x,UCLS] MBR[ORP[x,domain_of[x]],DOMR]
50: ~E.[x,y] E.[symmetric_difference[x,z],symmetric_difference[y,z]]
```

```

51: ~E.[x,y] E.[symmetric_difference[z,x],symmetric_difference[z,y]]
52: ~E.[x,y] E.[not_subclass_element[x,z],not_subclass_element[y,z]]
53: ~E.[x,y] E.[not_subclass_element[z,x],not_subclass_element[z,y]]
54: ~MBR[ORP[x,ORP[y,z]],application_function] E.[APY[x,y],z]
55: ~E.[x,y] E.[not_homomorphism1[x,z,u],not_homomorphism1[y,z,u]]
56: ~E.[x,y] E.[not_homomorphism1[z,x,u],not_homomorphism1[z,y,u]]
57: ~E.[x,y] E.[not_homomorphism1[z,u,x],not_homomorphism1[z,u,y]]
58: ~E.[x,y] E.[not_homomorphism2[x,z,u],not_homomorphism2[y,z,u]]
59: ~E.[x,y] E.[not_homomorphism2[z,x,u],not_homomorphism2[z,y,u]]
60: ~E.[x,y] E.[not_homomorphism2[z,u,x],not_homomorphism2[z,u,y]]
61: ~MBR[ORP[ORP[x,y],z],rotate[u]] MBR[ORP[ORP[y,z],x],u]
62: ~MBR[ORP[ORP[x,y],z],flip[u]] MBR[ORP[ORP[y,x],z],u]
63: ~MBR[ORP[x,y],DOMR] E.[domain_of[x],y]
64: ~single_valued_class[x] subclass[COMP[x,INV[x]],identity_relation]
65: ~subclass[COMP[x,INV[x]],identity_relation] single_valued_class[x]
66: ~MBR[ORP[x,y],COMP[z,u]] MBR[y,image[z,image[u, singleton[x]]]]
67: ~E.[x,y] E.[UNPR[x,z],UNPR[y,z]]
68: ~E.[x,y] E.[UNPR[z,x],UNPR[z,y]]
69: ~E.[restrict[x, singleton[y],UCLS],NULC] ~MBR[y,domain_of[x]]
70: ~MBR[ORP[x,y],compose_class[z]] E.[COMP[z,x],y]
71: ~MBR[x,CXP[y,z]] E.[ORP[first[x],second[x]],x]
72: ~MBR[ORP[x,y],SUCR] E.[SUC[x],y]
73: ~E.[x,y] E.[INT[x,z],INT[y,z]]
74: ~E.[x,y] E.[INT[z,x],INT[z,y]]
75: ~E.[x,y] E.[ORP[x,z],ORP[y,z]]
76: ~E.[x,y] E.[ORP[z,x],ORP[z,y]]
77: ~E.[x,y] E.[single_valued1[x],single_valued1[y]]
78: ~E.[x,y] E.[single_valued2[x],single_valued2[y]]
79: ~E.[x,y] E.[single_valued3[x],single_valued3[y]]
80: ~function[x] subclass[x,CXP[UCLS,UCLS]]
81: ~compatible[x,y,z] subclass[range_of[x],domain_of[domain_of[z]]]
82: ~E.[x,y] E.[CXP[x,z],CXP[y,z]]
83: ~E.[x,y] E.[CXP[z,x],CXP[z,y]]
84: ~compatible[x,y,z] E.[domain_of[domain_of[y]],domain_of[x]]
85: ~E.[x,y] E.[restrict[x,z,u],restrict[y,z,u]]
86: ~E.[x,y] E.[restrict[z,x,u],restrict[z,y,u]]
87: ~E.[x,y] E.[restrict[z,u,x],restrict[z,u,y]]
88: ~MBR[x,UCLS] MBR[power_class[x],UCLS]
89: ~E.[x,y] E.[compose_class[x],compose_class[y]]
90: E.[x,NULC] E.[INT[x,regular[x]],NULC]
91: ~E.[x,y] E.[power_class[x],power_class[y]]
92: ~MBR[ORP[x,y],CXP[z,u]] MBR[x,z]
93: ~MBR[ORP[x,y],CXP[z,u]] MBR[y,u]
94: ~MBR[x,UCLS] MBR[sum_class[x],UCLS]
95: ~function[x] subclass[COMP[x,INV[x]],identity_relation]
96: ~E.[x,y] E.[diagonalise[x],diagonalise[y]]
97: ~E.[x,y] E.[DOMN[x,z,u],DOMN[y,z,u]]
98: ~E.[x,y] E.[DOMN[z,x,u],DOMN[z,y,u]]
99: ~E.[x,y] E.[DOMN[z,u,x],DOMN[z,u,y]]
100: ~MBR[ORP[x,y],element_relation] MBR[x,y]
101: ~MBR[x,UCLS] MBR[x,UNPR[x,y]]
102: ~MBR[x,UCLS] MBR[x,UNPR[y,x]]
103: ~operation[x] subclass[range_of[x],domain_of[domain_of[x]]]
104: >MBR[not_subclass_element[x,y],x] subclass[x,y]
105: >>MBR[not_subclass_element[x,y],y] subclass[x,y]
106: ~E.[x,y] E.[domain_of[x],domain_of[y]]
107: ~E.[x,y] E.[range[x,z,u],range[y,z,u]]
108: ~E.[x,y] E.[range[z,x,u],range[z,y,u]]
109: ~E.[x,y] E.[range[z,u,x],range[z,u,y]]
110: ~E.[x,y] E.[singleton[x],singleton[y]]
111: ~E.[x,y] E.[SUC[x],SUC[y]]
112: ~E.[x,y] E.[sum_class[x],sum_class[y]]
113: ~E.[x,y] E.[complement[x],complement[y]]
114: ~E.[x,y] E.[range_of[x],range_of[y]]
115: ~E.[x,y] E.[COMP[x,z],COMP[y,z]]
116: ~E.[x,y] E.[COMP[z,x],COMP[z,y]]
117: ~E.[x,y] E.[image[x,z],image[y,z]]
118: ~E.[x,y] E.[image[z,x],image[z,y]]
119: ~E.[x,y] E.[INV[x],INV[y]]
120: ~E.[x,y] E.[regular[x],regular[y]]
121: ~E.[x,y] E.[U[x,z],U[y,z]]
122: ~E.[x,y] E.[U[z,x],U[z,y]]
123: ~inductive[x] subclass[image[SUCR,x],x]
124: ~E.[x,y] E.[rotate[x],rotate[y]]
125: ~E.[x,y] E.[second[x],second[y]]
126: ~HOM[x,y,z] compatible[x,y,z]
127: ~E.[x,y] E.[APY[x,z],APY[y,z]]
128: ~E.[x,y] E.[APY[z,x],APY[z,y]]
129: ~E.[x,y] E.[first[x],first[y]]
130: ~E.[x,y] E.[cantor[x],cantor[y]]
131: ~E.[x,y] E.[flip[x],flip[y]]

```

```

132: ~MBR[x,INT[y,z]] MBR[x,y]
133: >~MBR[x,INT[y,z]] MBR[x,z]
134: E.[x,NULC] MBR[regular[x],x]
135: ~HOM[x,y,z] operation[y]
136: ~HOM[x,y,z] operation[z]
137: ~inductive[x] subclass[omega,x]
138: ~E.[x,y] subclass[x,y]
139: ~E.[x,y] subclass[y,x]
140: ~one_to_one[x] function[INV[x]]
141: ~MBR[x,complement[y]] ~MBR[x,y]
142: ~compatible[x,y,z] function[x]
143: ~inductive[x] MBR[NULC,x]
144: ~maps[x,y,z] subclass[range_of[x],z]
145: ~maps[x,y,z] E.[domain_of[x],y]
146: ~one_to_one[x] function[x]
147: ~operation[x] function[x]
148: ~maps[x,y,z] function[x]
149: ~E.[x,y] E.[y,x]
150: E.[INT[CXP[UCLS,UCLS]],INT[CXP[UCLS,UCLS]],complement[COMP[complement[element_relation],
INV[element_relation]]]],SUBR]
151: E.[INT[complement[COMP[element_relation,complement[identity_relation]]],element_relation],
singleton_relation]
152: E.[INT[complement[INT[x,y]],complement[INT[complement[x],complement[y]]]],
symmetric_difference[x,y]]
153: subclass[CMPP,CXP[UCLS,CXP[UCLS,UCLS]]]
154: subclass[application_function,CXP[UCLS,CXP[UCLS,UCLS]]]
155: E.[DOMN[x,image[INV[x],singleton[single_valued1[x]]],single_valued2[x],single_valued3[x]]]
156: E.[second[not_subclass_element[COMP[x,INV[x]],identity_relation]],single_valued2[x]]
157: E.[first[not_subclass_element[COMP[x,INV[x]],identity_relation]],single_valued1[x]]
158: subclass[rotate[x],CXP[CXP[UCLS,UCLS],UCLS]]
159: E.[INT[domain_of[x],diagonalise[COMP[INV[element_relation],x]]],cantor[x]]
160: subclass[DOMR,CXP[UCLS,UCLS]]
161: subclass[flip[x],CXP[CXP[UCLS,UCLS],UCLS]]
162: subclass[compose_class[x],CXP[UCLS,UCLS]]
163: E.[INT[INV[SUBR],SUBR],identity_relation]
164: E.[domain_of[restrict[element_relation,UCLS,x]],sum_class[x]]
165: E.[UNPR[singleton[x],UNPR[x,singleton[y]]],ORP[x,y]]
166: subclass[SUCR,CXP[UCLS,UCLS]]
167: E.[complement[domain_of[INT[x,identity_relation]]],diagonalise[x]]
168: E.[first[not_subclass_element[restrict[x,y,singleton[z]],NULC]],DOMN[x,y,z]]
169: E.[complement[image[element_relation,complement[x]]],power_class[x]]
170: subclass[element_relation,CXP[UCLS,UCLS]]
171: E.[second[not_subclass_element[restrict[x,singleton[y],z],NULC]],range[x,y,z]]
172: subclass[COMP[x,y],CXP[UCLS,UCLS]]
173: E.[complement[INT[complement[x],complement[y]]],U[x,y]]
174: E.[domain_of[flip[CXP[x,UCLS]]],INV[x]]
175: MBR[UNPR[x,y],UCLS]
176: E.[INT[x,CXP[y,z]],restrict[x,y,z]]
177: E.[INT[CXP[x,y],z],restrict[z,x,y]]
178: E.[range_of[restrict[x,y,UCLS]],image[x,y]]
179: E.[sum_class[image[x,singleton[y]]],APY[x,y]]
180: E.[UNPR[x,x],singleton[x]]
181: subclass[x,UCLS]
182: E.[U[x,singleton[x]],SUC[x]]
183: MBR[omega,UCLS]
184: E.[domain_of[INV[x]],range_of[x]]
185: inductive[omega]
186: function[choice]
187: E.[x,x]

```

Negated conclusion:

```
188S>~subclass[INT[CXP[wC,xC],CXP[x,zC]],CXP[wC,zC]] ... ?
```

Phase 0 clauses used in proof:

```
196S>(188a*105b) ~MBR[not_subclass_element[INT[CXP[wC,xC],CXP[x,zC]],CXP[wC,zC]],CXP[wC,zC]] ... ?
197S>(188a*104b) MBR[not_subclass_element[INT[CXP[wC,xC],CXP[x,zC]],CXP[wC,zC]],INT[CXP[wC,xC],
CXP[x,zC]]] ... ?
```

Phases 1 and 2 clauses used in proof:

```
209S>(197a,133a) MBR[not_subclass_element[INT[CXP[wC,xC],CXP[x,zC]],CXP[wC,zC]],CXP[x,zC]] ... ?
210S>(209a,196a) [] ... ?
```

Inferences derived when variable is replaced by original constant

```
196 197 209
```

```

PHASE 0: 0 s PHASE 1: 0 s PHASE 2: -NA- Total Time: 0 s
NOD: 34 RES: 142 FAC: 0 T: 40 V: 32 L: 16
CTE: 219 CTH: 0 CTF: 0 CSZ: 16777216
UTE: 3323 UTH: 2022 UTF: 0 SBA: 23
BAS: 188 RED: 201 LEN: 2+2=4

```

Proof found to theorem by slave 6!

Axioms:

```
17 >-MBR[x,y] ~MBR[z,u] MBR[ORP[x,z],CXP[y,u]]
26 >-MBR[x,UCLS] MBR[x,complement[y]] MBR[x,y]
32 >-MBR[x,y] ~MBR[x,z] MBR[x,INT[y,z]]
38 >-E.[x,y] ~MBR[x,z] MBR[y,z]
43 >-subclass[x,y] ~MBR[z,x] MBR[z,y]
71 >-MBR[x,CXP[y,z]] E.[ORP[first[x],second[x]],x]
92 >-MBR[ORP[x,y],CXP[z,u]] MBR[x,z]
93 >-MBR[ORP[x,y],CXP[z,u]] MBR[y,u]
104S MBR[not_subclass_element[x,y],x] subclass[x,y]
105S>-MBR[not_subclass_element[x,y],y] subclass[x,y]
132 >-MBR[x,INT[y,z]] MBR[x,y]
141 >-MBR[x,complement[y]] ~MBR[x,y]
149 >-E.[x,y] E.[y,x]
181 >subclass[x,UCLS]
```

Negated conclusion:

```
188S>-subclass[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]
```

Phase 0 clauses used in proof:

```
190S>MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],INT[CXP[wC,xC],CXP[yC,zC]]]
191S>MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],CXP[yC,zC]]
197S>(191a*141b) ~MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],
complement[CXP[yC,zC]]]
198S>(191a*71a) E.[ORP[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],
second[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],not_subclass_element[INT[CXP[wC,xC],
CXP[yC,zC]],CXP[wC,zC]]]
200S>(190a*132a) MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],CXP[wC,xC]]
```

Phases 1 and 2 clauses used in proof:

```
209S>(200a,38b) ~E.[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],x] MBR[x,CXP[wC,xC]]
210S>(209b,92a) ~E.[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],ORP[x,y]] MBR[x,wC]
211S>(210a,149b) MBR[x,wC]
~E.[ORP[x,y],not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]]
212S>(211b,198a) MBR[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],wC]

213S>(200a,32b) ~MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],x]
MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],INT[x,CXP[wC,xC]]]
214S>(213b,43b) ~MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],x]
~subclass[INT[x,CXP[wC,xC]],y] MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],y]
215S>(214b,181a) ~MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],x]
MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],UCLS]
216S>(215a,200a) MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],UCLS]

217S>(198a,38a)
~MBR[ORP[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],second[not_subclass_element[
INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],x]
MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],x]
218S>(217b,105a) ~MBR[ORP[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],
second[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],CXP[wC,zC]]]
subclass[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]
219S>(218b,188a) ~MBR[ORP[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],
second[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],CXP[wC,zC]]]
220S>(219a,17c) ~MBR[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],wC]
~MBR[second[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],zC]
221S>(220a,212a) ~MBR[second[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],zC]

222S>(198a,149a) E.[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],
ORP[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],second[not_subclass_element[INT[
CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]]]]
223S>(222a,38a) ~MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],x]
MBR[ORP[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],second[not_subclass_element[
INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],x]
224S>(223a,216a)
MBR[ORP[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],second[not_subclass_element[
INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],UCLS]

225S>(198a,38a) ~MBR[ORP[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],
second[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],x]
MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],x]
226S>(225a,26b) MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],complement[x]]
~MBR[ORP[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],second[not_subclass_element[
INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],UCLS]
MBR[ORP[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],second[not_subclass_element[
INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]],x]
```

```
227S>(226b,224a) MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],complement[x]]
MBR[ORP[first[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],second[not_subclass_element
[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]]]],x]
228S>(227b,93a)
MBR[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],complement[CXP[x,y]]]
MBR[second[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],y]
229S>(228a,197a) MBR[second[not_subclass_element[INT[CXP[wC,xC],CXP[yC,zC]],CXP[wC,zC]],zC]
230S>(229a,221a) []
```

```
PHASE 0: 1 s PHASE 1: 3 s PHASE 2: 8 s Total Time: 12 s
NOD: 43210 RES: 336684 FAC: 6727 T: 40 V: 32 L: 16
CTE: 101628 CTH: 6173 CTF: 0 CSZ: 16777216
UTE: 401255 UTH: 683198 UTF: 0 SBA: 23
BAS: 188 RED: 201 LEN: 5+22=27
```

## SET237-6

Predicates: operation compatible equal homomorphism member function subclass single\_valued\_class inductive maps one\_to\_one  
Functions: yC xC xr universal\_class . subset\_relation omega domain\_relation singleton\_relation application\_function composition\_function choice null\_class identity\_relation element\_relation successor\_relation : successor compose\_class compose inverse singleton restrict image flip complement unordered\_pair intersection rotate range\_of symmetric\_difference not\_subclass\_element first second single\_valued1 single\_valued2 single\_valued3 power\_class regular sum\_class diagonalise domain range union cantor cross\_product domain\_of not\_homomorphism2 not\_homomorphism1 ordered\_pair apply

Renamed predicates: operation compatible E. HOM MBR function subclass single\_valued\_class inductive maps one\_to\_one  
Renamed functions: yC xC xr UCLS . SUBR omega DOMR singleton\_relation application\_function CMPF choice NULC identity\_relation element\_relation SUCC : SUC compose\_class COMP INV singleton restrict image flip complement UNPR INT rotate range\_of symmetric\_difference not\_subclass\_element first second single\_valued1 single\_valued2 single\_valued3 power\_class regular sum\_class diagonalise DOMN range U cantor CXP domain\_of not\_homomorphism2 not\_homomorphism1 ORP APY

C1: Spawned: 60 xvl 16.20.3 W0 188S ~subclass[restrict[x,xC,yC],CXP[UCLS,UCLS]] ... ?

Axioms:

```
1: ~operation[x] ~operation[y] ~compatible[z,x,y]
~E.[APY[y,ORP[APY[z,not_homomorphism1[z,x,y]],APY[z,not_homomorphism2[z,x,y]]],APY[z,APY[x,ORP[not_ho
morphism1[z,x,y],not_homomorphism2[z,x,y]]]]] HOM[z,x,y]
2: ~operation[x] ~operation[y] ~compatible[z,x,y]
MBR[ORP[not_homomorphism1[z,x,y],not_homomorphism2[z,x,y]],domain_of[x]] HOM[z,x,y]
3: ~function[x] ~E.[CXP[domain_of[domain_of[x]],domain_of[domain_of[x]]],domain_of[x]]
~subclass[range_of[x],domain_of[domain_of[x]]] operation[x]
4: ~function[x] ~E.[domain_of[domain_of[y]],domain_of[x]]
~subclass[range_of[x],domain_of[domain_of[z]]] compatible[x,y,z]
5: ~MBR[ORP[x,ORP[y,z]],CXP[UCLS,CXP[UCLS,UCLS]]] ~MBR[y,domain_of[x]]
MBR[ORP[x,ORP[y,APY[x,y]]],application_function]
6: ~MBR[ORP[ORP[x,y],z],u] ~MBR[ORP[ORP[z,x],y],CXP[CXP[UCLS,UCLS],UCLS]]
MBR[ORP[ORP[z,x],y],rotate[u]]
7: ~MBR[ORP[ORP[x,y],z],u] ~MBR[ORP[ORP[y,x],z],CXP[CXP[UCLS,UCLS],UCLS]]
MBR[ORP[ORP[y,x],z],flip[u]]
8: ~HOM[x,y,z] ~MBR[ORP[u,v],domain_of[y]] E.[APY[z,ORP[APY[x,u],APY[x,v]]],APY[x,APY[y,ORP[u,v]]]]
9: ~MBR[x,image[y,image[z,singleton[u]]] ~MBR[ORP[u,x],CXP[UCLS,UCLS]] MBR[ORP[u,x],COMP[y,z]]
10: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] ~E.[COMP[z,x],y] MBR[ORP[x,y],compose_class[z]]
11: ~E.[SUC[x],y] ~MBR[ORP[x,y],CXP[UCLS,UCLS]] MBR[ORP[x,y],SUCC]
12: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] ~MBR[x,y] MBR[ORP[x,y],element_relation]
13: ~subclass[x,CXP[UCLS,UCLS]] ~subclass[COMP[x,INV[x]],identity_relation] function[x]
14: ~MBR[x,UCLS] E.[restrict[y,singleton[x],UCLS],NULC] MBR[x,domain_of[y]]
15: ~E.[x,y] ~single_valued_class[x] single_valued_class[y]
16: ~function[x] ~MBR[y,UCLS] MBR[image[x,y],UCLS]
17: ~MBR[x,y] ~MBR[z,u] MBR[ORP[x,z],CXP[y,u]]
18: ~MBR[NULC,x] ~subclass[image[SUCR,x],x] inductive[x]
19: ~E.[x,y] ~HOM[x,z,u] HOM[y,z,u]
20: ~E.[x,y] ~HOM[z,x,u] HOM[z,y,u]
21: ~E.[x,y] ~HOM[z,u,x] HOM[z,u,y]
22: ~MBR[x,UCLS] E.[x,NULC] MBR[APY[choice,x],x]
23: ~E.[x,y] ~compatible[x,z,u] compatible[y,z,u]
24: ~E.[x,y] ~compatible[z,x,u] compatible[z,y,u]
25: ~E.[x,y] ~compatible[z,u,x] compatible[z,u,y]
26: ~MBR[x,UCLS] MBR[x,complement[y]] MBR[x,y]
27: ~MBR[x,UNPR[y,z]] E.[x,y] E.[x,z]
28: ~function[x] ~subclass[range_of[x],y] maps[x,domain_of[x],y]
29: ~E.[x,y] ~one_to_one[x] one_to_one[y]
30: ~E.[x,y] ~subclass[x,z] subclass[y,z]
31: ~E.[x,y] ~subclass[z,x] subclass[z,y]
32: ~MBR[x,y] ~MBR[x,z] MBR[x,INT[y,z]]
33: ~E.[x,y] ~inductive[x] inductive[y]
34: ~E.[x,y] ~operation[x] operation[y]
35: ~subclass[x,y] ~subclass[y,x] E.[x,y]
36: ~function[INV[x]] ~function[x] one_to_one[x]
37: ~E.[x,y] ~function[x] function[y]
38 >~E.[x,y] ~MBR[x,z] MBR[y,z]
39 >~E.[x,y] ~MBR[z,x] MBR[z,y]
40: ~E.[x,y] ~maps[x,z,u] maps[y,z,u]
41: ~E.[x,y] ~maps[z,x,u] maps[z,y,u]
42: ~E.[x,y] ~maps[z,u,x] maps[z,u,y]
43 >~subclass[x,y] ~MBR[z,x] MBR[z,y]
44: ~E.[x,y] ~E.[y,z] E.[x,z]
45: ~MBR[ORP[x,y],CXP[UCLS,UCLS]] MBR[ORP[x,ORP[y,COMP[x,y]]],CMPF]
46: ~MBR[ORP[x,ORP[y,z]],CMPF] E.[COMP[x,y],z]
47: ~MBR[ORP[x,ORP[y,z]],application_function] MBR[y,domain_of[x]]
48: ~operation[x] E.[CXP[domain_of[domain_of[x]],domain_of[domain_of[x]]],domain_of[x]]
49: ~MBR[x,UCLS] MBR[ORP[x,domain_of[x]],DOMR]
50: ~E.[x,y] E.[symmetric_difference[x,z],symmetric_difference[y,z]]
```

```

51: ~E.[x,y] E.[symmetric_difference[z,x],symmetric_difference[z,y]]
52: ~E.[x,y] E.[not_subclass_element[x,z],not_subclass_element[y,z]]
53: ~E.[x,y] E.[not_subclass_element[z,x],not_subclass_element[z,y]]
54: ~MBR[ORP[x,ORP[y,z]],application_function] E.[APY[x,y],z]
55: ~E.[x,y] E.[not_homomorphism1[x,z,u],not_homomorphism1[y,z,u]]
56: ~E.[x,y] E.[not_homomorphism1[z,x,u],not_homomorphism1[z,y,u]]
57: ~E.[x,y] E.[not_homomorphism1[z,u,x],not_homomorphism1[z,u,y]]
58: ~E.[x,y] E.[not_homomorphism2[x,z,u],not_homomorphism2[y,z,u]]
59: ~E.[x,y] E.[not_homomorphism2[z,x,u],not_homomorphism2[z,y,u]]
60: ~E.[x,y] E.[not_homomorphism2[z,u,x],not_homomorphism2[z,u,y]]
61: ~MBR[ORP[ORP[x,y],z],rotate[u]] MBR[ORP[ORP[y,z],x],u]
62: ~MBR[ORP[ORP[x,y],z],flip[u]] MBR[ORP[ORP[y,x],z],u]
63: ~MBR[ORP[x,y],DOMR] E.[domain_of[x],y]
64: ~single_valued_class[x] subclass[COMP[x,INV[x]],identity_relation]
65: ~subclass[COMP[x,INV[x]],identity_relation] single_valued_class[x]
66: ~MBR[ORP[x,y],COMP[z,u]] MBR[y,image[z,image[u, singleton[x]]]]
67: ~E.[x,y] E.[UNPR[x,z],UNPR[y,z]]
68: ~E.[x,y] E.[UNPR[z,x],UNPR[z,y]]
69: ~E.[restrict[x, singleton[y],UCLS],NULC] ~MBR[y,domain_of[x]]
70: ~MBR[ORP[x,y],compose_class[z]] E.[COMP[z,x],y]
71: ~MBR[x,CXP[y,z]] E.[ORP[first[x],second[x]],x]
72: ~MBR[ORP[x,y],SUCR] E.[SUC[x],y]
73: ~E.[x,y] E.[INT[x,z],INT[y,z]]
74: ~E.[x,y] E.[INT[z,x],INT[z,y]]
75: ~E.[x,y] E.[ORP[x,z],ORP[y,z]]
76: ~E.[x,y] E.[ORP[z,x],ORP[z,y]]
77: ~E.[x,y] E.[single_valued1[x],single_valued1[y]]
78: ~E.[x,y] E.[single_valued2[x],single_valued2[y]]
79: ~E.[x,y] E.[single_valued3[x],single_valued3[y]]
80: ~function[x] subclass[x,CXP[UCLS,UCLS]]
81: ~compatible[x,y,z] subclass[range_of[x],domain_of[domain_of[z]]]
82: ~E.[x,y] E.[CXP[x,z],CXP[y,z]]
83: ~E.[x,y] E.[CXP[z,x],CXP[z,y]]
84: ~compatible[x,y,z] E.[domain_of[domain_of[y]],domain_of[x]]
85: ~E.[x,y] E.[restrict[x,z,u],restrict[y,z,u]]
86: ~E.[x,y] E.[restrict[z,x,u],restrict[z,y,u]]
87: ~E.[x,y] E.[restrict[z,u,x],restrict[z,u,y]]
88: ~MBR[x,UCLS] MBR[power_class[x],UCLS]
89: ~E.[x,y] E.[compose_class[x],compose_class[y]]
90: E.[x,NULC] E.[INT[x,regular[x]],NULC]
91: ~E.[x,y] E.[power_class[x],power_class[y]]
92: ~MBR[ORP[x,y],CXP[z,u]] MBR[x,z]
93: ~MBR[ORP[x,y],CXP[z,u]] MBR[y,u]
94: ~MBR[x,UCLS] MBR[sum_class[x],UCLS]
95: ~function[x] subclass[COMP[x,INV[x]],identity_relation]
96: ~E.[x,y] E.[diagonalise[x],diagonalise[y]]
97: ~E.[x,y] E.[DOMN[x,z,u],DOMN[y,z,u]]
98: ~E.[x,y] E.[DOMN[z,x,u],DOMN[z,y,u]]
99: ~E.[x,y] E.[DOMN[z,u,x],DOMN[z,u,y]]
100: ~MBR[ORP[x,y],element_relation] MBR[x,y]
101: ~MBR[x,UCLS] MBR[x,UNPR[x,y]]
102: ~MBR[x,UCLS] MBR[x,UNPR[y,x]]
103: ~operation[x] subclass[range_of[x],domain_of[domain_of[x]]]
104: >MBR[not_subclass_element[x,y],x] subclass[x,y]
105: >>MBR[not_subclass_element[x,y],y] subclass[x,y]
106: ~E.[x,y] E.[domain_of[x],domain_of[y]]
107: ~E.[x,y] E.[range[x,z,u],range[y,z,u]]
108: ~E.[x,y] E.[range[z,x,u],range[z,y,u]]
109: ~E.[x,y] E.[range[z,u,x],range[z,u,y]]
110: ~E.[x,y] E.[singleton[x],singleton[y]]
111: ~E.[x,y] E.[SUC[x],SUC[y]]
112: ~E.[x,y] E.[sum_class[x],sum_class[y]]
113: ~E.[x,y] E.[complement[x],complement[y]]
114: ~E.[x,y] E.[range_of[x],range_of[y]]
115: ~E.[x,y] E.[COMP[x,z],COMP[y,z]]
116: ~E.[x,y] E.[COMP[z,x],COMP[z,y]]
117: ~E.[x,y] E.[image[x,z],image[y,z]]
118: ~E.[x,y] E.[image[z,x],image[z,y]]
119: ~E.[x,y] E.[INV[x],INV[y]]
120: ~E.[x,y] E.[regular[x],regular[y]]
121: ~E.[x,y] E.[U[x,z],U[y,z]]
122: ~E.[x,y] E.[U[z,x],U[z,y]]
123: ~inductive[x] subclass[image[SUCR,x],x]
124: ~E.[x,y] E.[rotate[x],rotate[y]]
125: ~E.[x,y] E.[second[x],second[y]]
126: ~HOM[x,y,z] compatible[x,y,z]
127: ~E.[x,y] E.[APY[x,z],APY[y,z]]
128: ~E.[x,y] E.[APY[z,x],APY[z,y]]
129: ~E.[x,y] E.[first[x],first[y]]
130: ~E.[x,y] E.[cantor[x],cantor[y]]
131: ~E.[x,y] E.[flip[x],flip[y]]

```

```

132 >~MBR[x,INT[y,z]] MBR[x,y]
133: ~MBR[x,INT[y,z]] MBR[x,z]
134: E.[x,NULC] MBR[regular[x],x]
135: ~HOM[x,y,z] operation[y]
136: ~HOM[x,y,z] operation[z]
137: ~inductive[x] subclass[omega,x]
138: ~E.[x,y] subclass[x,y]
139: ~E.[x,y] subclass[y,x]
140: ~one_to_one[x] function[INV[x]]
141: ~MBR[x,complement[y]] ~MBR[x,y]
142: ~compatible[x,y,z] function[x]
143: ~inductive[x] MBR[NULC,x]
144: ~maps[x,y,z] subclass[range_of[x],z]
145: ~maps[x,y,z] E.[domain_of[x],y]
146: ~one_to_one[x] function[x]
147: ~operation[x] function[x]
148: ~maps[x,y,z] function[x]
149 >~E.[x,y] E.[y,x]
150: E.[INT[CXP[UCLS,UCLS],INT[CXP[UCLS,UCLS],complement[COMP[complement[element_relation],
INV[element_relation]]]]],SUBR]
151: E.[INT[complement[COMP[element_relation,complement[identity_relation]]],element_relation],
singleton_relation]
152: E.[INT[complement[INT[x,y]],complement[INT[complement[x],complement[y]]]],
symmetric_difference[x,y]]
153: subclass[CMPP,CXP[UCLS,CXP[UCLS,UCLS]]]
154: subclass[application_function,CXP[UCLS,CXP[UCLS,UCLS]]]
155: E.[DOMN[x,image[INV[x],singleton[single_valued1[x]]],single_valued2[x],single_valued3[x]]]
156: E.[second[not_subclass_element[COMP[x,INV[x]],identity_relation]],single_valued2[x]]
157: E.[first[not_subclass_element[COMP[x,INV[x]],identity_relation]],single_valued1[x]]
158: subclass[rotate[x],CXP[CXP[UCLS,UCLS],UCLS]]
159: E.[INT[domain_of[x],diagonalise[COMP[INV[element_relation],x]]],cantor[x]]
160 >subclass[DOMR,CXP[UCLS,UCLS]]
161: subclass[flip[x],CXP[CXP[UCLS,UCLS],UCLS]]
162: subclass[compose_class[x],CXP[UCLS,UCLS]]
163: E.[INT[INV[SUBR],SUBR],identity_relation]
164: E.[domain_of[restrict[element_relation,UCLS,x]],sum_class[x]]
165: E.[UNPR[singleton[x],UNPR[x,singleton[y]]],ORP[x,y]]
166: subclass[SUCR,CXP[UCLS,UCLS]]
167: E.[complement[domain_of[INT[x,identity_relation]]],diagonalise[x]]
168: E.[first[not_subclass_element[restrict[x,y,singleton[z]],NULC]],DOMN[x,y,z]]
169: E.[complement[image[element_relation,complement[x]]],power_class[x]]
170: subclass[element_relation,CXP[UCLS,UCLS]]
171: E.[second[not_subclass_element[restrict[x,singleton[y],z],NULC]],range[x,y,z]]
172: subclass[COMP[x,y],CXP[UCLS,UCLS]]
173: E.[complement[INT[complement[x],complement[y]]],U[x,y]]
174: E.[domain_of[flip[x,UCLS]],INV[x]]
175: MBR[UNPR[x,y],UCLS]
176 >E.[INT[x,CXP[y,z]],restrict[x,y,z]]
177: E.[INT[CXP[x,y],z],restrict[z,x,y]]
178: E.[range_of[restrict[x,y,UCLS]],image[x,y]]
179: E.[sum_class[image[x,singleton[y]]],APY[x,y]]
180: E.[UNPR[x,x],singleton[x]]
181: subclass[x,UCLS]
182: E.[U[x,singleton[x]],SUC[x]]
183: MBR[omega,UCLS]
184: E.[domain_of[INV[x]],range_of[x]]
185: inductive[omega]
186: function[choice]
187 >E.[x,x]

```

Negated conclusion:

```
188S>~subclass[restrict[x,xC,yC],CXP[UCLS,UCLS]] ... ?
```

-----

Phase 0 clauses used in proof:

```
196S>(188a*105b) ~MBR[not_subclass_element[restrict[x,xC,yC],CXP[UCLS,UCLS]],CXP[UCLS,UCLS]] ... ?
197S>(188a*104b) MBR[not_subclass_element[restrict[x,xC,yC],CXP[UCLS,UCLS]],restrict[x,xC,yC]] ... ?
```

Phases 1 and 2 clauses used in proof:

```
209S>(196a,38c) ~E.[x,not_subclass_element[restrict[y,xC,yC],CXP[UCLS,UCLS]]]
~MBR[x,CXP[UCLS,UCLS]] ... ?
210S>(209b,43c) ~E.[x,not_subclass_element[restrict[y,xC,yC],CXP[UCLS,UCLS]]]
~subclass[z,CXP[UCLS,UCLS]] ~MBR[x,z] ... ?
211S>(210b,160a) ~E.[x,not_subclass_element[restrict[y,xC,yC],CXP[UCLS,UCLS]]] ~MBR[x,DOMR] ... ?
212S>(211a,187a) ~MBR[not_subclass_element[restrict[x,xC,yC],CXP[UCLS,UCLS]],DOMR] ... ?

213S>(197a,39b) ~E.[restrict[x,xC,yC],y]
MBR[not_subclass_element[restrict[x,xC,yC],CXP[UCLS,UCLS]],y] ... ?
214S>(213b,132a) ~E.[restrict[x,xC,yC],INT[y,z]]
MBR[not_subclass_element[restrict[x,xC,yC],CXP[UCLS,UCLS]],y] ... ?

```

```

215S>(214a,149b) MBR[not_subclass_element[restrict[x,xC,yC],CXP[UCLS,UCLS]],y]
~E.[INT[y,z],restrict[x,xC,yC]] ... ?
216S>(215b,176a) MBR[not_subclass_element[restrict[x,xC,yC],CXP[UCLS,UCLS]],x] ... ?
217S>(216a,212a) [] ... ?

```

Inferences derived when variable is replaced by original constant  
209 210 211 212 213 214 215 216

```

PHASE 0: 0 s PHASE 1: 2 s PHASE 2: 0 s Total Time: 2 s
NOD: 3868 RES: 27767 FAC: 316 T: 16 V: 32 L: 16
CTE: 16859 CTH: 1117 CTF: 0 CSZ: 16777216
UTE: 360213 UTH: 403464 UTF: 0 SBA: 23
BAS: 188 RED: 201 LEN: 2+9=11

```

Proof found to theorem by slave 1!

Axioms:

```

17 >~MBR[x,y] ~MBR[z,u] MBR[ORP[x,z],CXP[y,u]]
38S>~E.[x,y] ~MBR[x,z] MBR[y,z]
39S ~E.[x,y] ~MBR[z,x] MBR[z,y]
43S>~subclass[x,y] ~MBR[z,x] MBR[z,y]
71 >~MBR[x,CXP[y,z]] E.[ORP[first[x],second[x]],x]
92 >~MBR[ORP[x,y],CXP[z,u]] MBR[x,z]
93 >~MBR[ORP[x,y],CXP[z,u]] MBR[y,u]
104S MBR[not_subclass_element[x,y],x] subclass[x,y]
105S ~MBR[not_subclass_element[x,y],y] subclass[x,y]
132S ~MBR[x,INT[y,z]] MBR[x,y]
149S>~E.[x,y] E.[y,x]
160S subclass[DOMR,CXP[UCLS,UCLS]]
176S E.[INT[x,CXP[y,z]],restrict[x,y,z]]
177 >E.[INT[CXP[x,y],z],restrict[z,x,y]]
181 >subclass[x,UCLS]
187S E.[x,x]

```

Negated conclusion:

```

188S ~subclass[restrict[xr,xC,yC],CXP[UCLS,UCLS]]

```

-----

Phase 0 clauses used in proof:

```

189S>~E.[x,not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]] ~MBR[x,CXP[UCLS,UCLS]]
195S>MBR[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]],x] ~E.[INT[x,y],restrict[xr,xC,yC]]
204S>(195b*177a) MBR[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]],CXP[xC,yC]]

```

Phases 1 and 2 clauses used in proof:

```

209S>(204a,38b) ~E.[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]],x] MBR[x,CXP[xC,yC]]
210S>(209b,93a) ~E.[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]],ORP[x,y]] MBR[y,yC]
211S>(210a,149b) MBR[x,yC] ~E.[ORP[y,x],not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]]
212S>(211b,71b) MBR[second[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],yC]
~MBR[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]],CXP[x,y]]
213S>(212b,204a) MBR[second[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],yC]

214S>(204a,38b) ~E.[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]],x] MBR[x,CXP[xC,yC]]
215S>(214b,92a) ~E.[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]],ORP[x,y]] MBR[x,xC]
216S>(215a,149b) MBR[x,xC] ~E.[ORP[x,y],not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]]
217S>(216b,71b) MBR[first[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],xC]
~MBR[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]],CXP[x,y]]
218S>(217b,204a) MBR[first[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],xC]

219S>(189b,17c) ~E.[ORP[x,y],not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]] ~MBR[x,UCLS]
~MBR[y,UCLS]
220S>(219a,71b) ~MBR[first[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],UCLS]
~MBR[second[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],UCLS]
~MBR[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]],CXP[x,y]]
221S>(220c,204a) ~MBR[first[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],UCLS]
~MBR[second[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],UCLS]
222S>(221b,43c) ~MBR[first[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],UCLS]
~subclass[x,UCLS] ~MBR[second[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],x]
223S>(222b,181a) ~MBR[first[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],UCLS]
~MBR[second[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],x]
224S>(223b,213a) ~MBR[first[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],UCLS]
225S>(224a,43c) ~subclass[x,UCLS]
~MBR[first[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],x]
226S>(225a,181a) ~MBR[first[not_subclass_element[restrict[xr,xC,yC],CXP[UCLS,UCLS]]],x]
227S>(226a,218a) []

```

```

PHASE 0: 0 s PHASE 1: 216 s PHASE 2: 122 s Total Time: 340 s
NOD: 979136 RES: 10094731 FAC: 69551 T: 16 V: 32 L: 16
CTE: 5243871 CTH: 379091 CTF: 438858 CSZ: 16777216
UTE: 10855834 UTH: 18268632 UTF: 512253 SBA: 1011
BAS: 188 RED: 201 LEN: 3+19=22

```