# Excursions in Computing Science:
# Book 8d. Rocket Science.
# Part IV Spaceship Earth.

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39. Speeds. What can provide all the protection and sustenance we’ve been discussing in the previous Part?

Our planet has a surface gravity of 1 gee. It has a magnetic field and an atmosphere which protect us from solar and cosmic radiation. We can breathe the atmosphere. Seventy percent of the surface is water and although very little of that is fresh water the weather systems provide enough. The ecosystems provide food and shelter. The mineral resources provide shelter and energy.

The Earth is also speeding through space. It orbits the Sun, 1 AU (astronomical unit) out, once a year. That is $2\pi$AU per year.

We are 8 light minutes from the Sun, so 1 AU $= 8 \times 60$s $\times$ 300K Km/s and 1 AU per year is this divided by $3600 \times 24 \times 365$ s, giving 4.75 Km/s. (Think 5 Km/s for short. I’ve actually used 1 AU = 0.15 Tm because the 8 light minutes is also approximate.)

The circumference of $2\pi$ AU we travel in a year makes our speed 30 Km/s around the Sun. That’s bigger than the rocket delta-Vs we discussed in Parts I and II.

And that’s not all. The Solar System in turn orbits the centre of the Milky Way galaxy. We are about 26,000 light years from the galactic centre and orbit it about every 225 million years. That’s a speed of $2\pi \times 26/22.5 = 7.3$ light years per 10,000 years.

I picked that time unit because 1 light year per 10,000 years is 1/100 of 1 percent of lightspeed: 300,000 Km/s divided by 10,000 is 30 Km/s. It is also the speed of Earth in our orbit around the Sun. Since a *myriad* was an ancient military formation of 10,000 soldiers, we can call 10,000 years a *myriennium* (abbreviated *myr*—not to be confused with a million years, sometimes Myr but conventionally Ma).

So in addition to our 30 Km/s around the Sun, we are also moving at 220 Km/s around the centre of the galaxy.

| unit | approx Km/s | our speed | in Km/s |
|---|---:|---|---:|
| AU/year | 5 | 1 AU/year | 30 |
| ly/myr | 30 | 7 ly/myr | 220 |

These two velocities are more or less perpendicular to each other in direction (*velocities* have *direction*; *speeds* have only *magnitudes*) so they don't simply add or subtract but combine in a more complicated way.

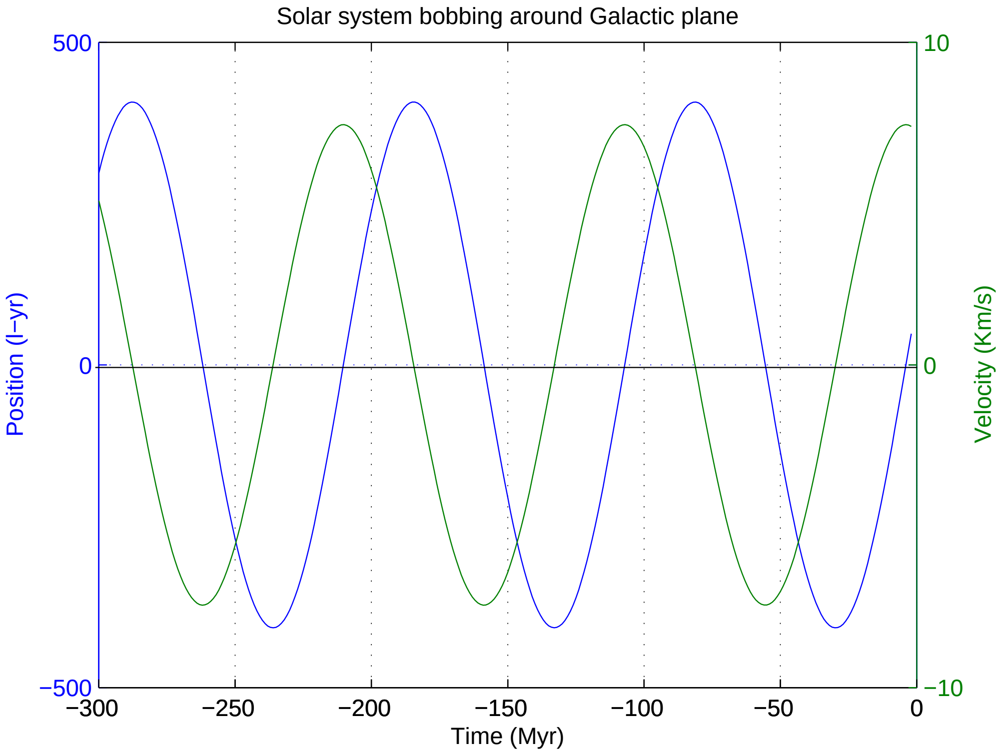
As it orbits the galaxy, our Solar System does not keep following a circle. The galaxy revolves in a more complicated way than planets orbit a star. So there are other components of motion.

The most significant one is a bobbing motion perpendicular to the "plane of the galaxy". The galaxy is a disc 100,000 light years in diameter, but it is not 2-dimensional, nor is it uniform. It has a bar with two spiral arms of stars. But a flat disc bisecting this third dimension can be thought of as a plane which gravitationally attracts stars lying off the plane. These stars and their planetary systems will then "bob" back and forth through the plane of the galaxy, the way a cork might bob in a sink or a buoy on the ocean.

As far as I know, such bobbing has not been measured. But it can be calculated given a certain degree of understanding of the galaxy and given certain assumptions. The most recent study I've found argues for a period of 4 × 25.79 million years for the Solar System, which amounts to just over two complete cycles in a "galactic year" of 225 million years.

That study also says that we are now 48.9 light years "above" the Galactic plane and moving away from it at 7.4 Km/s with the furthest distance to be 407.5 light years away before we turn around and start back.

If we suppose the solar system is really bobbing like a cork, we can picture its motion over the past 300 megayears.



Here the blue curve gives our position and the green curve our speed.

With a little trigonometry and a little calculus of a new type from what we've mentioned before, we don't need to be told about the speed (although we do need to be told that we are now moving away from rather than towards the Galactic plane).

The blue curve has the form

$$407.5\sin(\omega t) = 407.5\sin\left(\frac{\pi t}{2\times 25.79}\right)$$

—it's a "sine curve", the mathematical name for a wave. The sine function, sin(), is a trigonometric function.

The green curve can be found from this using calculus.

$$407.5\omega\cos(\omega t) = \frac{407.5\pi}{2\times 25.79}\cos\left(\frac{\pi t}{2\times 25.79}\right)$$

The fraction in front of cos() gives the maximum velocity in light years per megayear. To convert that to kilometers per second we need to reduce megayears by 100 to myriennia and then multiply light years per myriennium by 30 to get kilometers per second:

$$\frac{407.5\pi}{2\times 25.79}\frac{30}{100} = 7.44\ \text{Km/s}$$

This is the maximum speed, reached just as we cross the Galactic plane, and pretty close to our current speed of 7.4 Km/s—because we're pretty close to that midpoint. (And the only reason I kept the third significant figure, that last 4, which is not actually justified by the precision of the other terms I've used.)

This is only an exercise based on one claim a quarter century ago. The calculations are very tricky, as are the observations, and although what we are saying is probably qualitatively true the numbers are almost certainly not correct.

We are interested in the speed. The maximum speed we got of 7 Km/s does not contribute a whole lot to the 30 and 220 Km/s we found earlier. As for direction, it would probably add to and subtract from the 30 Km/s orbital speed of the Earth.

40. Extinctions. Species on Earth do not last very long. They are going extinct all the time. But there have been five notable periods in which not only very large numbers of species but whole *families* (the next level up grouping of species) have disappeared.

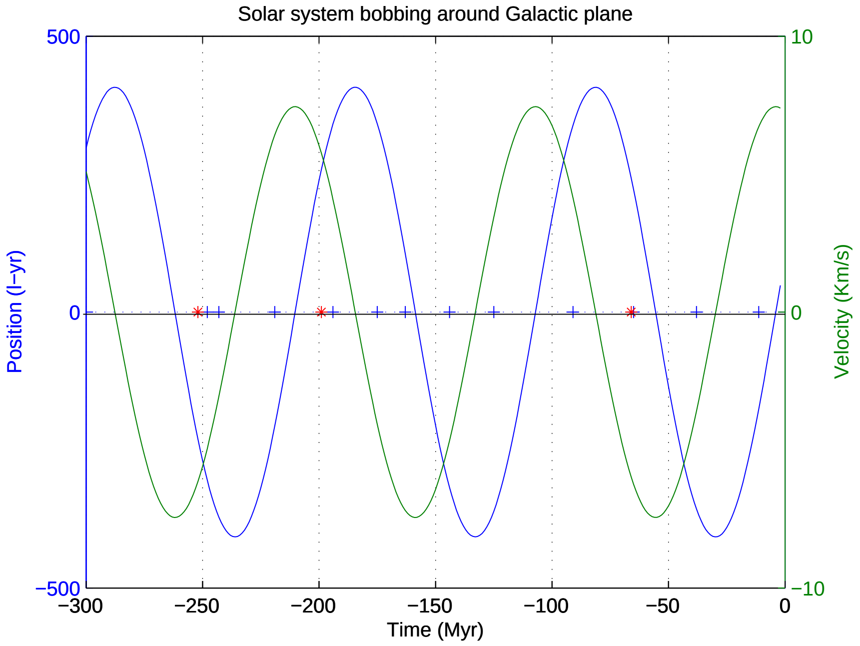
| Myr | period | possible cause |
|---|---|---|
| −66 | end Cretaceous | Mexico asteroid |
| −199 | late Triassic | Atlantic volcanoes |
| −252 | end Permian | Siberia volcanoes |
| −378 | late Devonian | not known |
| −447 | late Ordovician | global cooling |

The most recent of these was the famous extinction of the dinosaurs. The probable cause was an extraterrestrial body—asteroid or comet—which hit the Yucatan peninsula and deposited a world-wide layer of iridium enabling it to be dated. There is still disagreement about this cause, and arguments that the dinosaurs hung in for hundreds of thousands of years after that date. Geology and paleontology are intricate sciences.

But let's go with the collision hypothesis. We already discussed (in Note 25 of Part II) what might cause an asteroid to leave the belt between Mars and Jupiter and come zinging towards Earth. What would send a comet our way?

The home of comets is the Oort cloud, which forms a sphere around the Sun at 2,000 to 100,000 AU. That goes as far as 1.5 light years.

One event which could disturb the Oort cloud would be the Solar System passing through the Galactic midplane, which is more densely populated than the outskirts, with both other star systems and dust clouds. Indeed a dozen extinctions over the past 250 Myr, including three of the above big five, have been observed to have a periodicity of 26 Myr, which is pretty close to the 25.79 Myr quarter-period of "bobbing" discussed in the previous Note.



The plot shows the dozen extinctions (black ticks) and the three big ones (red asterisks) superimposed on the periodic sine waves of the previous Note. (My data is from two different sources. The red asterisks should coincide with the ticks nearest them, but do so in only one of three cases. The chronology of the fossil record is very difficult to pin down.)

We note that there is a sort of correlation. The first two red asterisks indeed occur a few million years after a mid-plane crossing, and the third a similar lag after the peak position has been reached and the Sun starts to turn back. Maybe the disturbance takes this long to generate an Earth-crossing comet.

This is all pretty much speculation. There are other reasons for extinctions, some of which are listed in the table above for the big five. "Nonlinearities" in the dynamics of populations—akin to the nonlinearities that produce chaos—can spontaneously lead to population collapse. Indeed, we are apparently at the start of a "sixth extinction" which we are causing ourselves and has nothing to do with close comets (not even Shoemaker-Levy which hit Jupiter in 1994).

The point is that Spaceship Earth, despite its size and advantages, is also vulnerable and we must look after it.

41. Herd science. What must we learn to be successful stewards of Spaceship Earth? Our evolution as social animals has given us benefits we could not have enjoyed as solitary individuals. We have gained remarkable achievements through cooperation. But there is more. Beyond herdthink there is education.

Our brief discussion of swarms in Note 31 of Part III made two major assumptions. First, communications are local (although my own swarm there was so small that I simply included it all). Second, individuals are identical.

"Local communications" can mean a variety of things. If the swarm is a flock of birds or a school of fish, the communication is observing the directions and magnitudes of motion of the nearest so many neighbours in three dimensions and reacting accordingly. If it is a herd, we normally think of it as moving in two dimensions (with obstacles and varied terrain). But communications can transcend spatial dimensions, as, for humans, with telephones and the Internet, when we must begin to describe it as topological, as in a network.

We use the metaphor "being in touch" for kinships and friendships. If you've ever had the experience of being put in the middle of a flock (herd) of sheep by sheepdogs instructed to surround you, you will know that sheep prefer literally to be in touch. The metaphor is suggestive. The head of the Clarendon Laboratory when I was a physics student knew what to do with the kind of people who would write at length on what was wrong with physicists and on what they *should* be thinking: he put them in touch with each other. The Internet now does this routinely and without intervention.

So the first component of herdthink is, if you have a (big) decision to make, talk to people you know. When I had a flat tire on a weekend in a place where I don't know the garages very well but do know the community, as well as using a search engine, I telephoned a friend whom I would expect to have had some garage experience.

But when your friends are not in a position to know about the issue, their advice is less helpful. For instance, I write in the midst of a plague. Not the Black Plague that sent Newton home from university to build mechanical devices and to connect the gravity that makes apples fall with the gravity that keeps the Moon in its orbit, but a pandemic for which we have vaccines.

If I were hesitant to be vaccinated, because of false information or because I was just concerned by the speed with which the vaccine had been developed, pooling my anxieties with those of my friends would not be so constructive. We would need to break out of the herd.

This can be hard to do. We are social animals. Ideologies can be remarkably resistant to reason. Indeed we could define ideology to be socialized opinion—reinforced by talking to friends—and opinion to be a set of ideas reinforced by evidence to the contrary.

That's because the evidence to the contrary can be seen as an attack on the ideology. So one cannot "correct" an ideology from the outside but must come from the inside. That is, the converter must become a friend of the herd.

The second assumption we made about swarms in Note 31 of Part III was that there are no individuals. All birds or fish are interchangeable.

But that is not true for humans or most animals or even birds or fish. An active topic of zoological research is "dominance". In the first instance this is about who gets most of the resources—say, food at the kill or mates in breeding season. The domination-submission order is also called the "pecking order" because it was first investigated in domestic chickens. Note that it distinguishes two individuals, the dominant and the submissive, and so is a binary relationship which may extend into a hierarchy. It is easy to see this aspect of dominance in human society. Just think of income.

Understandably, then, a dominant individual is a focus of attention from the submissive. That can become leadership.

Leadership can be significant for humans, too. During this pandemic we have depended on our leaders. Societies with corrupt or disorganized government have fared badly; those with honest, open, trusted leaders have done relatively well.

But we notice in this example that the leaders who have done well have also taken scientific advice. They didn't just watch their social media pages but appointed advisors who are in a position to know. These are people who are outside the herd in a significant way.

They are people who have been educated (*e ducere* = to lead out).

What it means to be led out of herdthink is to have been liberated into being able to say "I was wrong" (or, more scarily, "I am wrong"). In particular, that is how science works. We've seen that in the scientists' responses to the pandemic: very little has been sure right away; only as evidence trickles in does the fluidity of the situation stabilize.

The uncertainty of experts can be very hard to take for people for whom to learn is to lose face. It is more acceptable to those educated in science, who know that scientific statements deliberately court error because error found is error contained, and error can thereby eventually be eliminated. (That is why a scientific theory is never "just a theory": theories are systems of ideas that contain their own potential disproof. If it doesn't tell you how to try to disprove it, it is not a theory but a much lesser thing.) The touchstone built into science for detecting error is disagreement with observation or with repeatable experiment.[1][2] Truth is hard-earned, after many mistakes.

Attention, influence and leadership are power. Power is the opportunity to mess up. Education speaks truth to power and so is threatening and liable to be resisted.

So the zoologists are studying herd behaviour and dominance and leadership—scientifically—but we must go beyond being just in the herd when it comes to pandemics and climate change - looking after Spaceship Earth.

42. Climate. Herd thinking has been especially prevalent on the topic of global warming—climate change. This is understandable since the process is immensely complicated, and educated thought, by its nature, has not leaped to conclusions but has been converging gradually since first inklings in 1824 and 1896.

Apart from climate change there are many less likely and less predictable threats to Spaceship

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[1] For example, the theory that the seasons are caused by Earth being alternately closer to and further from the Sun in its elliptical orbit, is refuted by the observation that we don't get two winters every year. The theory that the phases of the Moon are caused by Earth's shadow is refuted by any daytime observation of a phased Moon and the Sun together in the sky.

[2] However, if you want to unmask a scientific charlatan, you might do well also to be a trained magician: Houdini was enlisted to check spiritualists.

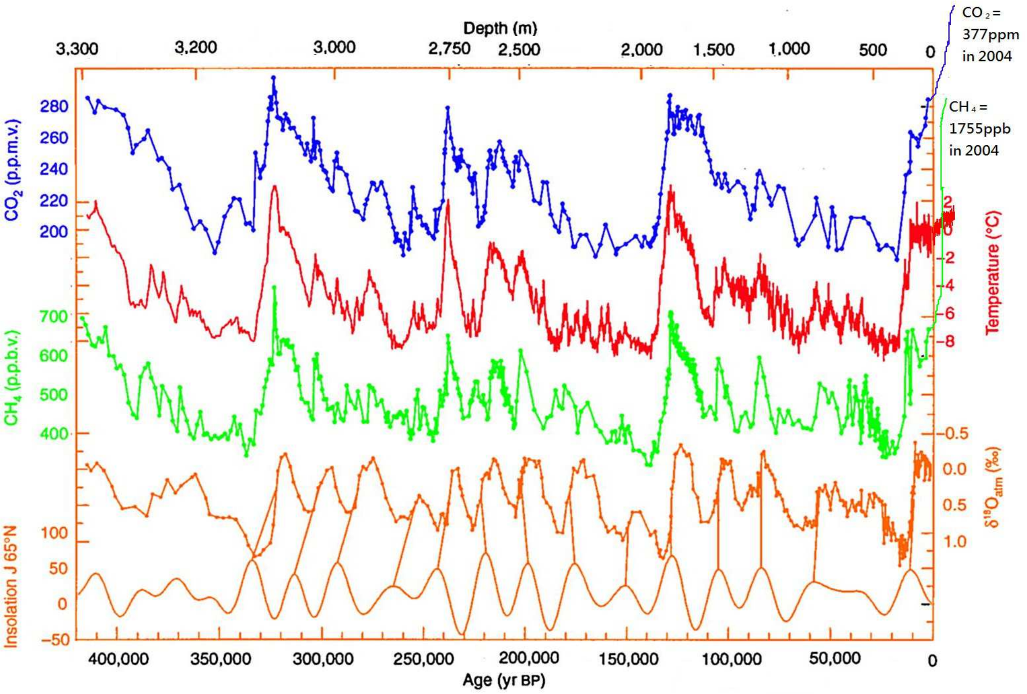
Earth: nearby supernova or gamma-ray burst, supervolcano eruption, or asteroid or comet strike. The eruption of the Laki fissure in Iceland for eight months in 1783–4 has been blamed for triggering the French Revolution by darkening European skies long enough to devastate the harvest. This was by no means a supervolcano such as the tuff and dome eruptions in New Mexico's Valles Caldera from 1.25 million to only about 40,000 years ago. Comet Shoemaker-Levy hit Jupiter on July 16 1994 at 61 Km/s. Its mass has been estimated at 4–40 teragrams (Tg, $10^{12}$g). So the energy released $mv^2/2 = 7.5$–$75\times 10^{18}$ joules. This is an explosion of 2 to 20 thousand megatonnes of TNT. The biggest man-made explosion was the Soviet Tsar Bomba of 58 megatonnes.

But climate change is certain, immediately upon us and man-made.

Jean Baptiste Joseph Fourier calculated in the 1820s that the Earth's average temperature should be $-18^o$C to keep it in equilibrium with radiation from the Sun. So he wondered if the fact that Earth is $15^o$C on average was due to atmospheric trapping of heat, like a greenhouse. Svante Arrhenius in 1896 identified $CO_2$, carbon dioxide, as a heat trapper and pointed out that coal burning since the industrial revolution in the 1750s would increase the gas in the atmosphere and warm the globe. Guy Callendar in 1938 published data showing a half-degree warming since 1890 and that $CO_2$ levels had risen ten percent. Charles David Keeling started detailed monitoring of atmospheric $CO_2$ in 1958, which has continued to this day.

To distinguish science from opinion in this discussion we must constantly ask *how* do we *know*? And we must recognize that knowing is doing—the checking of what we think we know, increasingly by calculation. The healthy attitude is to rejoice in what we (know we) don't know, especially when the means exist to find out.[3]

How do we know what temperatures and $CO_2$ levels have been, before we started monitoring? Ice cores are one way. Glaciers and ice caps go back thousands of years. They trap bubbles of atmospheric gases which can be analyzed. And they are built in layers of annual snowfall which can be counted for dates and whose thicknesses give snowfall and hence temperature. Here is 420,000 years of data from ice cores from the Vostok region of Antarctica.



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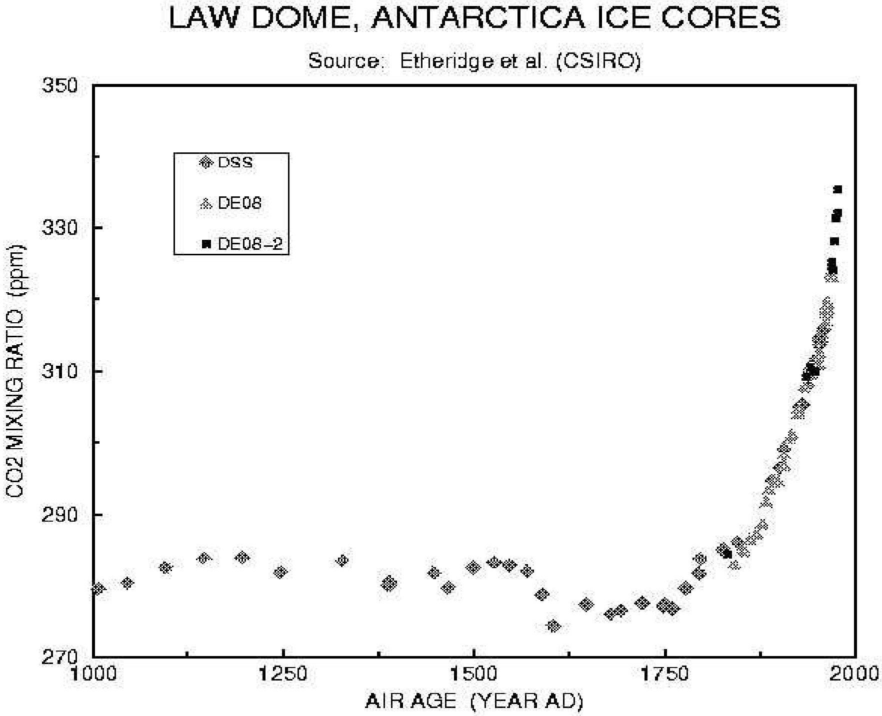
[3] The opposite of intellectual modesty is intellectual arrogance. That is risky, like the mouse which has lost its fear of cats: *Toxoplasma gondii* is a parasite which reproduces in cat intestines; it gets into a mouse brain and causes it to decide to go and kill a cat. Or at least cuddle. If you ever experience that kind of existential self-certainty, beware!

> 420,000 years of ice core data from Vostok, Antarctica research station. Current period is at right. From top to bottom: Levels of carbon dioxide ($CO_2$) in parts per million by volume; Relative temperature; Levels of methane (CH4) in parts per billion by volume; $^{18}O$ isotope of oxygen per mil ($\%_o$); Solar variation at $65^oN$ due to Milankovitch cycles (connected to $^{18}O$).

As well as $CO_2$ and temperature, levels of methane (a greenhouse gas 30 times more potent then carbon dioxide) are reported, and the heavier isotope of oxygen (which we'll come to).

$CO_2$ in parts per million does not seem much. But 1 ppm is somewhere between 7.8 and 12 gigatonnes ($GtCO_2$).[4] We must be careful in quoting gigatonnes. These figures are $GtCO_2$ as opposed to GtC. Carbon dioxide consists of a carbon atom (atomic mass 12) and two oxygen atoms (atomic mass 16 each)—normally: when we are not considering isotopes $^{13}C$, $^{17}O$ or $^{18}O$—so $CO_2$ masses 12+16+16 = 44 while atomic carbon masses 12. Thus 1 $GtCO_2$ is 44/12 GtC.

Running off the chart are plots from contemporary monitoring of $CO_2$, $CH_4$ (methane) and temperature. If the scales were the same as the rest of the plots the first two lines would be vertical. Here is $CO_2$ for the last thousand years, from an ice core at Law Dome, Antarctica.



What happened after 1750 is clear. Human industry is increasing $CO_2$ immensely.

The effect on temperature is harder to tease out. The Vostok ice core is not dramatic about temperature the way it and Law Dome are about $CO_2$. We need sophisticated statistics and a

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[4] Atmospheric pressure at sealevel is 100,000 pascal, which is $g \times 10$ tonnes (with gravitational acceleration $g \approx 10m/s^2$) of atmosphere above each square meter. The circumference of the Earth is $2\pi r = 40$ megameters, so its surface area $4\pi r^2$ is this squared divided by $\pi$, or 500 square terameters. Thus the atmosphere masses 5 petatonnes, a millionth of which is 5 gigatonnes. Right ballpark. Then adjust for volumes by comparing moles of $CO_2$ with moles of $N_2$ and $O_2$.

wider range of data—tree rings and coral reefs, which also have temperature-dependent growth layers—as well as ice cores.

The now famous "hockey stick" curve was published [MBH98] in 1998. The implication of their results is that temperature was also horizontal from 1400 to 1750, then turned sharply upwards. You can imagine a hockey stick on the ground. I don't reproduce their graphs here because they need the context of the paper to be understood. Instead I set the precedent of citing a reference directly in the text of these Notes rather than in an Excursion. It is worth at least skimming some of the basic original papers, if only to appreciate how involved the work of finding these things out really is. (The authors did not use the phrase "hockey stick". If their curve on p.783 is a hockey stick, we're playing hockey using a living flamingo.)

For even more involvement there is, for instance, the 2018 IPCC report of 630 pages [IPC19]. The Intergovernmental Panel on Climate Change reports attempt to cover all the scientific literature on climate change, which is now immense. At the risk of taking this out of context I'll quote only one result, from p.31 (p.45 in the .pdf):

> Human-induced warming reached approximately $1^o$C (likely between $0.8^o$C and $1.2^o$C) above pre-industrial levels in 2017, increasing at $0.2^o$C (likely between $0.1^o$C and $0.3^o$C) per decade (high confidence).

Note the qualified language: these qualifications have precise ranges incorporating scientific agreement and confidence in the results.

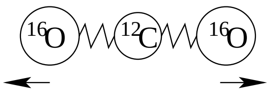
The *basic* science is fairly easy to understand.

A greenhouse (glass house, hothouse) works by allowing energy from the Sun in the form of visible light through the glass into the plants and soil inside. Absorbing this energy warms them up, and they in turn re-radiate the energy, but at infrared wavelengths. The glass does not transmit these wavelengths, so that re-radiated energy stays inside, warming the whole interior, by absorption by the glass among other mechanisms.

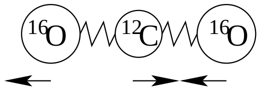
"Greenhouse gases" do the same thing as the glass—although without also providing walls the wind (and snow) can't get through. They absorb infrared energy because their molecules vibrate internally at those frequencies.

Although you will see, if you have looked at the IPCC report, that there are other significant man-made greenhouse gases, such as methane and nitrous oxide ($N_2O$) (from agriculture, for instance) and all the "Montreal protocol" gases (which include exceedingly strong absorbers), carbon dioxide is the dominant one and illustrates the processes.

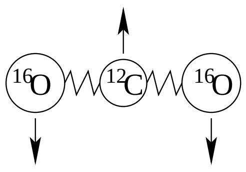
$CO_2$ has four modes of vibration, three of which can be triggered by infrared photons.



Symmetric stretch (not IR)



Asymmetric stretch (IR)
2349 cm$^{-1}$ 4.25µ



Bending (2 modes) (IR)
667 cm$^{-1}$ 15µ

(The bending modes can be in two directions, up-and-down as shown, or back-and-forth, by rotating the whole molecule around its axis.)

The numbers give the frequencies at which the molecule vibrates and so absorbs, or rather the wavelengths, $\lambda$, of those frequencies, $f$, related by $f = c/\lambda$ where $c$ is the speed of the wave, in this case lightspeed. The wavelength is measured in microns, $\mu = 10^{-6}$m; the cm$^{-1}$ gives $1/\lambda$ as so many waves per centimeter.

We can ask how much the temperature of the molecule rises when it absorbs a photon of infrared radiation. The temperature change $T$ is related to the change in energy $E$ by Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ joules per Kelvin degree.

$$E = k_B T$$

The energy of a wave is proportional to its frequency by Planck's constant $h = 6.626 \times 10^{-34}$ joule-seconds.

$$E = hf$$

So the temperature change for the more energetic 4.25 $\mu$ absorption works out to be [5]

$$T = \frac{E}{k_B} = \frac{hf}{k_B} = \frac{hc}{\lambda k_B} = \frac{h \times 71\text{THz}}{k_B} = \frac{47 \times 10^{-21}\text{J}}{k_B} = 3390\text{K}^{\text{o}}$$

And the temperature change for the less energetic 15 $\mu$ absorption is

$$T = \frac{E}{k_B} = \frac{hf}{k_B} = \frac{hc}{\lambda k_B} = \frac{h \times 20\text{THz}}{k_B} = \frac{13 \times 10^{-21}\text{J}}{k_B} = 910\text{K}^{\text{o}}$$

But that's only one $CO_2$ molecule mixed in with lots of nitrogen $N_2$, oxygen $O_2$ and other, trace, gases. Crudely speaking, if the $CO_2$ is at 280 parts per million (the pre-industrial concentration of $CO_2$) and shares this heat energy equally, that temperature change amounts to just under 1 degree.

If we suppose that the pre-industrial atmosphere was in equilibrium, radiating back into space all that absorbed energy, then if industrial activity were to add another 280 ppm, we might loosely expect the atmospheric temperature to go up by one degree.

To improve this very faulty argument we must actually consider radiation, the way Fourier did.

Here, from [cnx21], are the "blackbody" spectra at four quite hot temperatures, including $5000^o$K, the approximate temperature of the Sun. The horizontal axis shows a range of wavelengths and the figure dramatically presents the range of visible electromagnetic radiation—light. The $5000^o$K spectrum peaks at yellow light, which is the colour of the Sun. (We can see that the $4.25\mu$ and the $15\mu$ wavelengths at which $CO_2$ absorbs are well into the infrared.)

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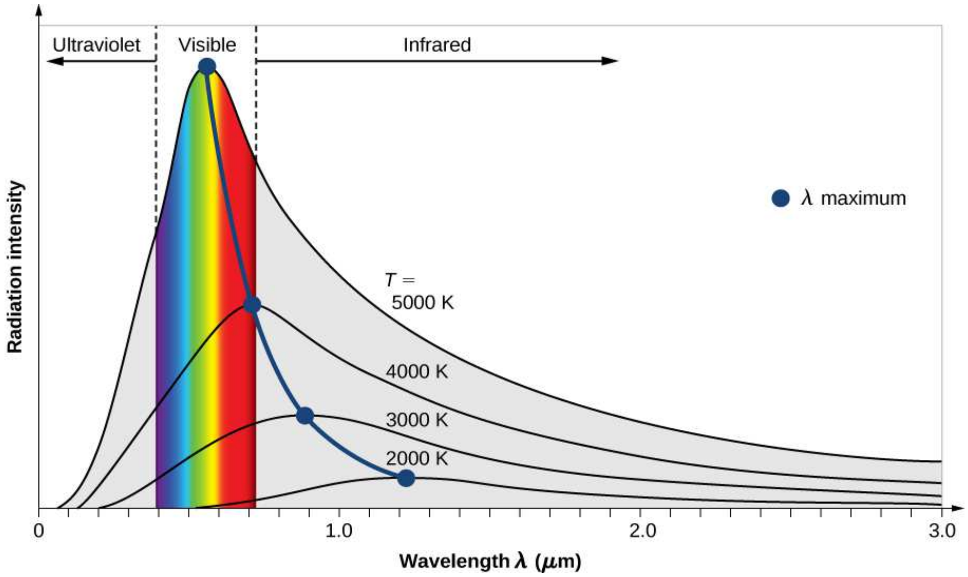
[5] The symmetric stretching mode does not absorb electromagnetic energy—it is not "IR -active" (although it is "Raman-active" but that's another topic)—because the two positively-charged oxygen atoms are moving in opposite direction. The other modes are IR-active because equal charges move in the same direction and opposite to the opposite charge.



The vertical axis is a bit more subtle and scale is not given. It is essentially given in terawatts/meter$^2$, power per unit area, but Planck's distribution also includes "per steradian" to take into account radiation that is not perpendicular to the surface of the "black body".[6]

This figure also shows "Wien's law" governing the relationship between the wavelength $\lambda_{\max}$ at the peak of the distributions, and the temperature. If you eyeball the graph you'll see that the product of wavelength in microns and the temperature in Kelvin is somewhere around 2800 (except for the 2000$^o$K peak which may be misplaced). The actual number is almost 2898, so

$$\lambda_{\max}T = 2.898 \times 10^{-6}\ {}^{o}\mathrm{K}$$

Wien's law enables us to determine the temperatures of stars from their colours.

The *Stefan-Boltzmann* law is more useful for our present need. It relates the total radiation per unit area ("flux", $F$) to the temperature

$$F = \sigma T^4$$

with $\sigma = 5.67 \times 10^{-8}\mathrm{W/m^2/K^4}$. This fourth-power dependence on temperature is remarkable and we should at least persuade ourselves from the plot that it is right.

We get the total radiated power from the area under the spectrum curve. Calculus calls this "integrating" but we can approximate it by the five rectangles shown for each of the 5000$^o$K and the 4000$^o$K distributions. (It was too messy to try to do all four curves.) I "eyeballed" the rectangles in such as way as to gain more or less as much area where they go beyond the curve as they lose where they fall short: as I say, crude.

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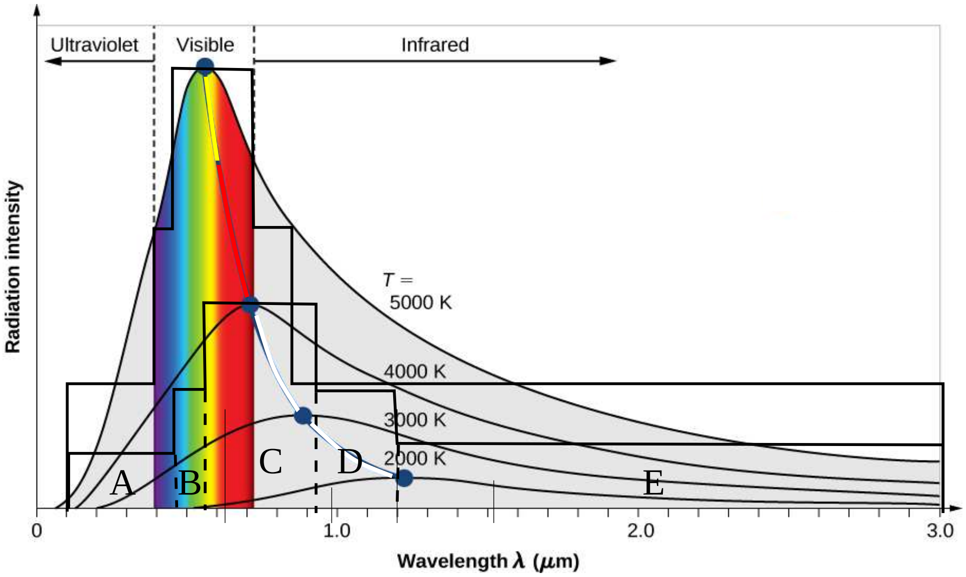
[6] Max Planck calculated the blackbody distribution in 1900 by supposing that radiation is emitted in "quanta"—units of $hf$ where $h$ is the Planck constant we've just encountered—as opposed to continuously, an error which led classical physics to immense discrepancies from observation especially in the ultraviolet parts of the spectra. Thereby began a significant era in physics.

NB. Fourier preceded Planck so didn't have the complete blackbody law. But so did Wien, Stefan and Boltzmann: their laws were not originally derived from theory but empirical.



Since I didn't have the vertical scale, I just measured the areas using sixteenths of an inch in both directions, on my screen display.

| | A | B | C | D | E | total | total$\times t^4$ |
|---|---|---|---|---|---|---|---|
| $4000^o$K | 8×17 | 17×4 | 29×16 | 17×11 | 9×77 | 1548 | 6.0 |
| $5000^o$K | 18×12 | 40×3 | 61×11 | 40×6 | 18×91 | 2896 | 4.6 |

The A,B,C,D,E rectangles are shown in the figure for the $4000^o$K curve; they have the corresponding meaning for the $5000^o$K distribution.

The $t$ is the temperature divided by 1000: 4 and 5 respectively. The numbers 6.0 and 4.6 should be the same. But note that I've left off the infrared tails of the distributions, a serious omission, especially for the higher temperature. So the $T^4$ dependence seems acceptable at the level of this bit of junior physics.

So if we accept the Stefan-Boltzmann law we can calculate flux from temperature or vice-versa.

The *effective temperature* of the Earth is the temperature it would have as a black body radiating all of the solar input. That we saw in Note 6 of Part I is 1366 watts/meter$^2$. The Earth absorbs 70% of this power (it reflects 30%, its *albedo*) across an area $\pi r^2$ where $r$ is Earth's radius. That heats up a surface area of $4\pi r^2$ which radiates almost like a black body. So the net radiated power is

$$\frac{1366 \times 0.7}{4} \text{ W/m}^2$$

and the effective temperature is

$$\left(\frac{1366 \times 0.7}{4 \times 5.67 \times 10^{-8}}\right)^{1/4} = 255\ ^o\text{K}$$

That is a chilly $-18^o$C.

(The average temperature of Earth is actually $288^o$K or $+15^o$C. This is the discrepancy that worried Fourier. The difference—33 C$^o$—is due partly to the greenhouse effect and partly to the radioactivity-induced heat that keeps our iron core molten (and generates our protective magnetic field).)

Knowing the effective temperature, $T$, we can also use the Stefan-Boltzmann law to find the effect of a small bit of "forced radiation" incoming due to extra CO$_2$ in the atmosphere.

First, a small change $f$ in the flux $F$ induces a small change $t$ in the temperature $T$:

$$F + f = \sigma(T+t)^4 = \sigma(T^4 + 4T^3t + 6T^2t^2 + 4Tt^3 + t^4) \approx \sigma(T^4 + 4T^3t) = F + 4\sigma T^3 t$$

The approximation holds since $t$ is much less than $T$.

That is,

$$f \approx 4\sigma T^3 t$$

or

$$t \approx \frac{f}{4\sigma T^3}$$

$T$ is the effective temperature, $255^o$K.

We need to know $f$, the additional effective flux due to extra $CO_2$. The Intergovernmental Panel on Climate Change has captured the measurements of this as $1.4 \times 10^{-5}$ W/m$^2$/ppb [FRA$^+$07, Table 2.14 on pp.212–13] so we can calculate the effect of an additional 280 ppm as

$$f = 1.4 \times 10^{-2} \times 280 = 3.92 \text{ W/m}^2$$

This gives our first estimate of *climate sensitivity*, the temperature increase due to an additional 280 ppm of $CO_2$ on top of the 280 ppm of $CO_2$ that was the norm before industrialization in 1750.

$$t \approx \frac{f}{4\sigma T^3} = \frac{3.92}{4 \times 5.67 \times 10^{-8} \times 255^3} = 1 \text{ K}^{\text{o}}$$

There's that 1 degree again. But we've already quoted another IPCC report which says that the 1-degree increase was reached in 2017, when the U.N. reported $CO_2$ at 405.5 ppm, an increase of only 125 ppm since 1750, not 280.

What we've left out is feedback.

Feedback brings us into territory much harder to quantify and hence predict. There are many feedbacks, some negative (slowing down the warming) but most positive (speeding it up).

The first obvious feedback is the reduction of the Earth's albedo by melting ice. We can see this in the decline of glaciers. Here is the Pedersen glacier in Alaska, viewed from the same point in 1917 and in 2005.

And here is the Careser glacier in Italy in 1933 and 2012.

Careser Glacier, Italy

We can see that the ground exposed after the ice has gone is darker and so will absorb heat that much more readily.

The significant ice-albedo feedback reinforcing climate warming is the loss of sea-ice in the Arctic, especially in summer when the Sun can shine nonstop. The water exposed by melted ice has a lower albedo (0.06 compared with 0.5 to 0.7) and so warms even faster.

Ice melting has a further positive reinforcement. The Greenland ice sheet, especially, is melting internally, with the water running down sinkholes in the ice to flow out to sea underneath. This lubricates the land the ice is sitting on and allows it to slide more readily towards the ocean, where it breaks into icebergs that float away and melt. Ice shelves that are already floating do not raise the sealevel, but ice floating away from land support does, even though it loses 10% of its volume by melting.

When the ice is underground as permafrost near the poles, it contains plant matter and especially methane. I've mentioned that methane is a greenhouse gas significantly more absorbent than $CO_2$, and releasing it provides a tremendous reinforcement to climate warming.

Water vapour is also a greenhouse gas, and one whose concentration in the atmosphere is highly sensitive to temperature. So it provides positive feedback to warming by other greenhouse gases. Fortunately it freezes out by the top of the troposphere at 11 or 12 km; the stratosphere is effectively freeze-dried.

In another positive feedback loop, warming dries out forests which become more susceptible to wildfire. California's fall 2020 fires released 1/10 $GtCO_2$. This is only a third of a percent of our global $CO_2$ emissions, but it is also only one fire, big though it was. Fire soot, landing on ice, reduces the albedo, also accelerating warming. However, fire aerosols may increase the albedo of the atmosphere somewhat, slowing warming down.[7]

Positive feedback is what happens in a sound system when the microphone picks up little sounds from the loudspeakers, feeding them back into the amplifier where they grow, in principle without

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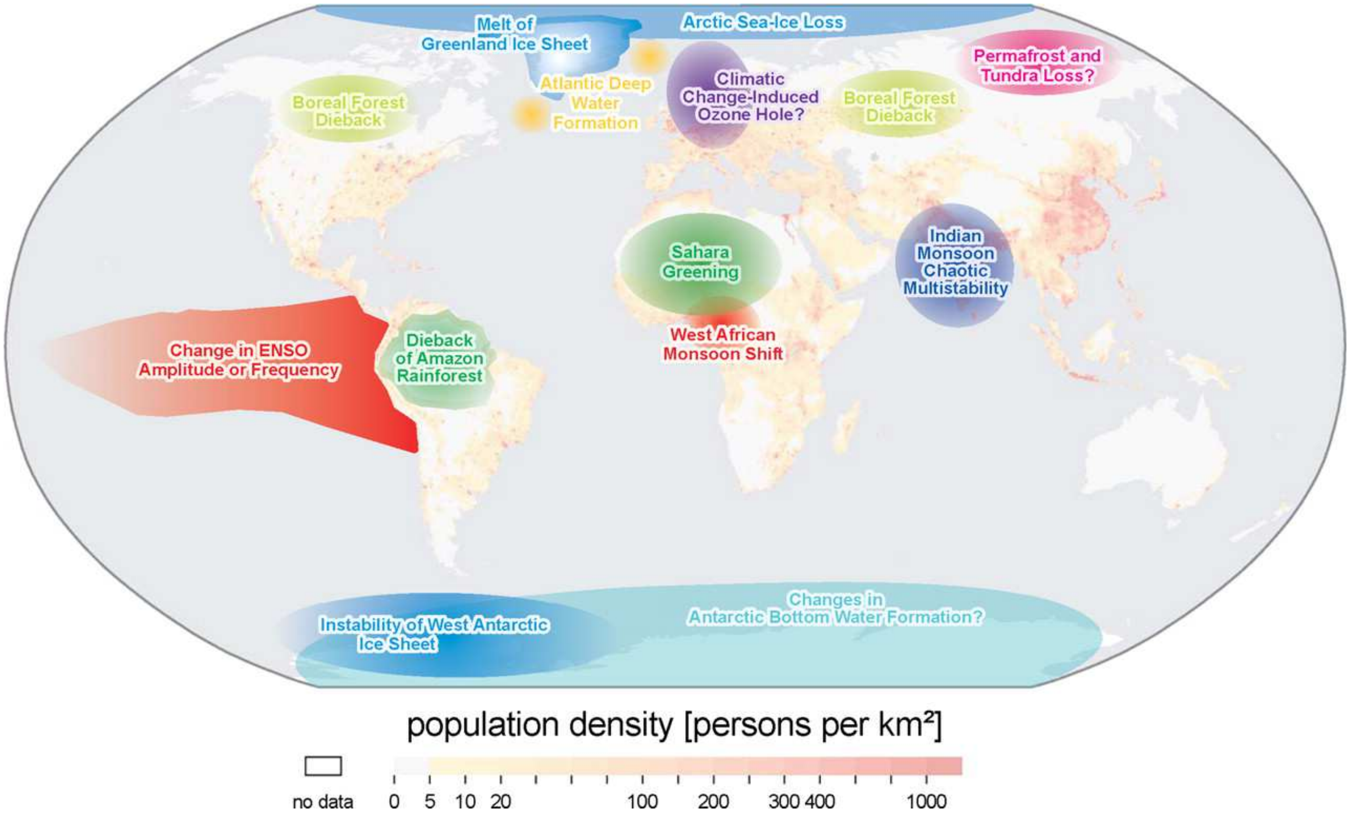
[7] Wildfires give a dramatc way to visualize a gigatonne of carbon dioxide. The U.S. West Coast fires in 2020 burned some million hectares, which is 10,000 square kilometers (100 hectares in a $(\mathrm{Km})^2$). A gigatonne would result from ten times this. So make a line from you to a place 300 Km away. (That's a 3-hour highway drive.) Square that. Pack the square with mature trees. Burn it. You've just generated a $GtCO_2$. Humanity is now doing that 36 times per year and increasing.
(This estimate needs a few times more trees than [NB08]'s figure of 5 to 10 GtC sequestered by reforestation of 500,000 $\mathrm{Km}^2$ in the tropical Americas when European conquest and pandemic decimated indigenous populations—a possible cause of the subsequent Little Ice Age.)

limit. This illustration may help you appreciate the idea of “tipping point” where the screech becomes unbearable and you must turn the system off.

Methane release may be an example of an abrupt, and after a certain point, irreversible, climate change. A more localized tipping point would be in the “thermohaline system” of ocean currents. Warm salty water is less dense than cold salty water, and so rises to the top of the ocean. The Gulf Stream is an example, bringing warm water from the eastern seaboard of North America across the north Atlantic to warm Iceland, Britain and all the way up the Baltic Sea to Scandinavia and Russia. The cold returning current is forced down and so returns south without affecting weather.

But cold fresh water, from Greenland ice melt, is less dense even than warm salty water, and so will interfere with this pattern. It runs the risk even of “turning the Gulf Stream off”. That would significantly affect northern Europe, with global warming paradoxically making a large population a lot colder.

Here are some more potential tipping points, from [LHK$^+$08].



The upshot is that science has been unable to calculate climate sensitivity. The best estimate is that, instead of 1 degree increase after 280 more ppm of $CO_2$ have been added, the increase will range from 1.5 to 4.5C$^o$. This range has not successfully been narrowed in three decades of research.

At our present annual rate of 3–4.6 ppm $CO_2$ we’ll have doubled our pre-industrial emissions to 560 ppm by 2050–2060. And the rate has not started to decrease.

One degree of warming, or even 4.5 degrees, does not immediately sound too dire. It even sounds comforting in northern countries. But, quite apart from this being but the thin end of the wedge, it brings with it unpredictable threats. Such as insect-borne disease vectors (Lyme’s disease, Zika), increasingly energetic hurricanes and cyclones due to ocean warming (wind, flooding) complemented by increasing drought in mid-latitudes as the climatic Hadley cells are driven polewards (wildfires), and the threat to coastal cities of sealevel rise.

Climate models are probably not ready to make full predictions of all of this, so we are left to look at past occurrences.

A seventh of the way through the geologically short period of time between the extinction of the dinosaurs and the present, paleontology reveals the Paleocene-Eocene Thermal Maximum (PETM). That was the last time $CO_2$ concentrations were as high as our present 400+ ppm, and they went to over 1000 ppm. Temperatures increased 5–8$C^o$. There was no ice or snow. The cause is still being debated, but carbon was released into the atmosphere over some 20,000 to 50,000 years, the warm period (no ice) lasted 200,000 years and the recovery back to temperatures humans have hitherto known took another 300,000 years.

Further estimates are that 12Tt (tera tonnes: exagrams) of carbon (equivalent to 44 Tt $CO_2$) were released over that, say, 20Kyr period. That averages to 2.2Gt$CO_2$ equivalent per year, and probably less. For comparison, in 2018 we put out 36Gt, 16 times that rate. During the PETM, species adapted and evolved. (Indeed our primate ancestors appeared then, as did the ancestors of horses.) This may not have time to happen in the anthropocene.

How do we measure temperatures and $CO_2$ concentrations going back 55 million years? By isotopes in seafloor sediments.

These sediments contain the calcium carbonate, $CaCO_3$, from shells and bones of creatures growing in seawater. Both carbon and oxygen have various stable *isotopes*—atoms with the same number of protons in their nucleii, and so having the same chemistry, but with different numbers of neutrons. Thus the usual $^{12}C_6$ and $^{16}O_8$ are accompanied by heavier $^{13}C_6$ and $^{18}O_8$. Heavier molecules of $H_2O$ and $CO_2$ do not evaporate quite as readily as the lighter versions, and so build up a concentration in the sea: glaciers, say, and seawater have different mixes of the oxygen isotopes, which can be detected. So a sudden melting of ice will show up as sedimentary layers with a sudden relative increase in the lighter isotopes. Conversely, we can also measure how much ice there was out of the oceans and on land.

(A similar analysis detects whether the $CO_2$ in ice core bubbles came from burning former plants—fossil fuels—with their slight surfeit of lighter carbon isotopes. It did.)

Before we can do anything about fouling our collective nest this way we should know *where* we are generating the most carbon. My Google search on “sources CO2” produced, first, a simple answer from a European 3-year consortium of unidentified authors:

| $CO_2$ Human Emissions, World 2017 (https://www.che-project.eu/) | |
|---|---|
| Fossil fuels | 87% |
| Land clearing & use | 9% |
| Industry, e.g., cement | 4% |

Further answers were conflicting, owing no doubt to different criteria and regional focus (e.g., U.S.A. and global). There is some agreement that agriculture is responsible for about 10% of human carbon emissions. The other economic sectors (transportation, industry, residential and commercial) divide the remaining 90% fairly equally, with transportation emitting somewhat more than the rest and commercial, and maybe residential, somewhat less.

Some of the political confusion about what to do may be due to lack of clarity in the data that might tell us where to begin. But it seems clear—without need for further studies—that we must start with fossil fuels.

Petroleum molecules, and natural gas and coal to a lesser extent, have wonderful potential, so burning them into $CO_2$ is woefully short-sighted. (Of course, we must also stop dumping the plastics they produce into the oceans.)

Solar power—and its derivative, wind power—can easily supply our needs for energy. The world consumed 0.4ZJ (zeta = $10^{21}$ joules) in 2017, which translates to 12.7TW (tera = $10^{12}$ watts =

joules/sec). A “Kardashev Type I” technical civilization would use the entire solar energy incident on Earth, which is 1KW per square meter (let’s accept the albedo and reduce the 1366W/m$^2$ to 70%) times $\pi r^2 = (20\text{M m})^2/\pi = 127\text{T m}^2$ (remember the definition of a meter) or 127PW (peta = $10^{15}$). That is 10,000 times what we are now using: there is no need to whine about running out of power. Nor about jobs: we have lots of work ahead.

To make this abundant energy portable, we can use it to dissociate water into hydrogen and oxygen, carry the hydrogen in fuel cells to power electric vehicles or burn it directly in modified internal combustion engines. The product is water in both cases—whose vapour is also a greenhouse gas, but a transient one. Hydrogen becomes an “energy currency”, a clean form of energy storage.

We can conclude this Note, and this chapter, optimistically. Although the news of global warming has been met with massive herdthink—understandably, because the evidence is subtle and nuanced—we have two examples of succesful political cooperation over similar issues, despite temporary economic downside and change of direction for various stakeholders.

The first was the 1970 U.S. Clean Air Act. Passed unanimously in the U.S. Senate and with one dissenting vote in the House of Representatives, it has made U.S. air 77% cleaner today (and car tailpipe pollution 99% cleaner)[8] with benefits forty times the costs. The issue of National Geographic that contains a brief summary [Gar21] of the Act and its effects also highlights the downside of dirty air throughout the world, and notes the counterproductive effect of wildfires.

The second successful agreement was the Montreal Protocol to phase out ozone depleting substances (ODS) which were opening a vast hole in the ozone layer, which protects Earth from ultraviolet and other carcinogenic radiation. “Adopted on 15 September 1987, the Protocol is to date the only UN treaty ever that has been ratified by every country on Earth - all 198 UN Member States” (https://www.unep.org/ozonaction/who-we-are/about-montreal-protocol last accessed 2021 May 12). An Encyclopedia Britannica article by John Rafferty says that by 2016 “scientists got sufficient data to confidently reveal proof that the ozone layer was indeed on a path to recovery .. [and] expected the ozone layer to fully heal sometime between 2040 and 2070.” Those gases (chlorofluorocarbons CFCs, hydrofluorocarbons HFCs, hydrochlorofluorocarbons HCFCs and others) are also exceedingly powerful greenhouse gases, although very sparse, so the Montreal Protocol is a direct start on halting climate warming.

## Appendix. Trigonometry and calculus.

43. Trigonometry.
44. Integral calculus.
45. Differential calculus.

## II. The Excursions

You’ve seen lots of ideas. Now *do* something with them!

1. The data for the graph in Note 39 come from [DR93]. Before quoting the results you should check out that paper and any subsequent papers that examine it critically. I am not an astrophysicist; don’t take my word for it.

2. Extinction data in Note 40 come from [RS84] (black ticks on the plot) and [Smi10] (red asterisks). Note that one asterisk coincides with a tick. The other two should also coincide with ticks. That they do not shows the uncertainty of paleontological dating. These two publications are separated by 35 years, so improvements may account for the discrepancies. Other publications show yet other dates. Getting to the bottom of it all would take more time than I have. I am not a paleontologist either; don’t take my word for it.

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[8] I imported a car into North America in 1970. The British manufacturer had had to build in elaborate emission control equipment to meet California standards. A friend at the British Motors special tuning division, who adjusted the system before I put the car on the boat, said that the car ran better because waste was reduced.

3. Elizabeth Kolbert [Kol14] elaborates on the "sixth extinction". Elizabeth Royte [Roy21] spells out what a few extra degrees of heat can do.

4. A reference for stars approaching the Solar System is [Ben21]. It shows 5 stars approaching to some 3 light years within the next 50,000 years, and a number of stars close enough to pass through the Oort cloud in ±5 megayears, especially some 2 Myr ago.

5. I have no background in ethology, the study of animal behaviour introduced in Note 41, so am particularly susceptible to being wrong, especially in pushing results past their realm of applicability, which is animals. (For most of the other topics in this Book I have been able to detect my own errors—but not guarantee correctness—by doing a calculation, mentally, pen-and-paper, or by writing a computer program. That approach is less available here—although zoology increasingly does modelling.) This Excursion invites you to make a career in ethology and correct me. A "mini-revue" which is still being cited links dominance with leadership but adds other considerations such as kinships and social bonds: [KC09]. It may give you a start.

6. An enlightening example, of vaccine hesitation reduced by making new friends with experienced people, was aired on the Canadian Broadcasting Corporation's *The Current* a year after the Covid pandemic started: [Gal16].

7. The history of vaccines in [Phi13] gives a perspective on the tremendous amount of work that has gone into vaccines and their enormous success in containing ravaging diseases such as smallpox (Edward Jenner, 1796; eradicated in 1980), rabies (Louis Pasteur, 1884), cholera (Jaime Ferrán, 1885, Waldemar Haffkine, 1911), diphtheria (William Park, 1914), yellow fever (Max Theiler, 1936), whooping cough (Pearl Kendrick and Grace Elderding, 1939), Japanese encephalitis (Maurice Hilleman, 1944), influenza (Thomas Francis Jr. and Jonas Salk, 1945), polio (Jonas Salk, 1952), measles (John Enders et al., 1963, Maurice Hilleman, 1968), mumps (Maurice Hilleman, 1967), influenza A (Maurice Hilleman, 1968) and rubella (Maurice Hilleman, 1969).
   Before Jenner, smallpox immunization was achieved by "variolation", the risky direct exposure to the smallpox pathogen itself (*variola* = smallpox). Jenner's innovation was to use cowpox, a less virulent (to humans) relative of smallpox. The procedure became known as vaccination (*vacca* = cow). Modern vaccines use a dead or weakened form of the pathogen, or parts of it. Two of the vaccines for SARS-CoV-2, the Covid-19 coronavirus, use messenger ribonucleic acid, mRNA, to induce the patient's cells to produce a protein specific to the virus for the immune system to recognize and target.
   Coronaviruses, as well as influenza, measles and some cold viruses, use RNA rather than DNA for their genomes. They mutate often and the advantage of mRNA-based vaccines is that they work from generic "platforms" which can be repurposed to deal with mutations or indeed any new mRNA. Chinese researchers decoded the SARS-CoV-2 genome, in Jan. 2020, ten days after the first cases were reported, and shared it. So Moderna, for instance, had a vaccine in Phase I clinical trials two months later. This very speed, unprecedented as it is, has led to mistrust of SARS-CoV-2 vaccines, as has government bungling in some jurisdictions. [Yon21]
   The timeline also lists setbacks, which would have given rise to misgivings about vaccination. In 1928 and 1929 vaccination disasters in Queensland, Australia, and Lübeck, Germany, for diptheria and tuberculosis, respectively, were caused by mistakes in the production processes. Similar oversights led to polio vaccination being temporarily suspended by the U.S. Surgeon General in 1955.
   And, of course, in 1997 a subsequently discredited and retracted article in The Lancet by Andrew Wakefield claimed that the measles, mumps and rubella (MMR) vaccine increased autism in children.
   But misinformation about vaccination predates even these excuses. The Anti-Vaccination

League of America held its first meeting in New York in 1882, claiming that smallpox was not contagious but caused by dirt. It would be interesting to know what motivated that antipathy to successful science, and what (contagion) spread those ideas.

8. Attribute the following quotes.
   - "It's hard to get someone to understand an idea whose job depends on not understanding it."
   - "It's hard to get someone to accept a fact who desperately needs to believe something else."
   - "If you think education is expensive then try ignorance." But "learning is impossible without ignorance."
   - "It's not what you don't know that's dangerous, but what you know that ain't so."
   - "If you have power, you don't need intelligence."
   - "It doesn't take much cleverness to camouflage the obvious."

9. If all the ice in the Antarctic and Greenland ice sheets were to melt (as in the PETM of Note 42) how high would the seas rise?

| | Area ($M(Km)^2$) | Max. thick (Km) | Volume ($M(Km)^3$) |
|---|---|---|---|
| Antarctica | 14 | 4.5 | 30 |
| Greenland | 1.7 | 3 | 2.6 |

   (You'll discover that I've applied a "fudge factor" of 1/2 to reduce maximum thickness to average thickness.)
   The world's oceans occupy 71% of the whole surface area, which is $4\pi r^2 = (2\pi r)^2/\pi$ where the circumference $2\pi r = 40$Mm by the Napoleonic definition of a meter (1/10,000,000 of the distance from pole to equator). This works out to be 362M(Km) so the oceans will rise by $(30 + 2.6)\text{M(Km)}^3/362\text{M(Km)}^2 = 90$m.
   Taking into account that water expands 10% on freezing (so that ice floats), this is reduced to 80m. I've found another estimate which says 65m. How many cities are not entirely higher than 65 meters?
   Calculating the annual rise is trickier because it depends on the feedbacks and tipping points of Note 42. So far, the sea seems to have risen 0.2 m—some of which is due, not to ice melt, but to expansion of the water as it warms.

10. A review of the science of the Paleocene-Eocene Thermal Maximum (PETM: Note 42) is in [MW11]. (Foraminifera are single-celled animals: the planktic kind float in the ocean; the benthic kind live on the seafloor.) They give different estimates of onset, duration and recovery times, and of $CO_2$ levels: how do these rates of carbon emission compare with anthropocene levels? (They also use numbers which imply that 1 ppm $CO_2$ in the atmosphere is equivalent to 12Gt$CO_2$. Their $CO_2$ estimate in the PETM of 16 times the 280 ppm preindustrial levels gives 4500 ppm, which at 12Gt$CO_2$ would require $(4500 - 280)/(36/12) = 1400$ years for us to reach at our present rate of $36/12 = 3$ ppm per year. I wouldn't get too relaxed: using 7.8Gt$CO_2$ per ppm gets us to 1000ppm by year 2150 at 36 Gt$CO_2$/year.

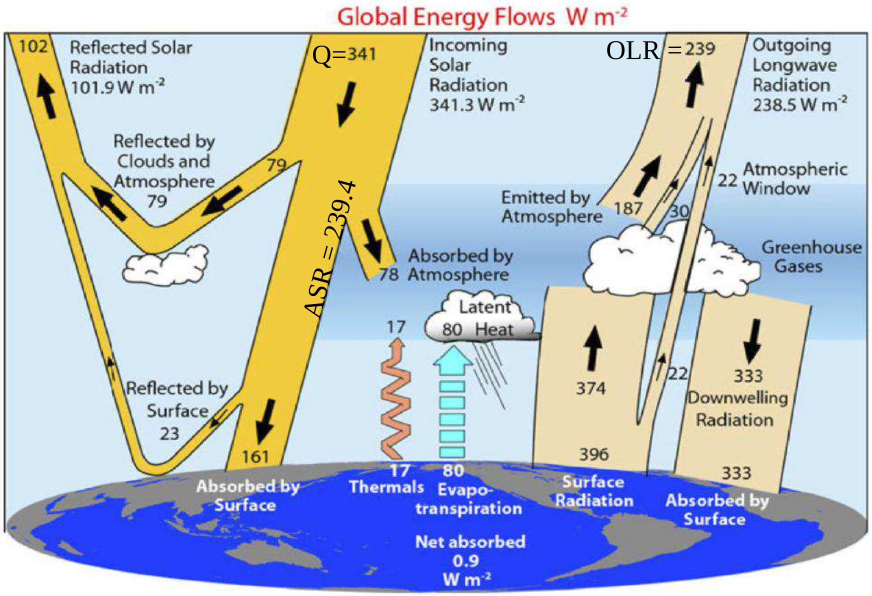
11. **Climate Science.**

    a. Climate models.

    The temperature calculation of Note 42 constitutes a climate model. The simplest climate models just consider energy radiation under the Stefan-Boltzmann law.

    A *model* is a computational formalism, based as far as possible on physics (its strength), but necessarily making simplifying assumptions (its weakness), which enables us to explore the effect of various processes.

Here are some data about our planet's atmosphere, in the form of a picture (not a model). The images and the discussion of this Excursion I owe to the online course [Ros22] by Rose. This picture, which Rose takes from Trenberth and Fasullo, I've added to.



$$\alpha = 1 - \frac{\text{ASR}}{\text{Q}} = 0.299 \ \text{albedo}$$

Trenberth and Fasullo (2012) Surv. Geophys.

$$\tau = \frac{\text{OLR}}{\text{Surface Radiation}} = 0.6 \ \text{transmissivity}$$

for balance: ASR = OLR

We will here explore the relationships among these numbers using various models.

Shortwave (high energy, mainly visible) solar radiation, $Q$, is absorbed (ASR absorbed shortwave radiation) by atmosphere and planetary surface. These in turn radiate back into space at longer wavelengths (mainly infrared), the total energy being OLR (outgoing longwave radiation).

For energy balance, ASR = OLR. That is, $161 + 78 = 187 + 30 + 22$. These are both 239. (More precisely, 239.4 should equal 238.5. The picture shows a slight imbalance, leaving 0.9 watts per square meter to warm the Earth. This would lead to a temperature correction which restores the energy balance.)

Some shortwave radiation is directly reflected from surface and clouds and does not participate in the energy balance. The proportion reflected from the planet is called its *albedo* $\alpha = (79{+}23)/341$ or, more precisely, $\alpha = 101.9/341.3 = 0.299$.

The *transmissivity* of the atmosphere tells us how much of the longwave radiation from the surface actually gets out, $\tau = 238.5/396 = 0.602$.

The *absorptivity* of the atmosphere tells us how much of the longwave radiation from the surface is absorbed by the atmosphere, *before* much of it is re-radiated out. It is called $\epsilon$ for *emissivity* because $\alpha$ is already in use and because *Kirchhoff's law* says absorptivity and emissivity are the same quantity. To contrast with the transmitivity it is interesting to calculate $1 - \epsilon = 22/396 = 0.056$.

### b. Stefan-Boltzmann.

We get slightly different numbers when we pick up the simplest Stefan-Boltzmann model from Note 42.

First, before we start, the "solar constant" (1366 watts/m$^2$ from Note 6), is $4 \times 341.3 = 1365$. (The 4 is the ratio of the radiating surface of the Earth, $4\pi r^2$, to the Earth's cross-section. $\pi r^2$, as seen by the Sun.)

We can re-do the temperature calculation from Note 42 of the Earth's temperature if there were no atmosphere.

$$\left(\frac{341.3 \times (1 - 0.299)}{5.67_{10}\text{–}8}\right)^{1/4} = 254.9$$

so still 255$^o$K.

Now this is a *model*, and since it is we can explore. What happens, for instance, if the albedo changes? For an ice-free world the albedo might be 0.289 (we're almost ice-free, but this would be the condition under runaway global warming) and the temperature becomes 255.8$^o$K, not even a Celcius degree warmer.

If the planet were "snowball Earth"—totally iced over, which has happened more than once in the geological past (715 and 635 million years ago, for instance), and, in very simple models, is a stable, locked-in state in which no further heating is possible—the albedo becomes 0.7 and the temperature drops drastically to 206.1$^o$K.

In fact, the planet is not as cold as any of these. We can find its temperature by what it actually radiates

$$\left(\frac{396}{5.67_{10}\text{–}8}\right)^{1/4} = 289.1$$

which is close to the measured 288$^o$K.

We can use $\alpha$ and $\tau$ as *parameters* in a calculation which uses the Stefan-Boltzmann model to find the temperature at equilibrium (OLR = ASR) for a fixed insolation $Q = 341.3$ W/m$^2$ and, of course, the Stefan-Boltzmann constant $\sigma = 5.67_{10}\text{–}8$. The temperature is

$$\left(\frac{341.3(1-\alpha)}{5.67_{10}\text{–}8\tau}\right)^{1/4}$$

We get, for example

| $\alpha$ | $\tau$ | equil. temp. $^o$K | comments |
|---|---|---|---|
| 0.299 | 0.602 | 289.3 | today |
| 0.7 | 0.602 | 234.0 | snowball ($-39^o$C !) |
| 0.299 | 1 | 254.9 | no atmosphere ($\tau = 1$) |
| 0.7 | 1 | 206.1 | snowball, no atmosphere |

### c. A Dynamic Model.

And we can do more. We can plot the transition to equilibrium after the parameters have suddenly been changed from today's settings.

To do this we need some more physics: how energy retained by the planet affects its temperature. (The Stefan-Boltzmann law tells us how the temperature affects the energy the planet radiates, or, precisely, the power radiated. Power is the *rate* of energy, or energy divided by time.)

This introduces the *heat capacity*, $C$, of the planet, by which the temperature, $T$, depends on the energy as

$$\text{energy}/m^2 = CT$$

We're going to be very simplistic and suppose that, for purposes of heat absorption, the planet is a 100-meter layer of water.

Water has a *specific heat* $c = 4_{10}3$ Joules per kg per Kelvin degree, and, to get a volume out of this mass, a *density* $\rho = 10^3$ kg/m$^3$.

So, for that 100-meter layer of water

$$C = 4_{10}3 \times 10^3 \times 100 = 4_{10}8 \;\; J/m^2/K^o$$

When the planet is out of equilibrium, as it is if it had been in equilibrium but we suddenly changed $\alpha$ or $\tau$, the power—that is, the energy per second—it retains is ASR − OLR.

So the change in temperature in time $\Delta t$ will be

$$\Delta T = \frac{ASR - OLR}{C} \Delta t$$

What we are going to do is assume that the ratio $\Delta T/\Delta t$ holds even for very small timesteps $\Delta t$ (and hence for small changes $\Delta T$ in temperature) so that starting at a temperature $T$ then waiting for a small time $\Delta t$ we get a new temperature

$$T' = T + \frac{ASR - OLR}{C} \Delta t$$

This is our *temparature step* and we can express it in computer code (Rose (Ch.2 §5ff) uses the programming language Python so I follow suit).

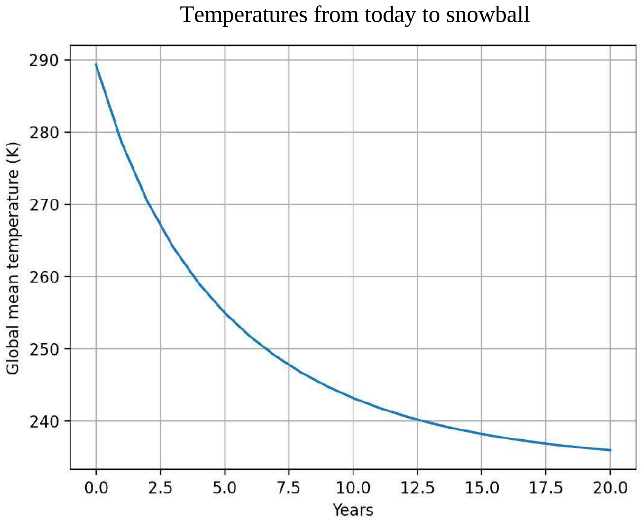
```
def tempStep(T,alpha,tau):
 Q = 341.3                       # insolation, W/m^2, 1365.2/4
 ASR = (1-alpha)*Q               # absorbed shortwave radiation
 sigma = 5.67e-8                 # Stefan-Boltzmann constant, W/m^2/Kdeg^4
 OLR = tau*sigma*T**4            # outgoing longwave radiation
 C = 4.0e8                       # heat capacity of 100 m of water, J/m^2/degK
 delt = 60. * 60. * 24. * 365.   # timestep 1 year = 31536000.0 seconds
 return T + (delt/C)*(ASR - OLR)
```

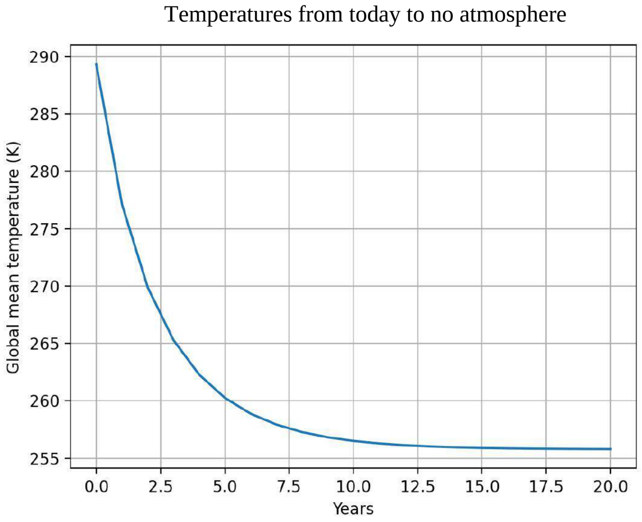
Now we can invoke `tempStep()` repeatedly in a "`for` loop" and plot the results. The programming for this assumes more knowledge of Python than the above, but Rose gives a helpful tutorial. (My Python configuration is slightly different from Rose's and requires, for instance, the `plt.show()` "method" to display the result.)

```
def plotTemp(initialT,numSteps,alpha,tau):
 Tsteps = np.zeros(numSteps+1)
 Tsteps[0] = initialT
 T = initialT
 for step in range(numSteps):
  n = step+1
  T = tempStep(T,alpha,tau)
```

```
  Tsteps[n] = T
 plt.plot(Tsteps)
 plt.xlabel('Years')
 plt.ylabel('Global mean temperature (K)')
 plt.grid(True)
 plt.show()
```

Here are two runs, expanding two of the runs of `equilibTemp()` above.


Temperatures from today to snowball


Temperatures from today to no atmosphere

The runs are, respectively, `plotTemp(289.3,20,0.7,0.602)` (converging at equilibrium to the snowball temperature of $234.0^o$K) and `plotTemp(289.3,20,0.289,1)` (converging to the no-atmosphere temperature of $254.9^o$K).

Incidentally, the choice of 100 meters of water to give the heat capacity of the planet, and hence its response to heating, determines the rate at which the cooling occurs in these two examples: about 20 years to snowball and about 10 to no atmosphere.

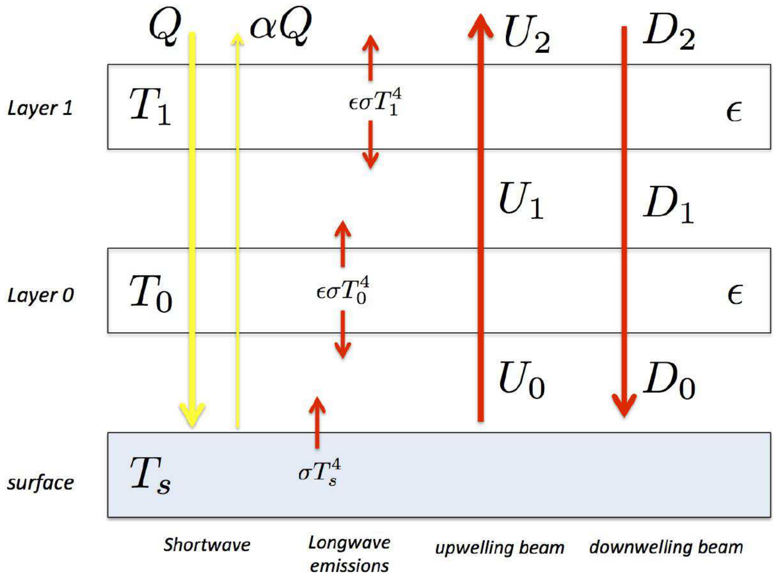
### d. Modelling atmosphere.

As we saw at the beginning, the atmosphere does not just transmit longwave radiation from the surface of the planet to space. It absorbs that radiation and re-emits it at its own temperature.

Here is a figure from Rose (Ch.7 §2) showing a 2-layer atmosphere, and, critically, that Kirchoff's law says that if a layer at temperature $T$ absorbs a fraction $\epsilon$ of the longwave radiation coming at it (from below or above), it in turn emits power

$$\epsilon\sigma T^4$$

both upwards and downwards.



$\epsilon$ is the emissivity/absorptivity we mentioned at the beginning.

Using the diagram we can follow the "upwelling" longwave radiation.

$$\begin{aligned} U_0 &= \sigma T_s^4 \\ U_1 &= (1-\epsilon)U_0 + \epsilon\sigma T_0^4 \\ U_2 &= (1-\epsilon)U_1 + \epsilon\sigma T_1^4 \end{aligned}$$

The last two lines capture the transmission $(1-\epsilon)$ as well as the absorption and re-radiation $(\epsilon\sigma T^4)$ of power at each of the two layers of the atmosphere.

Putting these three contributions together we have the outgoing longwave radiation

$$\begin{aligned} \mathrm{OLR} &= (1-\epsilon)^2\sigma T_s^4 + \epsilon(1-\epsilon)\sigma T_0^4 + \epsilon\sigma T_1^4 \\ &= \sigma(\epsilon^2(T_s^4 - T_0^4) + \epsilon(T_0^4 + T_1^4 - 2T_s^4) + T_s^4) \end{aligned}$$

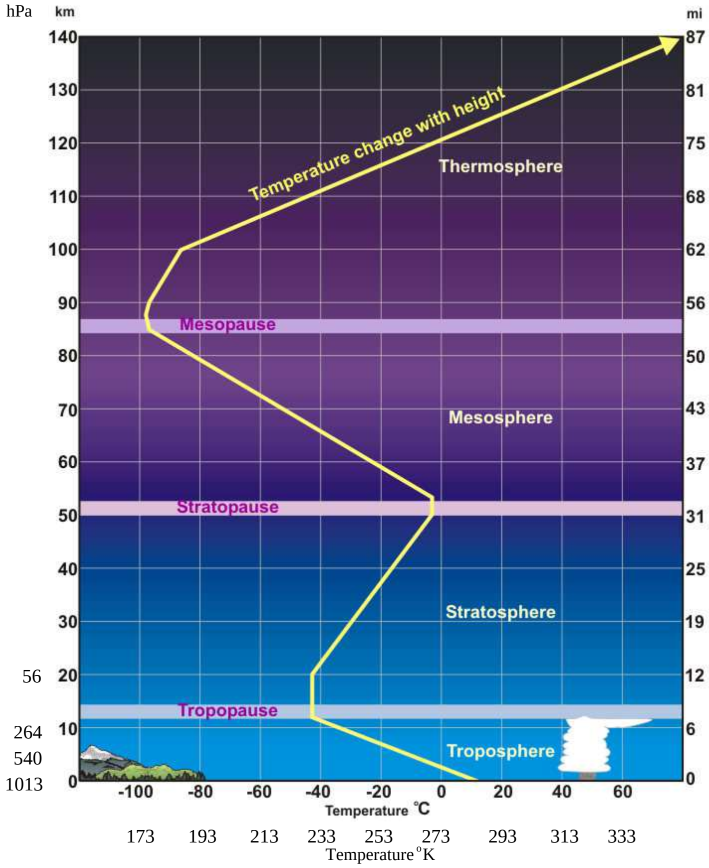
As well as the surface temperature, $T_s$, we are considering the possibility of two different temperatures, $T_0$ and $T_1$, at the two layers of this model atmosphere.

If $\epsilon = 0$ (no absorption), the first form makes it clear that OLR is just the surface radiation and the atmosphere has no effect.

If $\epsilon = 1$ (total absorption), the first form also makes it clear that OLR is just the radiation from the top layer.

If all the temperatures are the same (the *isothermal* model), the second form makes it clear that OLR is just the surface radiation and the atmosphere has no effect.

In fact, the atmosphere has a strong temperature dependence.



[9]

Most of the atmosphere, in terms of numbers of molecules, lies in the *troposphere.* I've added some numbers on the left which supplement the altitude (in kilometers) with pressure. At sea level, the atmosphere exerts a pressure of 1 *bar* = 101,300 Pascals. So a useful measure is millibars, and a millibar essentially equals a hectoPascal (100 Pascals), which is the present conventional measure.

At about 5 km, the pressure is down to 500 hPa, so half of the atmosphere is below this altitude. And the other half, of course, is everything above.

Since the troposphere is most of the atmosphere, it is understandable that the temperature drops with altitude (actually to rather lower than the "no-atmosphere" temperature modelled above).

In the stratosphere, just above, we have the ozone layer. Ozone strongly absorbs *shortwave* radiation from the Sun—we haven't included that in any of our discussions so far—and so warms the stratosphere, causing the temperature to rise again with altitude.

Incidentally, water vapour, like ozone a strong "greenhouse gas", does not survive the tem-

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[9] Source: `http://forecast.weather.gov/jetstream/atmos/atmprofile.htm`. Last accessed March 2022.

peratures of the upper troposphere, and so the stratosphere contains none—effectively freeze-dried.

It remains for us to estimate the temperatures, $T_0$ and $T_1$, of the two layers of our model atmosphere. From these we can solve the quadratic equation in $\epsilon$.

We can suppose each layer to contain half the atmosphere. So they separate at about 5 km, where the temperature is about $260^o$K. The mean temperature from this and the surface at $T_s = 288^o$K is, say, $275^o$K. So we can set $T_0 = 275^o$K.

The upper atmosphere, layer 1, is harder to estimate. I did a crude average (involving the "area" to the left of the temperature plot from 5 km to the top of the figure) without considering the extreme scarcity of atmospheric molecules in the upper reaches. This gave me $T_1 = 250^o$K.

Given the three temperatures we can solve

$$0 = a\epsilon^2 + b\epsilon + c$$

where

$$\begin{aligned} a &= \sigma(T_s^4 - T_0^4) \\ b &= \sigma(T_0^4 + T_1^4 - 2T_s^4) \\ c &= \sigma T_s^4 - \text{OLR} \end{aligned}$$

and get

$$\epsilon = 0.849$$

The other solution for $\epsilon$ is 2.71, but $\epsilon$ must be between 0 and 1. However, since the equation for $\epsilon$ is a parabola and $a$ is clearly positive, we know that the slope of this parabola is negative at $\epsilon = 0.849$.

That tells us that OLR will go *down* as $\epsilon$ goes up in that region.

So this two-layer model is telling us

(a) different atmospheric temperatures permit a "greenhouse" effect; and

(b) increasing the absorptivity of the atmosphere (more greenhouse gases) decreases the outgoing radiation and so increases the warmth of the atmosphere.

It is also instructive to calculate the contributions of each layer at this value of $\epsilon$.

$$\begin{aligned} \text{OLR}_1 &= \epsilon\sigma T_1^4 = 188.1 \\ \text{OLR}_0 &= \epsilon(1-\epsilon)\sigma T_0^4 = 41.6 \\ \text{OLR}_s &= (1-\epsilon)^2\sigma T_s^4 = 8.9 \end{aligned}$$

Almost all of the OLR comes from the top layer of the atmosphere.

Note that the surface contribution of 8.9 W/m$^2$ is tiny, as is the 22 W/m$^2$ given in the initial chart of global energy flows.

We can find that negative slope of OLR with respect to $\epsilon$

$$\text{slope}_\epsilon OLR = \text{slope}_\epsilon(a\epsilon^2 + b\epsilon + \sigma T_s^4) = 2a\epsilon + b$$

which is $-122.66$ for $\epsilon = 0.849$.

For a small change in $\epsilon$ this allows us to calculate the *radiative forcing* (the rate of warming the atmosphere), which is the negative of the change in OLR. For example, if we increase $\epsilon$ by 1%

$$\text{radiative forcing} = -0.01\epsilon \ \text{slope}_\epsilon OLR = 1.04W/m^2$$

For large changes, of course, we must calculate the new OLR and find the difference. So if $\epsilon$ increases to 0.977, to pick a number (a 15% increase), we actually get 14.6 $\mathrm{W/m^2}$ not 15.6.

However, changes even this size are not really valid because $\epsilon$ depends on the three temperatures, and these must change to give a different $\epsilon$.

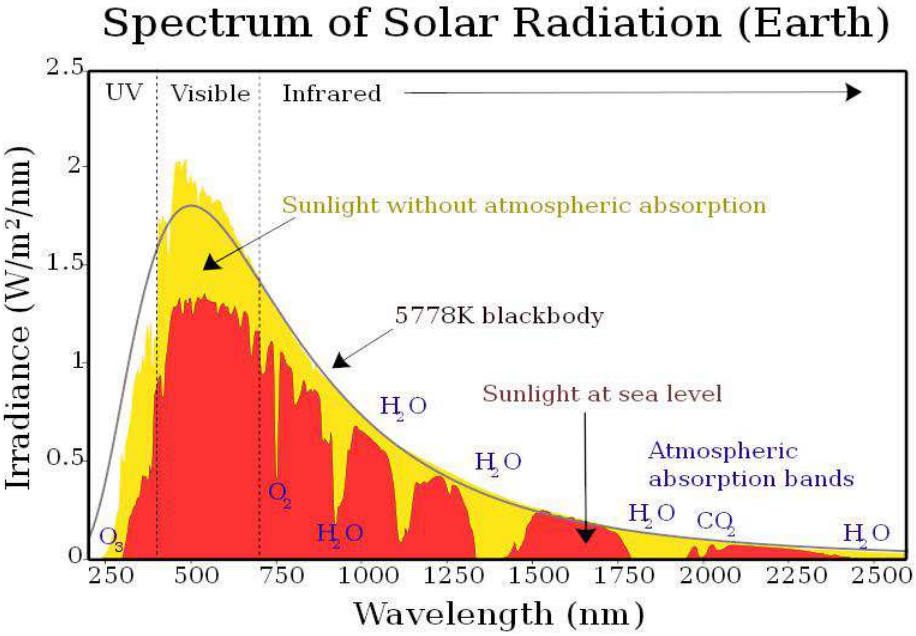
And if we wish to reverse the direction of the calculations and predict the changes of temperature with altitude, we will need multiple layers and much calculation. Even so, calculations based only on radiation will not succeed, because we have, at least in the troposphere where most of the atmosphere is, a temperature *inversion.* Warm air rises, so temperatures falling as we go up make an unstable state. It is corrected by *convection*, which we must take into account. This is a thermodynamic transport problem and I will not pursue it.

## e. Absorption spectra.

Even changes to $\epsilon$ are not straightforward because the greenhouse gases are not “grey”.

Here is the black line giving the familiar blackbody spectrum due to the Sun as seen from the top of the atmosphere. But what appears at sea level is the spectrum filled in in red: various gases (oxygen, water vapour and carbon dioxide) absorb certain wavelengths, so light at these energies does not reach the surface. Note that ozone, $O_3$, takes out all the shortest wavelengths, and this warms the stratosphere.

(The yellow distribution is the blackbody distribution after “Raleigh scattering”, which we can see shifts the peak towards higher energies (shorter wavelengths) and is what makes the sky appear blue.)



[10]

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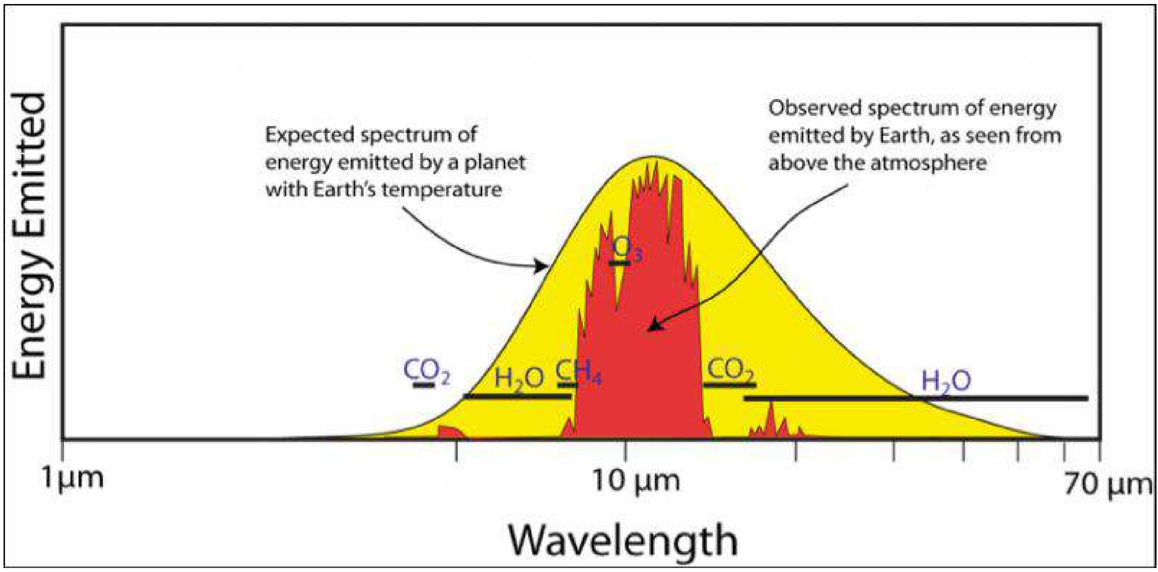
[10] Source: `http://commons.wikipedia.org/wiki/File:Solar spectrum en.svg - Wikimedia Commons.html.`

What is re-radiated to space from the surface is at a lower temperature and so in the infrared. That's the blackbody curve below, filled in in yellow. But again this radiation is absorbed at certain frequencies by water vapour, carbon dioxide and also by methane, so what gets to the top of the atmosphere is only what is coloured in red.



[11]

We can't take on the frequency-dependence of the absorption of the planet's re-radiation without elaborate computations. But we can get an upper bound if we ignore it and return to grey gases.

Something called Schwartzschild's equations say that $\epsilon$, the absorptivity/emissivity of the gases in the atmosphere, depends on the *absorption cross-section* $\kappa$ as follows.

$$\epsilon = 1 - c^{\kappa}$$

where $c$ is a constant. $\kappa$ and $\epsilon$ are also constants as long as we're ignoring their frequency dependence.

$\kappa$ is the quantity that varies directly with the amount of energy-absorbing gas. It doubles when the gas concentration doubles.

So doubling $\kappa$ gives

$$\begin{aligned}\epsilon_2 &= 1 - c^{2\kappa} \\ &= 1 - (c^{\kappa})^2 \\ &= 1 - (1 - \epsilon)^2 \\ &= 2\epsilon - \epsilon^2\end{aligned}$$

Thus, doubling the concentration of absorbing gas that gave us $\epsilon = 0.849$ would now give us $\epsilon_2 = 0.977$.

This we saw in the previous section adds 14.6 watts per meter$^2$ to global warming (the "radiative forcing").

Some climate models use a "net climate feedback" of 1.3 W/m$^2$ per K$^o$. That would give us

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Last accessed May 2022.

[11] Source: David Bice, Penn State University `http://www.education.psu.edu/earth103/node/1006`. Accessed via Rose, Ch.10, March 2022.

an “equilibrium climate sensitivity” of

$$14.6 \times 1.3 = 19^o$$

Fortunately, the frequency dependence of energy absorption reduces this climate sensitivity tremendously. It is nowhere near 19 Celcius degrees (which would increase the *average* temperature for the whole Earth to $34^o$C).

f. Feedback.

Most of the climate feedback mechanisms named in Note 42 are hard to quantify although we can say some general things about feedback.

To focus, let’s stick with Arrhenius’ identification of carbon dioxide as the driving culprit of global warming (and with the correlations in the ice core records), and add water vapour as an amplifier.

The warmer it gets, the more water vapour the atmosphere can hold. For instance, at 100% “specific humidity” (which is as much water vapour as air can hold at a given temperature, the “dew point” beyond which the grass gets wet) here are the absolute humidities (in grams per cubic meter) at a selection of temperatures (in degrees Celcius).

| Temp $^o$C | 50 | 40 | 30 | 20 | 10 | 0 | $-10$ | $-20$ |
|---|---|---|---|---|---|---|---|---|
| A.H. g/m$^3$ | 83.0 | 51.1 | 30.4 | 17.3 | 9.4 | 4.7 | 2.3 | 0.9 |

But water vapour is a greenhouse gas and so itself warms the air further.

If we double the $CO_2$ the atmosphere warms by an amount we can call $\Delta R$ watts per square meter (the $R$ stands for “radiative forcing”), which is given by the increase in the difference between the absorbed shortwave radiation, ASR, and the outgoing longwave radiation, OLR

$$\Delta R = (\mathtt{ASR}_{2\times CO_2} - \mathtt{OLR}_{2\times CO_2}) - (\mathtt{ASR} - \mathtt{OLR})$$

It is characterized by Rose (Ch.13 §1) as the “short-term change in the TOA [top-of-the-atmosphere] energy budget .. before the surface has a chance to warm up” as a result of suddenly doubling the carbon dioxide.

But the water vapour increase due to the rise in temperature adds a fraction $f = 0.565$ to this heating, so we have a heating of

$$\Delta R + f\Delta R = (1 + f)\Delta R$$

But the extra $f\Delta R$ adds more water, and we’re at

$$\Delta R + f\Delta R + f^2\Delta R = (1 + f + f^2)\Delta R$$

You can see where this is going:

$$(1 + f + f^2 + f^3 + \cdots)\Delta R$$

Fortunately, if $\mid f \mid< 1$, the infinite series converges and we can simplify to

$$\frac{1}{1-f}\Delta R$$

(Just multiply $(1-f)(1+f+f^2+f^3+\cdots+f^{n-1})$ and consider the result when $n$ is big enough that $f^n$ is negligible.)

The fraction $g = 1/(1-f)$ is called the *gain* of the system, and we see that $f = 0.565$ gives $g = 2.3$.

The feedback more than doubles the heating effect. If the temperature increase from doubling $CO_2$ with no feedback (the no-feedback "equilibrium climate sensitivity", ECS) were $1.3K^o$, then the $ECS_{H_2O}$ is 2.3 times this, or $3K^o$.

### g. Greenhouse.

If we need any more convincing about the magnitude of the greenhouse effect than provided by the 33-degree difference between Earth without and with atmosphere, we can just look at Venus.

Mars and Venus have surface temperatures of $-65^oC$ ($208^oK$) and $464^oC$ ($737^oK$) respectively, and are close enough to $\sqrt{2}$ times further from and closer to the Sun, respectively, than the Earth. They both have carbon dioxide atmospheres (95% and 96% respectively).

Are these temperature differences due entirely to the different distances?

Because of the distances, their respective insolations are half and twice that of Earth. Earth's no-atmosphere temperature is $255^oK$ and Stefan-Boltzmann says that the no-atmosphere temperature varies as the fourth root of the insolation. So, other things assumed equal (the albedo of 1-0.7 is the major other consideration for Earth: we'll simply keep this albedo for Mars and Venus), we have

| Mars | Venus |
|---|---|
| $255/2^{1/4} = 214^oK$ $(-59^oC)$ | $255 \times 2^{1/4} = 303^oK$ $(30^oC)$ |

The Mars result is pretty close to the true number. Why? Because Mars has effectively no atmosphere: 6/1000 Earth's sealevel pressure at the surface. But Venus has a tremendous atmosphere, 95 times our sealevel at the surface. So the density difference is what's relevant, not the distance from Sol.

The greenhouse effect is responsible for tens of degrees of difference on Earth and hundreds on Venus.

### h. Seasonal variation.

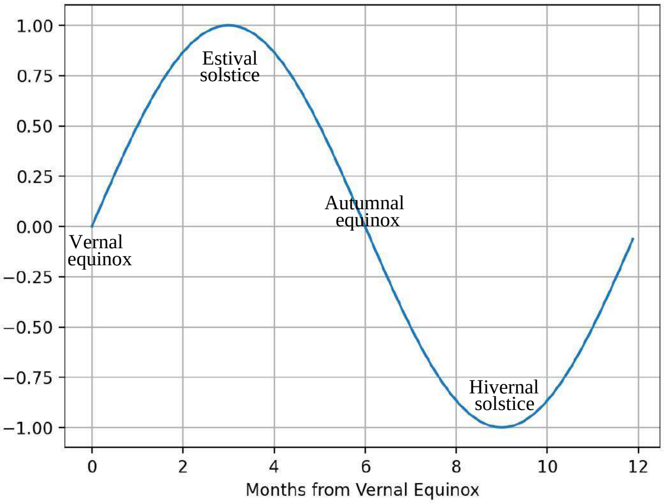
A "toy" model, like any attempt to simplify climate, is not very accurate but serves to illustrate the major factors.

We can suppose the seasonal variation is periodic over the year, starting, say, at zero at the Spring equinox. The incoming solar radiation then varies about the annual average $Q_0 = 341.3\text{W/m}^2$ as

$$Q = Q_0 + Q^* \sin(\omega t)$$

with an amplitude of $Q^*$ (which Rose gives as $180\text{W/m}^2$ for latitude 45N) (Ch. 21 §2) and an angular frequency of $\omega = 2\pi$ per year (or about 1/30 Gsec.). (By the way, this discussion applies without change to 45S.)

Here is the sin() part of this variation over the twelve months, showing the equinoxes and the solstices (and starting at the Spring equinox).



We can relate temperature to this heat input using a heat capacity, $C$, as in our dynamic model of Part c. Rose says (where the $A$ constant will appear only in $Q_0 - A$ and so could have been incorporated into $Q_0$)

$$C\partial_t T = Q - (A + BT)$$

where Rose gives the parameter $B$ as 2.0 $\mathrm{W/m^2/K^o}$.

The $\partial$ indicates a differential equation, but we can suppose a periodic solution

$$T(t) = T_0 + T^* \sin(\omega t - \Phi)$$

The $\Phi$ is important. It quantifies a *time lag* which will be due to the thermal inertia represented by $C$. In Part c that inertia was modelled using 100 meters of water. Here we will consider water depths of 0 to $\infty$ meters, and the inertia of the atmosphere itself.

If you know differential equations you will see that

$$\partial_t T = \omega T^* \cos(\omega T - \Phi)$$

and so the differential equation becomes the equation

$$C\omega T^* c'' = Q^* s + Q_0 - (A + BT_0 + BT^* s'')$$

where I've introduced some shorthand for the sin() and cos():

$$\begin{aligned} s &= sin(\omega t) \\ c &= cos(\omega t) \\ s'' &= sin(\omega t - \Phi) = s\cos(\Phi) - c\sin(\Phi) \\ c'' &= cos(\omega t - \Phi) = c\cos(\Phi) + s\sin(\Phi) \end{aligned}$$

where the last two expansions are trigonometric identities you can easily derive by multiplying the matrix representations of the two rotations.

Now $Q_0$ is just the average annual incident energy and $T_0$ is just the average annual temperature. Such averages must be found by getting rid of all the seasonal wriggles described by the $c''$, $s''$ and $s$. So

$$0 = Q_0 - A - BT_0$$

and that part of the equation cancels out.

On rearranging $B$, $C$, $T^*$ and $\omega$ in the equation to get rid of the physical measures,

$$\frac{C\omega}{B}c'' + s'' = \frac{Q^*}{BT^*}s$$

And, calling $\tilde{C} = C\omega/B$ and using the trigonometric identities,

$$\tilde{C}(c\cos\Phi + s\sin\Phi) + s\cos\Phi - c\sin\Phi = \frac{Q^*}{BT^*}s$$

From this we can get an equation with $\cos(\omega t)$ on one side and $\sin(\omega t)$ on the other.

$$c(\tilde{C}\cos\Phi - \sin\Phi) = s\left(\frac{Q^*}{BT^*} - \tilde{C}\sin\Phi - \cos\Phi\right)$$

So $\cos(\omega t)$ times some number which does not depend on $t$ equals $\sin(\omega t)$ times some similar number.

If you compare sin() and cos() curves (the cos() curve is just the sin() curve shifted, so that month 3 moves left to month 0 in the plot above), you can see that the only way for this equation to hold for all $t$ is for both numbers to be 0.

So we have two equations, and we can use the first in the second to replace the second by the third line below.

$$\begin{aligned}
\tilde{C}\cos\Phi &= \sin\Phi \\
\frac{Q^*}{BT^*} &= \tilde{C}\sin\Phi + \cos\Phi \\
\frac{Q^*}{BT^*} &= \cos\Phi(1 + \tilde{C}^2)
\end{aligned}$$

So $\tilde{C} = \tan\Phi$ and

$$T^* = \frac{Q^*}{B}\frac{1}{(1 + \tilde{C}^2)\cos\Phi}$$

For very large $C$, $\Phi \approx \pi/2$—the temperature is totally out of phase with the insolation, so it's hottest at the Fall equinox and coldest at the Spring equinox—except that the temperature does not change because of the infinite inertia:

$$\begin{aligned}
T^* &= \frac{Q^*}{B}\frac{1}{\left(1 + \left(\frac{\sin\Phi}{\cos\Phi}\right)^2\right)\cos\Phi} \\
&= \frac{Q^*}{B}\frac{\cos\Phi}{1} = 0
\end{aligned}$$

the last because $\cos(\pi/2) = 0$.

For small $C$, $\Phi \approx \tilde{C}$ and

$$\begin{aligned}
T^* &\approx \frac{Q^*}{B}\frac{1}{(1 + \tilde{C}^2)(1 - \tilde{C}^2/2)} \\
&\approx \frac{Q^*}{B}\frac{1}{1 + \tilde{C}^2/2} \\
&\approx \frac{Q^*}{B}\left(1 - \frac{\tilde{C}^2}{2}\right)
\end{aligned}$$

and we can put in some numbers, if we use Rose's $C = 10^7 \mathrm{J/m^2/K^o}$ for the atmosphere.

$$\tilde{C} = \frac{C\omega}{B} = \frac{10^7 2\pi}{2\frac{1}{30}10^9} = \frac{3\pi}{10} = 0.94$$

So $\tilde{C}^2/2 = 0.44$ and, at 45N ($Q^* = 180$),

$$T^* = \frac{Q^*}{B}\left(1 - \frac{\tilde{C}^2}{2}\right) = 90 \times 0.56 = 50$$

So we get a seasonal temperature variation of $\pm 50\mathrm{C}^o$. (If we set $C = 0$ this would be $\pm 90\mathrm{C}^o$.)

Of course, $C$ is not just due to the atmosphere (which Rose argues is the same as the $C$ due to 2.5 meters of water) but to water as well, and $\tilde{C}$ will be bigger than the 0.94, reducing the temperature amplitude, hopefully to something realistic.

Since the small-$\tilde{C}$ approximation is invalid for $\tilde{C} = 0.94$, we should do the full calculation at various $\tilde{C}$.

$$\begin{aligned} \Phi &= \arctan \tilde{C} \\ T^* &= \frac{Q^*}{B}\frac{1}{(1+\tilde{C}^2)\cos\Phi} \end{aligned}$$

| $\tilde{C}$ | 0.01 | 0.1 | 0.5 | 1 | 2 | 5 | 10 |
|---|---|---|---|---|---|---|---|
| $\Phi$ | 0.01 | 0.1 | 0.46 | 0.79 | 1.11 | 1.37 | 1.47 |
| $T^*$ | 90.0 | 89.6 | 80.5 | 63.6 | 40.2 | 17.7 | 8.96 |

I leave it to the climate scientists to find the correct temperature amplitude at 45N (or 45S) and thereby set the parameter $\tilde{C}$.

### i. Ice ages.

The ice ages of the Pleistocene are a significant variation of Earth's climate in which most of North America and much of northern Europe were covered in kilometers of ice—much as Antarctica is today—at least four times in a row. Will the glaciations repeat, even offsetting global warming?

The most widely accepted explanation are the "Milankovitch cycles" in three parameters characterizing Earth's orbit—eccentricity, obliquity and precession—which may be due to gravitational influences from other planets in the Solar system (the intractable "$n$-body problem") notably the four outer giants.

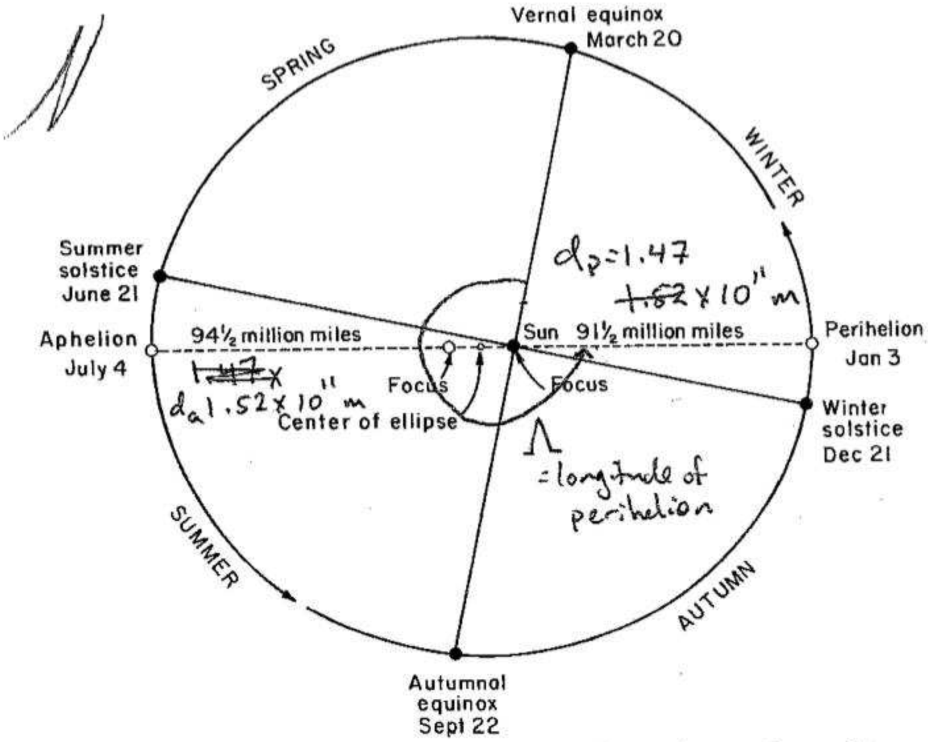


*Figure 14.* Dates of equinox and solstice. At the equinoxes, the earth's axis is pointed at right angles to the sun, and the day and night are of equal length all over the globe. At the summer solstice, the North Pole is tipped in the direction of the sun and the northern hemisphere has the longest day of the year. At the winter solstice, the North Pole is tipped away from the sun, and the northern hemisphere has the shortest day of the year. [12]

(The seasons in this diagram are those of the northern hemisphere.)

The *eccentricity* measures how far from a circle the elliptical orbit of Earth is around the Sun:

$$e = \frac{d_a - d_p}{d_a + d_p}$$

is presently 0.017 but has varied between 0.01 and 0.05 over the past million years. ($d_a$, the distance to the Sun from *aphelion*, our furthest out, is currently 0.152 Tm. $d_p$, the distance to the Sun from *perihelion*, our closest in, is currently 0.147Tm.)

Our *obliquity*, the tilt of our axis of daily rotation from perpendicular to our orbital plane, is currently $\Phi = 23.5^o$, but has varied from 22.0 to 24.5 degrees.

Because of this tilt—it doesn't matter how many degrees as long as not zero—we have seasons, in particular summer and winter *solstices* (longest and shortest days) and spring and autumn *equinoxes*. When the northern hemisphere is tilted towards the Sun the days will be longer and the nights shorter, with the extreme days when we are halfway between the equinoxes, at which night equals day because we are tilted neither towards nor away from Sol.

That is, if the tilt preserves its direction in space, which, like a top, it tends to do. However, also like a top, it *precesses* under external influences—the top under Earth's gravity, the

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[12] Source: Imbrie & Imbrie, 1986 Harvard University Press *Ice Ages: Solving the Mystery* via Rose Ch.18, May 2022

tilt under the influences of the Sun and other planets. This is called the *precession of the equinoxes*, because they can be thought of as precessing around our orbit as a result.

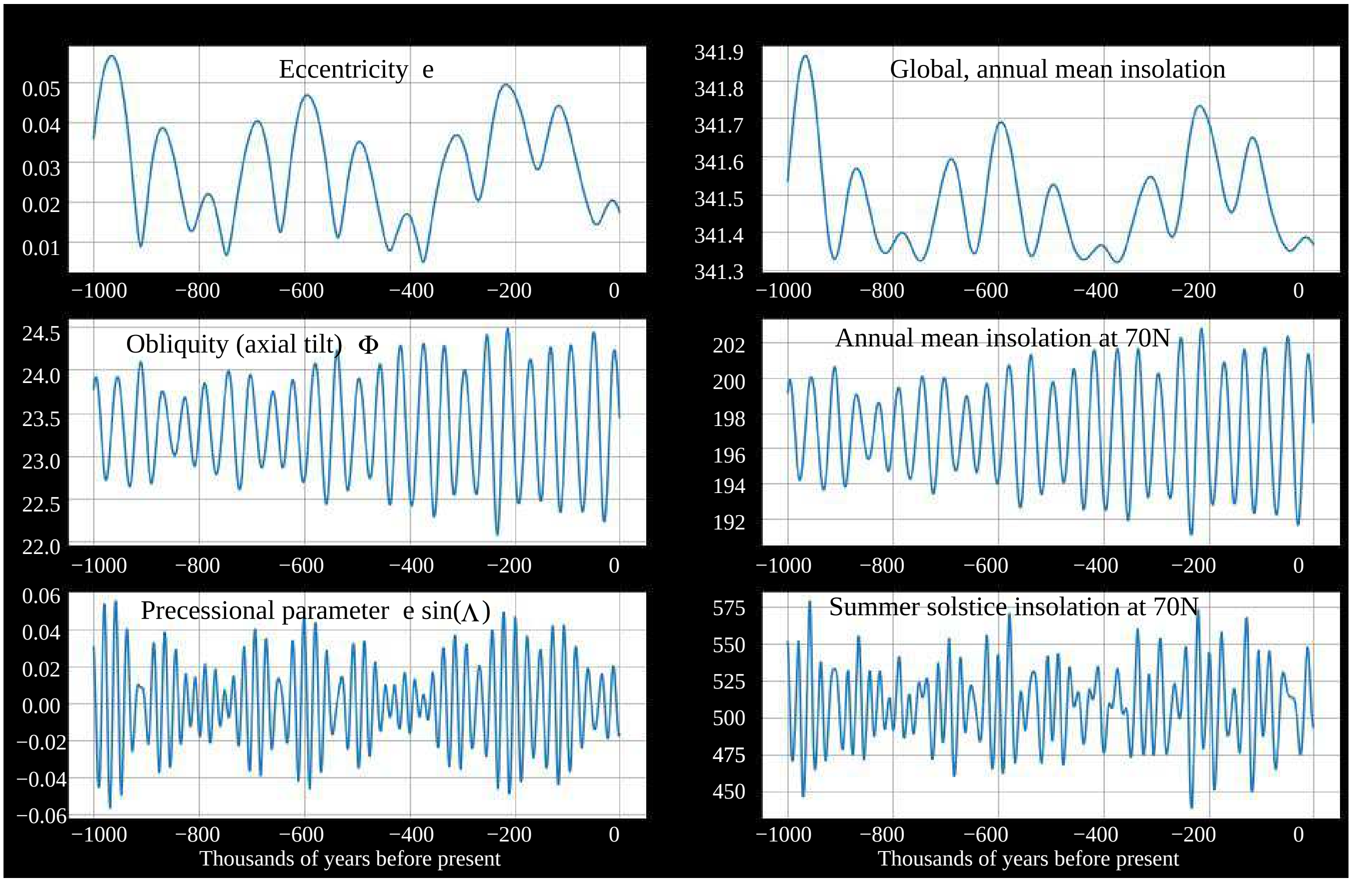
So the angle $\Lambda$, called "the longitude of perihelion", changes with time and measures this precession. The parameter used is the *precessional parameter* $e \sin \Lambda$, currently $0.017 \sin(281^o)$ which is just over $-0.017$.

We can calculate the changes in insolation (solar energy reaching Earth) due to these changes. The total insolation is affected only by the eccentricity of the orbit, and then only in miniscule ways, because the changes in $e$ are so small.

A more sophisticated calculation than we have done finds insolation as a function of latitude, which determines (along with the tilt and the time of year) how far the Sun appears from straight overhead (the "zenith").

These calculations require data on the variations of the three parameters (eccentricity, obliquity and precession) which comes from records such as the geology of distribution of $^{14}$C isotope of carbon. It would be nice to go straight to the $n$-body problem to find the orbital parameters, but this, as we said, is intractable.

So I've lifted the result from Rose's calculation (Ch.18 §7) from the historical data.



Note first that the overall insolation (global annual mean) varied by less than 0.2% and is affected only by the eccentricity.

The other insolation effects—and therefore ice ages—must be due to seasonal and latitudinal *redistributions* of solar energy.

If we look at a specific latitude—70N in this case—the annual mean depends on the tilt (obliquity) but not on the precession. A 10% variation in obliquity yields a 5% variation in annual mean insolation at 70N latitude.

And obliquity and precession both affect insolation at a particular time of year such as the summer solstice. As $e\sin(\Lambda)$ varies from $-0.06$ to $0.06$, this insolation can change over 24%, again at 70N latitude.

While we may not have a handle on the cause of the ice ages we have learned that redistributions of energy, without changing the total, may have a significant effect on climate.

12. Any part of the Preliminary Notes that needs working through.

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