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The author gratefully acknowledges support from the taxpayers of Québec and of Canada who have paid his salary and research grants while this work was developed at McGill University, and from his students (who built the implementations and investigated the data structures and algorithms) and their funding agencies.

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The Pointerless Representation of Tries

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I Tries

- Compression down to $2 / \lg n$ on n data items
e.g., 90% (1 Mbyte) 93% (1 Gbyte) 95% (1 Tbyte)
 - Good for suffix trees
better than suffix arrays [FODO'93]
 - Support regex and approximate matching
- Variable resolution
- Multidimensional tries and Z-order
- Dynamic

The Pointerless Representation of Tries

II Pointerless representation

1. RAM: main memory
2. SS: secondary storage

Orenstein 1983

www.cs.mcgill.ca/~cs420/logarithmicTxt.ps

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I Tries

Compression

Raw data: 2^h items of h bits each.

Trie: $2^h - 1$ nodes of 2 bits each
(pointerless representation).

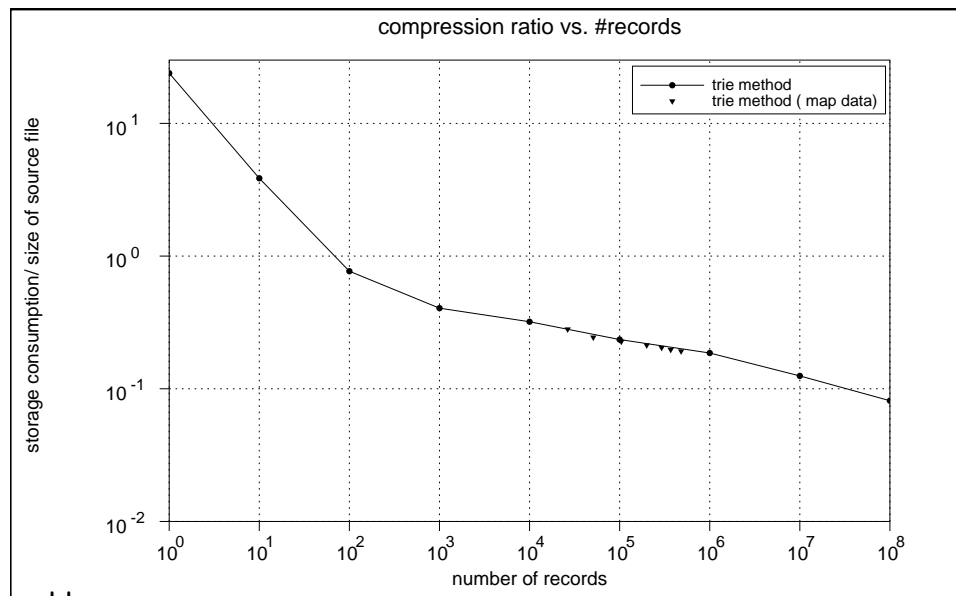
Compression: $h \rightarrow 2$

For n items, $h = \lg n$.

Theoretical best:

90% (1 Mbyte) 93% (1 Gbyte) 95% (1 Tbyte)

Experiment (log-log scale; 90% at 10^7 records):



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I Tries

Suffix tries vs. suffix arrays

Simplistic suffix array size

$n \lg N / 8$ for n suffixes, N bytes

E.g., $3.4n$ for 100 Mbytes

[FODO '93]:

“For an index of 100 million entries, our experiments show size factors of less than 3, as compared with 3.4 for the best previous method.

Our measurements show expected access costs of 0.1 sec., and construction times of 18 to 55 hours, depending on the text characteristics.”

www.cs.mcgill.ca/~tim/cv/theses/shang.ps.gz

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I Tries

Regex and approximate matching

[FODO '93]:

“Our organization .. supports searches for general patterns, as well as a variety of special searches, such as proximity, range, longest repetitions and most frequent occurrences.”

[IEEE TKDE 8 '96]:

“We discuss a variety of applications and extensions, including best match (for spelling checkers), case insensitivity, and limited approximate regular expression matching.”

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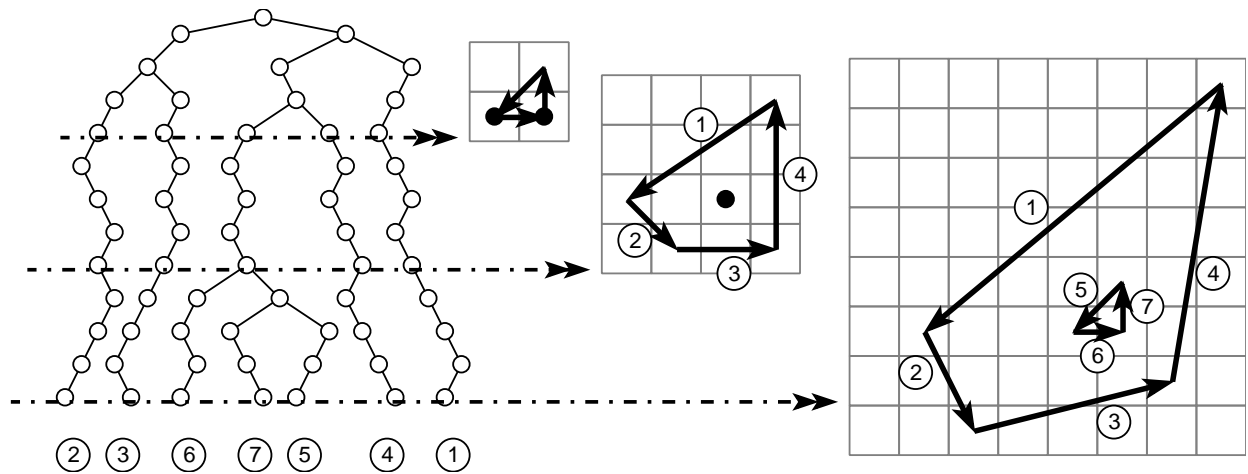
I Tries

Variable resolution

For low resolution, access only top of trie.

For higher resolution, go deeper.

E.g., a simple map:



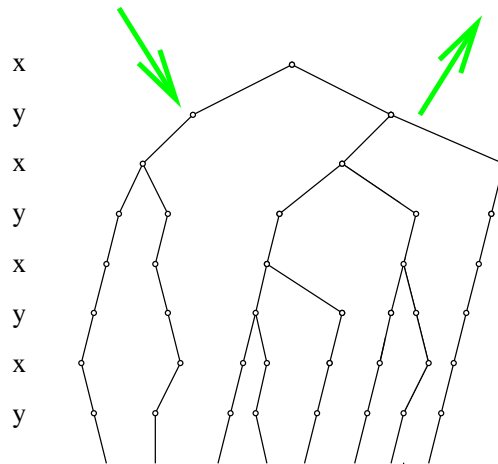
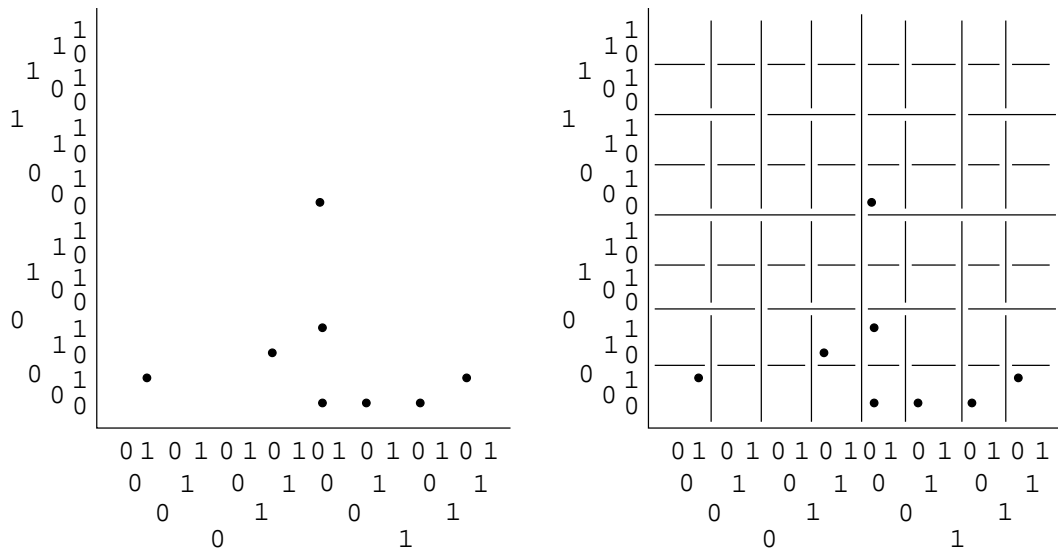
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I Tries

Multidimensional tries and Z-order

Trie interpreted in 2D:
Bentley's "discriminator"

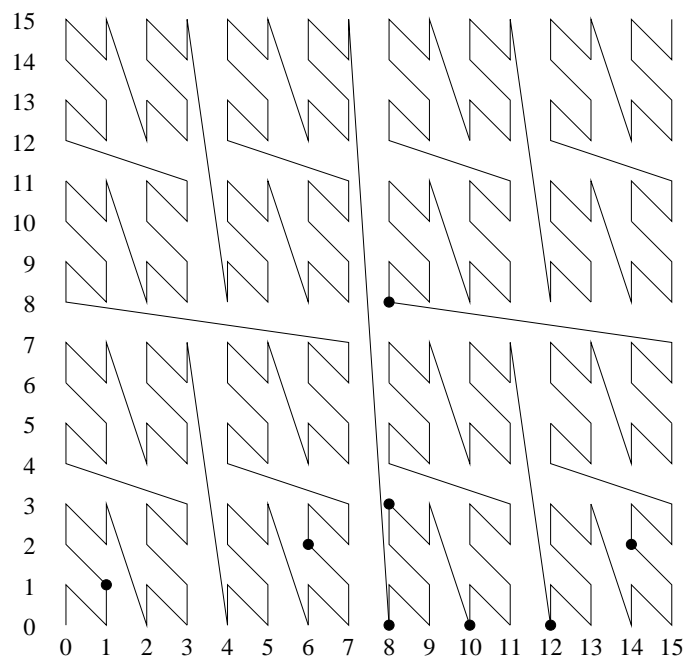


Bit interleaving, e.g.,: $(6,2) = (0110,0010) \Rightarrow 00101100$

I Tries

Multidimensional tries and Z-order, cont.

A 1D ordering of the same 2D data



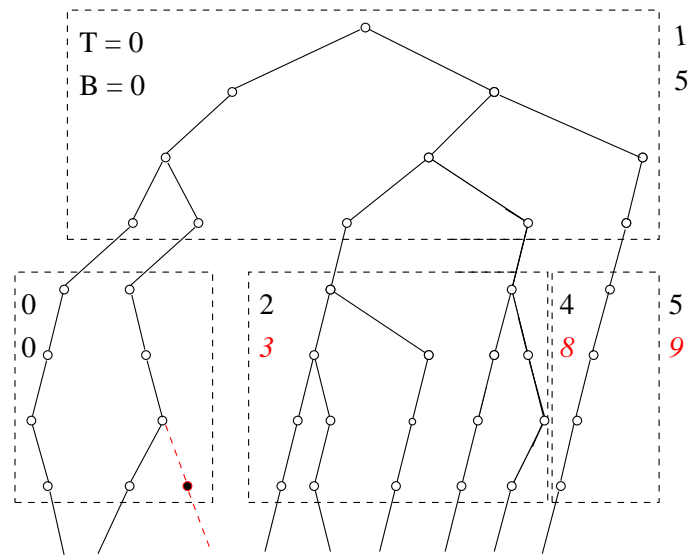
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I Tries

Dynamic

E.g., adding 00101111



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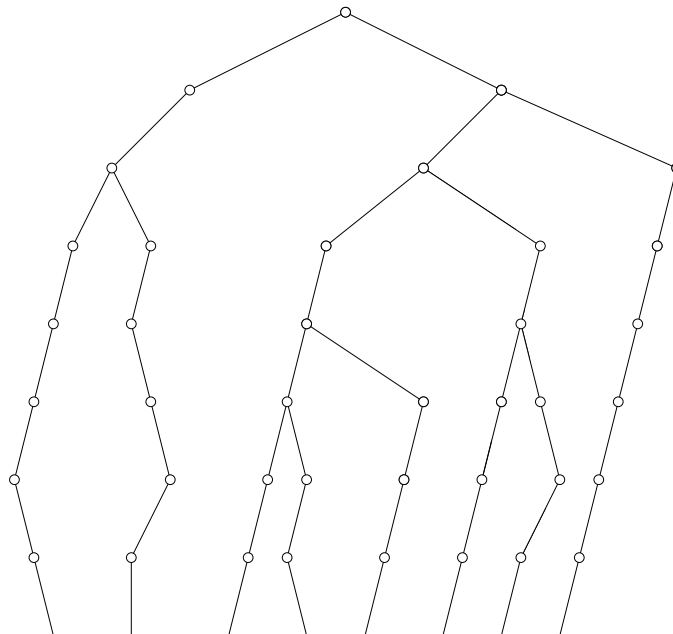
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II Pointerless representation

Trie in RAM

E.g., eight data values

00000011	00101100	10000000	10000101
10001000	10100000	10101100	11000000

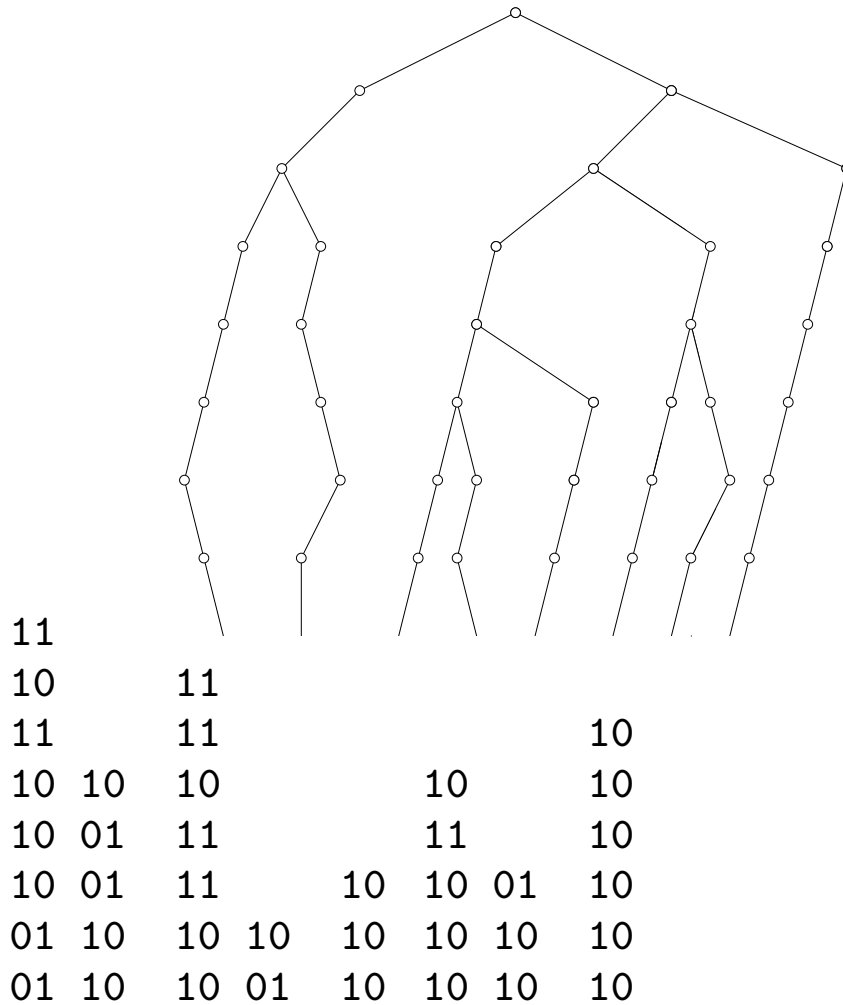


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II Pointerless representation: RAM

Two bits per node



II Pointerless representation: RAM

Two bits per node, cont.

```
11
10      11
11      11      10
10 10   10      10      10
10 01   11      11      10
10 01   11      10 10 01  10
01 10   10 10   10 10 10  10
01 10   10 01   10 10 10  10
```

```
11                2 on-bits mean 2 nodes next level
10 11            3 on-bits mean 3 nodes next level
11 11 10        5 on-bits mean 5 nodes next level
10 10 10 10 10 ..
10 01 11 11 10
10 01 11 10 10 01 10
01 10 10 10 10 10 10 10
01 10 10 01 10 10 10 10
```

```
11 10 11 11 11 10 10 10 10 10 10 01 11 11 10 10 01 11 10
10 01 10 01 10 10 10 10 10 10 01 10 10 01 10 10 10 10
```

II Pointerless representation: RAM

Searching

Search for 10001000

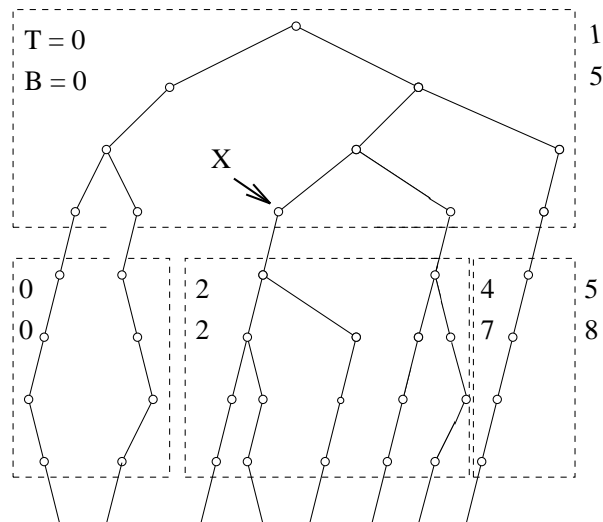
```
11
10 11
11 11 10
10 10 10 10 10
10 01 11 11 10
10 01 11 10 10 01 10
01 10 10 10 10 10 10 10
01 10 10 01 10 10 10 10
```

```
1x
10 x1
11 x1 10
10 10 x0 10 10
10 01 1x 11 10
10 01 11 x0 10 01 10
01 10 10 10 x0 10 10 10
01 10 10 01 x0 10 10 10
```

look in 2nd node, next level
look in 2nd node, next level
look in 3rd node, next level
..

II Pointerless representation

Trie on SS



T 0 11
B 0 10 11
 11 11 10
 10 10 10 10 10

T 1
B 5

T 0 10 01
B 0 10 01
 01 10
 01 10

T 2 11 11
B 2 11 10 10 01
 10 10 10 10 10
 10 01 10 10 10

T 4 10 *T* 5
B 7 10 *B* 8
 10
 10

II Pointerless representation: SS

Search [Orenstein, '83]

```
1x
10 x1
11 x1 10
10 10 x0 10 10
10 01 1x 11 10
10 01 11 x0 10 01 10
01 10 10 10 x0 10 10 10
01 10 10 01 x0 10 10 10
```

```
T 0  1x           T 1
B 0  10 x1       B 5
      11 x1 10
      10 10 x0 10 10
```

.. look in 3rd node, next level ..

3rd node must be in page headed $T=2$:

```
T 0  10 01       T 2  1x 11           T 4  10       T 5
B 0  10 01       B 2  11 x0 10 01       B 7  10       B 8
      01 10           10 10 x0 10 10           10
      01 10           10 01 x0 10 10           10
```


The Pointerless Representation of Tries

Conclusion

Two bits per node, shared storage of prefixes, hence compression.

Multidimensional tries and variable resolution both follow.

Paged representation (Orenstein) adds only 2 integers per page.

Dynamic tries follow.

- Orenstein, *Algorithms for Implementing Relational Databases*, 1983, Ph.D. Thesis, McGill University, School of Computer Science
- Merrett & Shang, *Trie Methods for Representing Text*, FODO'93 LNCS 730, 1993, 130–45.
- Shang, *Trie Methods for Text and Spatial Data on Secondary Storage*, 1995, www.cs.mcgill.ca/~tim/cv/students.html
- Shang & Merrett, *Tries for Approximate String Matching*, IEEE TKDE 8 (4), 1996, 540–7.
- www.cs.mcgill.cs/~cs420/logarithmicTxt.ps